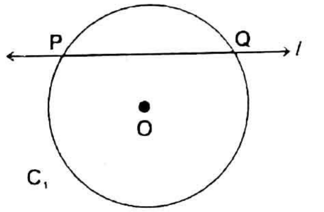
Unit 10

TANGENT TO A CIRCLE

Secant Line:

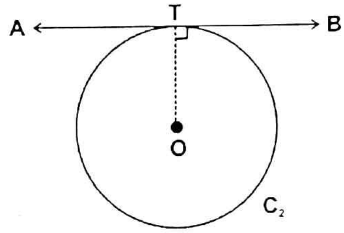
A secant is a straight line which cuts the circumference of a circle in two distinct points.

In the figure ℓ indicates the secant line to the circle C_1.



Tangent Line: A tangent to a circle is the straight line which touches the circumference at a single point only and perpendicular at the outer end of radial segment obtained by joining the centre and point of tangency. The point of tangency is also known as the point of contact.

In the figure line AB indicates the tangent line to the circle C_2.



Length of a tangent segment

The distance between the given point outside the circle and point of tangency is called length of tangent segment.

THEOREM 1

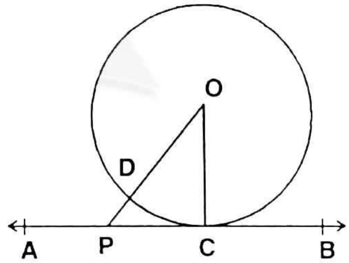
If a line is drawn perpendicular to a radial segment of a circle at its outer end point, it is tangent to the circle at that point.

Given: A circle with centre O and $\overline{OC}$ is the radial segment.

$\overleftrightarrow{AB}$ is perpendicular to $\overline{OC}$ at its outer end C.

To Prove: $\overleftrightarrow{AB}$ is a tangent to the circle at C.

Construction: Take any point P other than C on $\overleftrightarrow{AB}$. Join O with P.

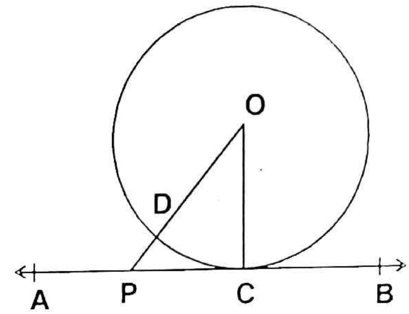


Proof:

Statements	Reasons
In ΔOCP,	
$m\angle OCP = 90^o$	$\overleftrightarrow{AB} \perp \overline{CD}$ (given)
and $m\angle OPC < 90^o$	Acute angle of right angled triangle.
$m\overline{OP} > m\overline{OC}$	Greater angle has greater side opposite to it.
$\therefore$ P is a point outside the circle.	$\overline{OC}$ is the radial segment.
Similarly, every point on $\overleftrightarrow{AB}$ except C lies out side the circle.	
Hence $\overleftrightarrow{AB}$ intersects the circle at one point C only.	
i.e., $\overleftrightarrow{AB}$ is a tangent to the circle at one point only.	

THEOREM 2

The tangent to a circle and the radial segment joining the point of contact and the centre are perpendicular to each other.



Given:

In a circle with centre O and radius $\overline{OC}$.

Also $\overleftrightarrow{AB}$ is the tangent to the circle at point C.

To Prove:

$\overleftrightarrow{AB}$ and radial segment $\overline{OC}$ are perpendicular to each other.

i.e $\overline{OC} \perp \overleftrightarrow{AB}$.

Construction:

Take any point P other than C on the tangent line $\overleftrightarrow{AB}$.

Join O with P so that $\overline{OP}$ meets the circle at D.

Proof:

Statements	Reasons
$\overleftrightarrow{AB}$ is the tangent to the circle at point C.	Given
$\overline{OP}$ cuts the circle at D.	Construction
$\therefore$ $m\overline{OC} = m\overline{OD}$ .... (i)	Radii of the same circle
But $m\overline{OD} < m\overline{OP}$ .... (ii)	Point P is outside the circle.
$\therefore$ $m\overline{OC} < m\overline{OP}$	Using (i) and (ii)
So radius $\overline{OC}$ is shortest of all lines segments that can be drawn from O to the tangent line $\overleftrightarrow{AB}$.	
Also $\overline{OC} \perp \overleftrightarrow{AB}$	
Hence, radial segment $\overline{OC}$ is perpendicular to the tangent $\overleftrightarrow{AB}$.	

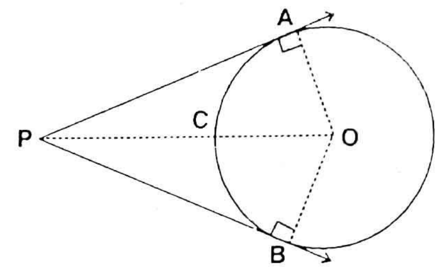
Corollary:

There can only be one perpendicular draw to the radial segment $\overline{OC}$ at the point C of the circle. It follows that one and only one tangent can be drawn to the circle at the given point C on its circumference.

THEOREM 3

Two tangents drawn to a circle from a point outside it, are equal in length.

Given:



Two tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are drawn from an external point P to the circle with centre O.

To prove:

$m\overline{PA} = m\overline{PB}$

Construction:

Join O with A, B and P, So that we form $\angle rt\Delta^s$ OAP and OBP.

Proof:

Statements	Reasons
In $\angle rt\Delta^s$ OAP $\leftrightarrow$ OBP	
$m\angle OAP = m\angle OBP = 90^\circ$	Radii $\perp$ to the tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$
$hyp.\overline{OP} = hyp.\overline{OP}$	Common
$m\overline{OA} = m\overline{OB}$	Radii of the same circle.
$\therefore \Delta OAP \cong \Delta OBP$	In $\angle rt\ \Delta^s$ H.S $\cong$ H.S
Hence, $m\overline{PA} = m\overline{PB}$	Corresponding sides of congruent triangles.

Note:

The length of a tangent to a circle is measured from the given point to the point of contact.

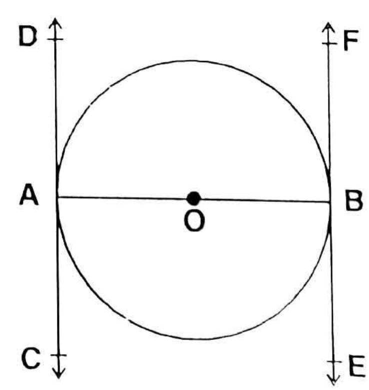
Corollary:

If O is the centre of a circle and two tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are drawn from an external point P then $\overline{OP}$ is the right bisector of the chord of contact $\overline{AB}$.

Example 1:

$\overline{AB}$ is a diameter of a given circle with centre O. Tangents are drawn at the end points A and B. Show that the two tangents are parallel.

Given:



$\overline{AB}$ is a diameter of a given circle with centre O. $\overleftrightarrow{CD}$ is the tangent to the circle at point A and $\overleftrightarrow{EF}$ is an other tangent at point B.

To Prove:

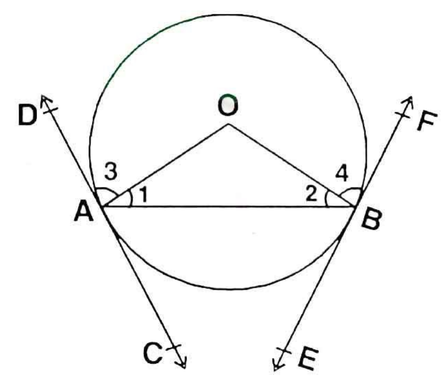
$\overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

Proof:

Statements	Reasons
$\overline{AB}$ is the diameter of a circle with centre O.	Given
$\therefore \overline{OA}$ and $\overline{OB}$ are radii of the same circle.	
Moreover $\overleftrightarrow{CD}$ is a tangent to the circle at A.	Given
$\therefore \quad \overline{OA} \perp \overleftrightarrow{CD}$	By Theorem 1
$\overline{AB} \perp \overleftrightarrow{CD}$ ....(i)	Given
Similarly $\overleftrightarrow{EF}$ is tangent at point B.	By Theorem 1
So $\overline{OB} \perp \overleftrightarrow{EF}$	
$\Rightarrow \quad \overline{AB} \perp \overleftrightarrow{EF}$ .... (ii)	From (i) and (ii) ($\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ are perpendicular to same $\overline{AB}$)
Hence $\overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$	

Example 2:

In a circle, the tangents drawn at the ends of a chord, make equal angles with that chord.



Given: $\overline{AB}$ is the chord of a circle with centre O.

$\overleftrightarrow{CAD}$ is the tangent at point A and $\overleftrightarrow{EBF}$ is an other tangent at point B.

To Prove: $m\angle BAD = m\angle ABF$

Construction: Join O with A and B so that we form a ΔOAB then write $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$ as shown in the figure.

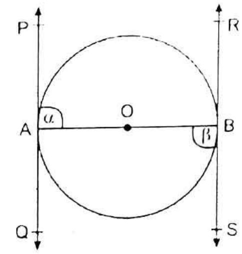
Proof:

Statements	Reasons
In ΔOAB	Construction
$\because \quad m\overline{OA} = m\overline{OB}$	Radii of the same circle.
$\therefore m\angle 1 = m\angle 2$........(i)	Angles opposite to equal sides of ΔOAB
Also $\because \overline{OA} \perp \overleftrightarrow{CD}$	Redial segment $\perp$ to the tangent line
$\therefore m\angle 3 = m\angle OAD = 90^{\circ}$ ....(ii)	
Similarly $\overline{OB} \perp \overleftrightarrow{EF}$	Redial segment $\perp$ to the tangent
$\therefore m\angle 4 = m\angle OBF = 90^{\circ}$.............(iii)	
Hence $m\angle 3 = m\angle 4$ .... (iv)	Using (ii) and (iii)
$\Rightarrow \quad m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$	Adding (i) and (iv)
i.e., $\quad m\angle BAD = m\angle ABF$	

Exercise 10.1

Q.1 Prove that the tangents drawn at the ends of a diameter in a given circle must be parallel.

Given: A circle with centre O, has $\overline{AB}$ as diameter. $\overleftrightarrow{PAQ}$ is tangent at point A. $\overleftrightarrow{RBS}$ is tangent at point B



To Prove: $\overleftrightarrow{PAQ} \parallel \overleftrightarrow{RBS}$

Proof:

Statements	Reasons
$\overleftrightarrow{PAQ} \perp \overline{OA}$	Tangent is $\perp$ at the outer end of radial segment.
$\Rightarrow$ $\overleftrightarrow{PAQ} \perp \overline{AB}$	
$\therefore$ $m\angle\alpha = 90^\circ$ ……… (i)	
$\overleftrightarrow{RBS} \perp \overline{OB}$	
$\Rightarrow$ $\overleftrightarrow{RBS} \perp \overline{AB}$	
$\therefore$ $m\angle\beta = 90^\circ$ ……… (ii)	
Thus, $m\angle\alpha = m\angle\beta$	
Therefore $\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$	If alternate Angles are equal in measurement, then lines are parallel.

Q.2 The diameters of two concentric circles are 10cm and 5cm respectively. Look for the length of any chord of the outer circle which touches the inner one.

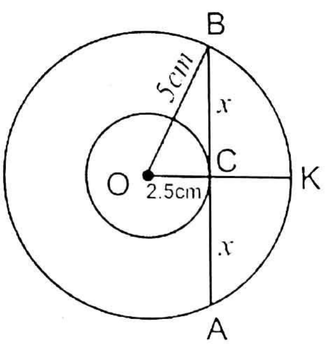
Solution: Let $\overline{AB}$ be any chord of the outer circle that touches the inner circle.

Diameter of outer circle = 10 cm

Radius of outer circle = $m\overline{OB} = \dfrac{10cm}{2} = 5cm$

Diameter of inner circle = 5 cm

Radius of inner circle = $m\overline{OC} = \dfrac{5cm}{2} = 2.5cm$



ΔOCB is right angled triangle with right angle at C. $(\because \overline{OC} \perp \overline{AB})$

By Pythagoras theorem

$$(m\overline{OB})^2 = (m\overline{BC})^2 + (m\overline{OC})^2$$

$$(5\text{ cm})^2 = (x)^2 + (2.5\text{cm})^2$$

$\Rightarrow$ $x^2 = (5\text{cm})^2 - (2.5\text{cm})^2$

$x^2 = 25\text{cm}^2 - 6.25\text{ cm}^2$

$x^2 = 18.75\text{cm}^2$

Taking Square root of both sides.

$$\sqrt{x^2} = \sqrt{18.75\text{cm}^2}$$

$$x = \sqrt{18.75}\text{cm}$$

Length of Chord = $m\overline{AB} = 2x$

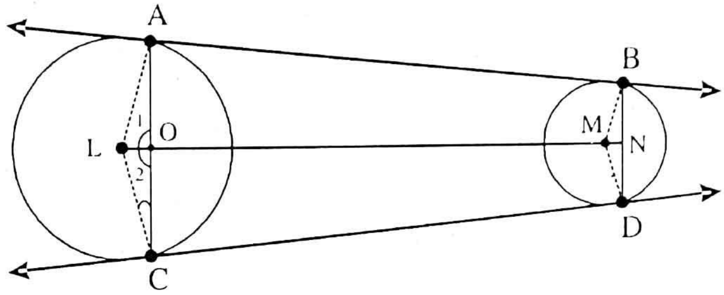
$m\overline{AB} = 2(\sqrt{18.75}\text{cm})$

$m\overline{AB} = 8.66\text{cm}$ $\Rightarrow$ $\boxed{m\overline{AB} \simeq 8.7\text{cm}}$

Q.3 $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are the common tangents drawn to the pair of circles.

If A and C are the points of tangency of 1st circle where B and D are the points of tangency of 2nd circle, then prove that $\overline{AC} \parallel \overline{BD}$.

Given: Two circles with centre L and M. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are their common tangents. A is joined with C and B is joined with D.



To prove:

$\overline{AC} \parallel \overline{BD}$

Construction:

Join L to A and C. Join M to B and D. Join L to M and produce it to meet the $\overline{BD}$ at N.

Proof:

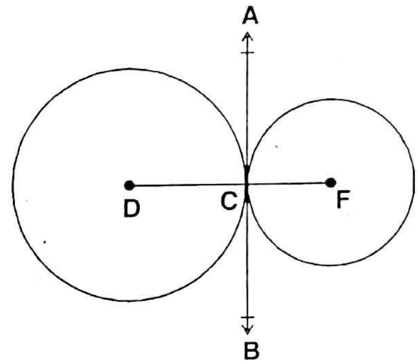
Statements	Reasons
In $\Delta AOL \leftrightarrow \Delta COL$	
$\overline{AL} \cong \overline{CL}$	Radii of the same circle
$\angle A \cong \angle C$	angles opposite to congruent sides
$\overline{LO} \cong \overline{LO}$	common side.
$\therefore$ $\Delta AOL \cong \Delta COL$	S.A.S $\cong$ S.A.S
$m\angle 1 = m\angle 2$ ....(i)	Corresponding angles of congruent triangle.
$m\angle 1 + m\angle 2 = 180^o$ ....(ii)	O is the point on line segment $\overline{AC}$.
$\Rightarrow$ $m\angle 1 = m\angle 2 = 90^o$	
$\overline{LO} \perp \overline{AO}$	
or $\overline{LO} \perp \overline{AC}$	
or $\overline{AC} \perp \overline{LOMN}$ ....(iii)	
Similarly in the circle with centre M, it can be proved that	
$\overline{BD} \perp \overline{MN}$	
or $\overline{BD} \perp \overline{LOMN}$ ....(iv)	
Both $\overline{AC}$ and $\overline{BD}$ are $\perp$ to the same line segment	
$\therefore$ $\overline{AC} \parallel \overline{BD}$	Two line segments making same angle with a line are parallel to each other.

THEOREM 4 (A)

If two circles touch externally then the distance between their centres is equal to the sum of their radii.

Given:

Two circles with centres D and F respectively touch each other externally at point C. So that $\overline{CD}$ and $\overline{CF}$ are respectively the radii of the two circles.



To Prove:

Point C lies on the join of centres D and F and $m\overline{DF}=m\overline{DC}+m\overline{CF}$

Construction:

Draw $\overleftrightarrow{ACB}$ as a common tangent to the pair of circles at C.

Proof:

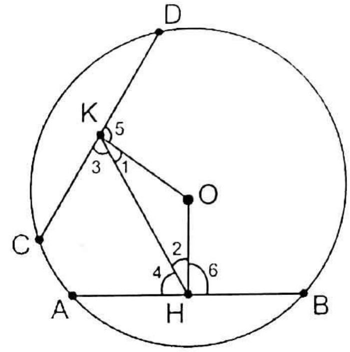
Statements	Reasons
Both circles touch externally at C whereas $\overline{CD}$ is radial segment and $\overleftrightarrow{ACB}$ is the common tangent.	
$\therefore\ m\angle ACD = 90^{\circ}$ ....(i)	Radial segment $\overline{CD} \perp$ the tangent line $\overline{AB}$
Similarly $\overline{CF}$ is radial segment and $\overleftrightarrow{ACB}$ is the common tangent	
$\therefore\ m\angle ACF = 90^{\circ}$ ....(ii)	Radial segment $\overline{CF} \perp$ the tangent line $\overline{AB}$
$m\angle ACD + m\angle ACF = 90^{\circ} + 90^{\circ}$	Adding (i) and (ii)
$m\angle DCF = 180^{\circ}$ ....(iii)	Sum of supplementary adjacent angles
Hence DCF is a line segment with point C between D and F	
and $m\overline{DF}=m\overline{DC}+m\overline{CF}$	

Exercise 10.2

Q. 1 $\overline{AB}$ and $\overline{CD}$ are two equal chords in a circle with centre O. H and K are respectively the mid points of the chords. Prove that $\overline{HK}$ makes equal angles with $\overline{AB}$ and $\overline{CD}$.

Given:

A circle with centre 'O'. Two chords such that $m\overline{AB} = m\overline{CD}$. H and K are mid points of chords AB and CD respectively. H is joined with K



To Prove:

(i) $m\angle AHK = m\angle CKH$

(ii) $m\angle BHK = m\angle DKH$

Proof:

Statements	Reasons
In ΔHOK	
$m\overline{OH} = m\overline{OK}$	Two equal chords are equidistant from the center.
$\therefore$ $m\angle 1 = m\angle 2$ .... (i)	Angles opposite to the equal line segments
And $m\angle 5 = m\angle 6$ .... (ii)	Each 90°
$m\angle 1 + m\angle 5 = m\angle 2 + m\angle 6$	Adding (i) and (ii)
Thus, $m\angle DKH = m\angle BHK$	
or $m\angle BHK = m\angle DKH$ Proved	
$m\angle AHO = m\angle CKO$	Each 90°
$m\angle 2 + m\angle 4 = m\angle 1 + m\angle 3$	
But $m\angle 2 = m\angle 1$	Proved in (i)
$\cancel{m\angle 1} + m\angle 4 = \cancel{m\angle 1} + m\angle 3$	
$m\angle 4 = m\angle 3$	By cancellation property
$m\angle AHK = m\angle CKH$	

Q.2 The radius of a circle is 2.5 cm. $\overline{AB}$ and $\overline{CD}$ are two chords 3.9cm apart. If $m\overline{AB} = 1.4$ cm, then measure the other chord.

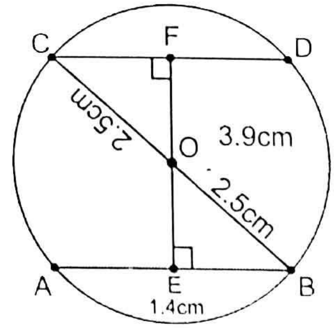
Given:

O is the centre of a circle.

(i) $m\overline{OB} = m\overline{OC} = 2.5$cm

(ii) $m\overline{AB} = 1.4$cm

(iii) $m\overline{EF} = 3.9$cm



To Find:

$m\overline{CD} = ?$

Construction:

Join O with B and C.

Calculations:

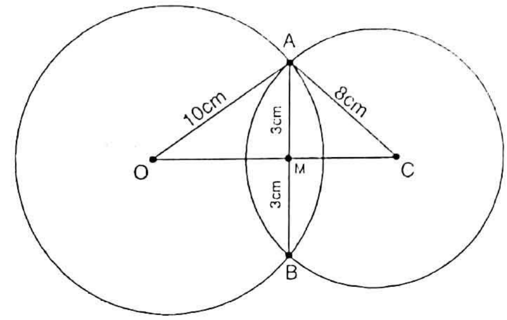
Steps	Reasons
In ΔOEB	Given
$m\overline{EB} = \frac{1}{2} m\overline{AB} = \frac{1}{2}(1.4cm) = 0.7\ cm$ ....(i)	
$m\overline{OB} = 2.5cm$ ....(ii)	
$(m\overline{OB})^2 = (m\overline{OE})^2 + (m\overline{EB})^2$	By Pythagoras theorem in right angled ΔOEB
$(2.5cm)^2 = (m\overline{OE})^2 + (0.7cm)^2$	From (i) and (ii)
$\Rightarrow (m\overline{OE})^2 = (2.5cm)^2 - (0.7cm)^2$	
$(m\overline{OE})^2 = 6.25cm^2 - 0.49cm^2$	
$(m\overline{OE})^2 = 5.76cm^2$	
$\sqrt{(m\overline{OE})^2} = \sqrt{5.76cm^2}$	
$m\overline{OE} = 2.4cm$ .... (iii)	
Now	
$m\overline{OF} = m\overline{EF} - m\overline{OE}$	
$m\overline{OF} = 3.9cm - 2.4cm$	
$m\overline{OF} = 1.5cm$ .... (iv)	
In right angled triangle ΔOCF	
$(m\overline{OC})^2 = (m\overline{OF})^2 + (m\overline{CF})^2$	
$\Rightarrow (m\overline{CF})^2 = (2.5cm)^2 - (1.5cm)^2$	From (iv)
$(m\overline{CF})^2 = 6.25cm^2 - 2.25cm^2$	
$(m\overline{CF})^2 = 4cm^2$	
$\therefore \sqrt{(m\overline{CF})^2} = \sqrt{4cm^2}$	
$(m\overline{CF}) = 2\ cm$ .... (v)	
$\therefore (m\overline{CD}) = 2(m\overline{CF})$	$\because m\overline{CF} = \frac{1}{2} m\overline{CD}$
$m\overline{CD} = 2(2cm)$	From (v)
$m\overline{CD} = 4cm$	

Q.3. The radii of two intersecting circles are 10cm and 8cm. If the length of their common chord is 6cm then find the distance between the centres.

Solution:

Given: Two intersecting circles with centers O and C having radius $m\overline{OA} = 10cm$, radius $m\overline{AC} = 8cm$ respectively.

Length of common chord $m\overline{AB} = 6cm$



To Find:

Distance between the centers $m\overline{OC} = ?$

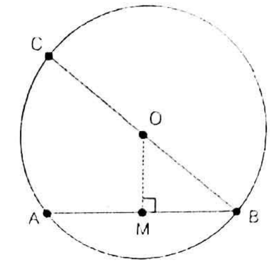
Construction:

Join the point A to the centers O and C. Joint O to C which meets the chord $\overline{AB}$ at its midpoint.

Calculations:

Steps	**Reasons**
$\overline{OM} \perp \overline{AB}$	
$m\overline{AM} = \frac{1}{2} m\overline{AB} = \frac{1}{2}(6cm) = 3cm$ ....(i)	$\perp$ from the center to the chord bisect it.
By Pythagoras theorem	
$(m\overline{OM})^2 = (m\overline{OA})^2 - (m\overline{AM})^2$	
$= (10cm)^2 - (3cm)^2$	From given and from (i)
$= 100cm^2 - 9cm^2$	
$(m\overline{OM})^2 = 91\,cm^2$	
$\sqrt{(m\overline{OM})^2} = \sqrt{91cm^2}$	
$m\overline{OM} = 9.54cm$ ....(ii)	
In ΔAMC	
By Pythagoras theorem	
$(m\overline{MC})^2 = (m\overline{CA})^2 - (m\overline{AM})^2$	
$= (8cm)^2 - (3cm)^2$	$m\overline{CA} = 8cm$, $m\overline{AM} = 3cm$
$= 64cm^2 - 9cm^2$	
$(m\overline{MC})^2 = 55\ cm^2$	
$\sqrt{(m\overline{MC})^2} = \sqrt{55cm^2}$	
$m\overline{MC} = 7.42\,cm$ ....(iii)	
We know that the distance between the centers	
$(m\overline{OC}) = (m\overline{OM}) + (m\overline{MC})$	From (ii) and (iii)
$= 9.54mc + 7.42cm$	
$= 16.96cm$	
Distance between the centres is 16.96cm	

Q.4. Show that greatest chord in a circle is its diameter.



Given: O be the centre of the circle, $m\overline{BC}$ is central chord and $\overline{AB}$ be any chord of the circle

To prove: Central chord $m\overline{CB}$ > Any chord $m\overline{AB}$

Construction: Draw $\overline{OM} \perp \overline{AB}$ to make right angled triangle OMB.

Proof:

Statements	Reasons
In right angled triangle OMB $\left(m\overline{OB}\right)^2 = \left(m\overline{OM}\right)^2 + \left(m\overline{MB}\right)^2$	By Pythagoras theorem
It means $m\overline{OB} > m\overline{MB}$ $\therefore \quad 2\left(m\overline{OB}\right) > 2\left(m\overline{MB}\right)$	The length of hypotenuse is greater than the length of other two sides.
As $2\left(m\overline{OB}\right)$ is length of the central chord and $2\left(m\overline{MB}\right)$ is length of the chord $\overline{AB}$ thus, Central chord $m\overline{CB}$ > Any chord $m\overline{AB}$.	
It means central chord of the circle i.e. diameter is greater than any other chord of the circle, which proved that the greatest chord in a circle is its diameter.	

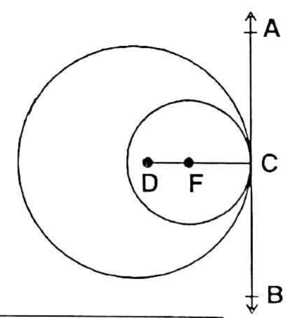
THEOREM 4(B)

If two circles touch each other internally, then the point of contact lies on the line segment through their centres and distance between their centres is equal to the difference of their radii.

Given: Two circles with centres D and F touch each other internally at point C.

So that $\overline{CD}$ and $\overline{CF}$ are the radii of two circles.

To Prove: Point C lies on the join of centres D and F extended, and $m\overline{DF} = m\overline{DC} - m\overline{CF}$



Construction: Draw $\overleftrightarrow{ACB}$ as the common tangent to the pair of circles at C.

Proof:

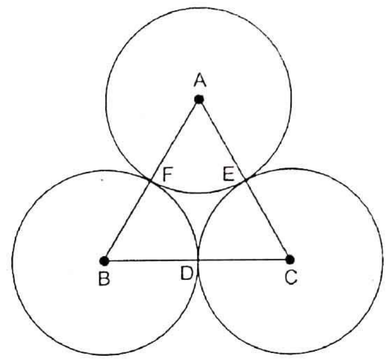
Statements	Reasons
Both circles touch internally at C whereas $\overleftrightarrow{ACB}$ is the common tangent and $\overline{CD}$ is the radial segment of the first circle.	
$\therefore m\angle ACD = 90^\circ$ ....(i)	Radial segment $\overline{CD} \perp$ the tangent line $\overline{AB}$
Similarly $\overleftrightarrow{ACB}$ is the common tangent and $\overline{CF}$ is the radial segment of the second circle. $\therefore m\angle ACF = 90^\circ$ .... (ii)	Radi
$\Rightarrow \quad m\angle ACD = m\angle ACF = 90^\circ$	al segment $\overline{CF} \perp$ the tangent line $\overline{AB}$. Using (i) and (ii)
Where $\angle ACD$ and $\angle ACF$ coincide each other with point F between D and C. Hence $m\overline{DC} = m\overline{DF} + m\overline{FC}$ i.e., $\quad m\overline{DC} - m\overline{FC} = m\overline{DF}$ or $\quad m\overline{DF} = m\overline{DC} - m\overline{FC}$	

Example 1:

Three circles touch in pairs externally. Prove that the perimeter of a triangle formed by joining centres is equal to the sum of their diameters.

Given:

Three circles have centres A, B and C their radii are r_1, r_2 and r_3 respectively. They touch in pairs externally at D, E and F. So that ΔABC is formed by joining the centres of these circles.



To Prove:

Perimeter of $\Delta ABC = 2r_1 + 2r_2 + 2r_3$

$= d_1 + d_2 + d_3$

= Sum of the diameters of these circles.

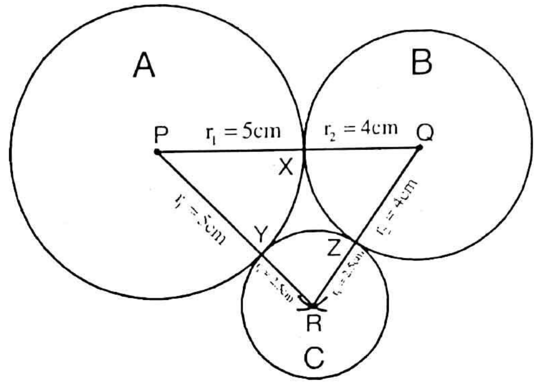
Proof:

Statements	Reasons
Three circles with centres A, B and C touch in pairs externally at the points, D, E and F.	Given
$\therefore \quad m\overline{AB} = m\overline{AF} + m\overline{FB}$ ....(i)	
$m\overline{BC} = m\overline{BD} + m\overline{DC}$ ....(ii)	
And $m\overline{CA} = m\overline{CE} + m\overline{EA}$ ....(iii)	
$m\overline{AB} + m\overline{BC} + m\overline{CA} = m\overline{AF} + m\overline{FB} + m\overline{BD}$ $+ m\overline{DC} + m\overline{CE} + m\overline{EA}$	Adding (i), (ii) and (iii)
$P = (m\overline{AF} + m\overline{EA}) + (m\overline{FB} + m\overline{BD}) + (m\overline{CD} + m\overline{CE})$	Sum of three sides of a triangle is equal to its perimeter (P).
Perimeter of $\Delta ABC = 2r_1 + 2r_2 + 2r_3$ $= d_1 + d_2 + d_3$	
Perimeter of ΔABC = Sum of diameters of the circles.	$d_1 = 2r_1$, $d_2 = 2r_2$ and $d_3 = 2r_3$ are diameters of the circles.

EXERCISE 10.3

Q.1 **Two circles with radii 5cm and 4cm touch each other externally. Draw another circle with radius 2.5cm touching the first pair, externally.**

Solution:



Radius of Circle A = $r_1 = 5cm$

Radius of Circle B = $r_2 = 4cm$

Radius of Circle C = $r_3 = 2.5cm$

Steps of construction:

Step 1: Draw a line segment $m\overline{PQ}$ $5cm + 4cm = 9cm$ long.

Step 2: Take 'P' as a centre and draw a circle of radius 5cm.

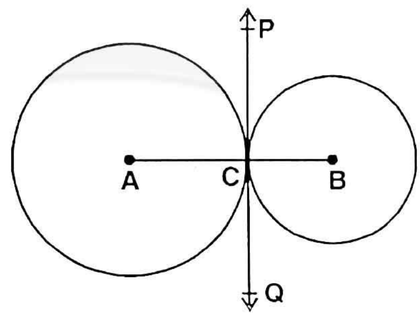
Step 3: Take 'Q' as a centre and draw a circle of radius 4cm, which intersects the circle of radius 4cm at point x.

Step 4: Take P as a centre and draw an arc of radius $(5cm + 2.5cm = 7.5cm)$

Step 5: Take Q as a centre and draw an arc of radius $(4cm + 2.5cm = 6.5cm)$, which intersects the previous arc at point R.

Step 6: Take R as centre and draw a circle of radius 2.5cm which touches externally the circles of centre P and Q at the points Y and Z respectively.

Q.2 **If the distance between the centres of two circles is the sum or the difference of their radii they will touch each other.**



Given: Two circles with centre A and B. $\overline{AC}$ and $\overline{BC}$ are radial segments of these circles such that $m\overline{AB} = m\overline{AC} + m\overline{BC}$ or

To Prove: Both circle touch each other.

Construction: Join A to B. Draw a tangent $\overleftrightarrow{PQ}$ of circle A at point C i.e. $m\angle PCA = 90^\circ$

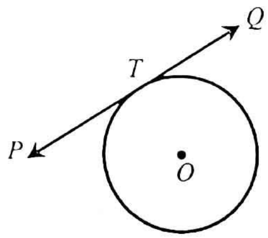
Proof:

Statements	Reasons
$m\overline{AB} = m\overline{AC} + m\overline{BC}$ ....(i)	Given
Points A, C and B are collinear such that C is between A and B.	From (i)
$m\angle PCA + m\angle PCB = 180^\circ$ ....(ii)	Supplementary angles
As $m\angle PCA = 90^\circ$ ....(iii)	Construction
$\therefore m\angle PCB = 90^\circ$ ....(iv)	From (ii) and (iii)
$\overline{PC} \perp \overline{BC}$ at C i.e. $\overleftrightarrow{PQ}$ is $\perp$ $\overline{BC}$ at C	From (iv)
$\overleftrightarrow{PQ}$ is also a tangent of circle B	
$\overleftrightarrow{PQ}$ is common tangent of both circles. i.e. Both circles have a common point C. Thus both circles touch each other at a point C. Similarly the same results can be proved when distance between the centers of two circles is equal to the difference of their radii	

MISCELLANEOUS EXERCISE – 10

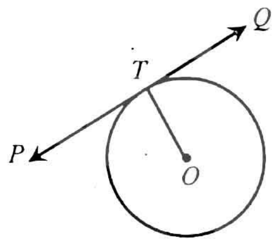
Q. 1 Four possible answers are given for the following questions.

1. In the adjacent figure of the circle, the line $\overleftrightarrow{PQ}$ is named as.



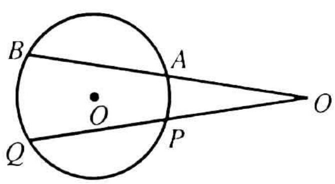
(a) an arc
(b) a chord
(c) a tangent
(d) a secant

2. In a circle with centre O, $\overline{OT}$ is the radical segment and $\overleftrightarrow{PTQ}$ is the tangent line, then



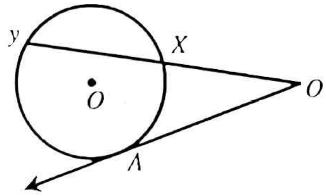
(a) $\overline{OT} \perp \overleftrightarrow{PQ}$
(b) $\overline{OT} \not\perp \overleftrightarrow{PQ}$
(c) $\overline{OT} \parallel \overleftrightarrow{PQ}$
(d) $\overline{OT}$ is the right bisector of $\overleftrightarrow{PQ}$

3. In the given diagram find m $\overline{OA}$ if m $\overline{OB}$ =8cm, m $\overline{OP}$ =4cm and m$\overline{OQ}$=12cm



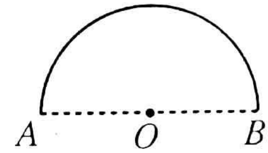
(a) 2cm
(b) 2.67
(c) 2.8 cm
(d) 3cm

4. In the given diagram find m $\overline{OX}$ if m$\overline{OA}$ =6cm and m$\overline{OY}$ =9cm



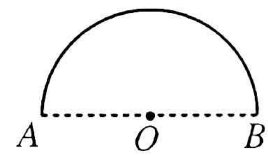
(a) 4cm
(b) 6cm
(c) 9cm
(d) 12cm

5. In the adjacent figure find semicircular area if $\pi \simeq 3.1416$ and m$\overline{OA}$ =20cm.



(a) 62.83sq cm
(b) 314.16sq cm
(c) 436.20sq cm
(d) 628.32sq cm

6. In the adjacent figure find half the perimeter of circle with center O if $\pi = 3.1416$ and m$\overline{OA}$=20cm.



(a) 31.42 cm
(b) 62.832 cm
(c) 125.65 cm
(d) 188.50 cm

7. A line which has two points in common with a circle is called.

(a) sine of a circle
(b) cosine of a circle
(c) tangent of a circle
(d) secant of a circle

8. A line which has only one point in common with a circle is called

(a) sine of a circle
(b) cosine of a circle
(c) tangent of a circle
(d) secant of a circle

9. Two tangents drawn to a circle from a point outside it are ....in length

(a) half (b) equal
(c) double (d) triple

10. A circle has only one.

(a) secant (b) chord
(c) diameter (d) centre

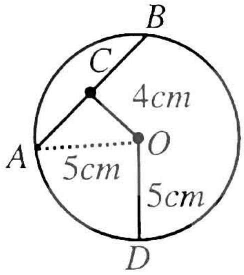
11. A tangent line intersects the circle at.

(a) three points (b) two points
(c) single point (d) no point at all

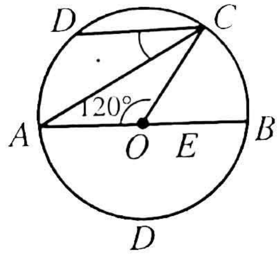
12. Tangents drawn at the ends of diameter of a circle are...... to each other.
(a) parallel (b) non-parallel
(c) collinear (d) perpendicular

13. The distance between the centres of two congruent touching circles externally is
(a) of zero length
(b) the radius of each circle
(c) the diameter of each circle
(d) twice the diameter of each circle

14. In the adjacent circular figure with centre O and radius 5cm. The length of the chord intercepted at 4cm away from the centre of this circle is
(a) 4cm
(b) 6cm
(c) 7cm
(d) 9cm



15. In the adjoining figure there is a circle with centre O. If $\overline{DC} \parallel$ diameter $\overline{AB}$ and $m\angle AOC = 120°$, then $m\angle ACD$ is
(a) 40°
(b) 30°
(c) 50°
(d) 60°



ANSWER KEY

1.	c	2.	a	3.	b	4.	a	5.	d
6.	b	7.	d	8.	c	9.	b	10.	d
11.	c	12.	a	13.	c	14.	b	15.	b