Unit 04

PARTIAL FRACTIONS

Fraction:

The quotient of two numbers or algebraic expressions is called a fraction. The quotient is indicated by a bar (—). We write, the dividend on top of the bar and the divisor below the bar.

For example, $\frac{x^2+2}{x-2}$ is a fraction with $x - 2 \neq 0$. If $x - 2 = 0$, then the fraction is not defined because $x-2=0 \Rightarrow x=2$ which make the denominator of the fraction zero.

Rational Fraction:

An expression of the form $\frac{N(x)}{D(x)}$, where N(x) and D(x) are polynomials in x with real coefficients, is called a rational fraction. The polynomial $D(x) \neq 0$ in the expression.

For example, $\frac{x^2+3}{(x+1)^2(x+2)}$ and $\frac{2x}{(x-1)(x+2)}$ are rational fractions.

Proper fraction:

A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called a proper fraction if degree of the polynomial N(x) in the numerator is less than the degree of the polynomial D(x) in the denominator. For example, $\frac{2}{x+1}, \frac{2x-3}{x^2+4}$ and $\frac{3x^2}{x^3+1}$ are proper fractions.

Improper Fraction:

A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called an improper fraction if degree of the polynomial N(x) is greater or equal to the degree of the polynomial D(x).

e.g., $\frac{5x}{x+2}, \frac{3x^2+2}{x^2+7x+12}, \frac{6x^4}{x^3+1}$ are improper fractions.

How we can we reduce the improper fraction into proper fraction?

Every improper fraction can be reduced by division to the sum of a polynomial and a proper fraction. This means that if degree of the numerator is greater or equal to the degree of the denominator, then we can divide N(x) by D(x) obtaining a quotient polynomial Q(x) and a remainder polynomial R(x), whose degree is less than the degree of D(x).

Thus $\frac{N(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$, with $D(x) \neq 0$,

where Q(x) is quotient polynomial and $\frac{R(x)}{D(x)}$ is a proper fraction.

Example 1: **Resolve the fraction $\frac{x^3 - x^2 + x + 1}{x^2 + 5}$ into proper fraction.**

Solution:

Let $N(x) = x^3 - x^2 + x + 1$ and $D(x) = x^2 + 5$

By long division, we have

$$\begin{array}{r|l} & x - 1 \\ x^2+5 & x^3 - x^2 + x + 1 \\ & \pm x^3 \qquad \pm 5x \\ \hline & -x^2 - 4x + 1 \\ & \mp x^2 \qquad \pm 5 \\ \hline & -4x + 6 \end{array}$$

$$\frac{x^3 - x^2 + x + 1}{x^2+5} = (x-1) + \frac{-4x+6}{x^2+5}$$

Activity:

Separate proper and improper fractions

(i) $\frac{x^2+x+1}{x^2+2}$ (ii) $\frac{2x+5}{(x+1)(x+2)}$

(iii) $\frac{x^3+x^2+1}{x^3-1}$ (iv) $\frac{2x}{(x-1)(x-2)}$

Ans.

(i) Improper Fraction
(ii) Proper Fraction
(iii) Improper Fraction
(iv) Proper Fraction

Activity: **Convert the following improper fractions into proper fractions:**

(i) $\frac{3x^2-2x-1}{x^2-x+1}$ (ii) $\frac{6x^3+5x^2-6}{2x^2-x-1}$

(i) Solution: $\frac{3x^2-2x-1}{x^2-x+1}$

Let $N(x) = 3x^2-2x-1$ and $D(x) = x^2-x+1$

By long division, we get

$$
\begin{array}{r|l}
 & 3 \\
x^2-x+1 & \cancel{3x^2}-2x-1 \\
 & \pm\cancel{3x^2}\mp 3x\pm 3 \\
\hline
 & x-4
\end{array}
$$

Thus $\frac{3x^2-2x-1}{x^2-x+1} = 3+\frac{x-4}{x^2-x+1}$

(ii) Solution: $\frac{6x^3+5x^2-6}{2x^2-x-1}$

Let $N(x) = 6x^3+5x^2-6$ and $D(x) = 2x^2-x-1$

By long division, we get

$$
\begin{array}{r|l}
 & 3x+4 \\
2x^2-x-1 & \cancel{6x^3}+5x^2-6 \\
 & \pm\cancel{6x^3}\mp 3x^2\mp 3x \\
\hline
 & \cancel{8x^2}+3x-6 \\
 & \pm\cancel{8x^2}\mp 4x\mp 4 \\
\hline
 & 7x-2
\end{array}
$$

Thus

$$\frac{6x^3+5x^2-6}{2x^2-x-1} = (3x+4)+\frac{7x-2}{2x^2-x-1}$$

Resolution of Fraction into Partial Fractions:

Every proper fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ can be resolved into an algebraic sum of components fractions. These components fractions of a resultant fraction are called its partial fractions. Here four cases of partial fractions are discussed, as follows:

Rule I:

Resolution of an algebraic fraction into partial fractions, when D(x) consists of non-repeated linear factors:

If linear factor $(ax + b)$ occurs as a factor of $D(x)$, then there is a partial fraction of the form $\frac{A}{ax+b}$, where A is a constant to be found.

In $\frac{N(x)}{D(x)}$, the polynomial $D(x)$ may be written as,

$D(x) = (a_1x + b_1)(a_2x + b_2) \ldots (a_nx + b_n)$ with all factors distinct.

We have

$$\frac{N(x)}{D(x)} = \frac{A_1}{a_1x+b_1}+\frac{A_2}{a_2x+b_2}+\frac{A_3}{a_3x+b_3}+\cdots+\frac{A_n}{a_nx+b_n}$$

Where $A_1A_2 \ldots A_n$ are constants to be determined.

Example 1:

Resolve $\frac{5x+4}{(x-4)(x+2)}$ into partial fractions.

Solution:

Let $\frac{5x+4}{(x-4)(x+2)} = \frac{A}{x-4}+\frac{B}{x+2}$ ....(i)

Multiplying throughout by $(x-4)(x+2)$,

we get $5x + 4 = A(x+2) + B(x-4)$ .... (ii)

Equation (ii) is an identity, which holds good for all values of x and hence for x = 4 and x = −2

Put x − 4 = 0 i.e x = 4 in equation (ii) We get

$$5(4) + 4 = A(4 + 2)$$

$$20+4 = A(6)$$

$$6A = 24 \quad \Rightarrow \quad \boxed{A = 4}$$

Put x+2 = 0 i.e, x = −2 in equation (ii) We get

$$5(-2)+ 4= B(-2 - 4)$$

$$-10+4 = B(-6)$$

$$\Rightarrow -6B = -6 \Rightarrow \boxed{B=1}$$

Putting the value of A and B in equation (i)

Thus required partial fractions are $\frac{4}{x-4}+\frac{1}{x+2}$

Hence $\frac{5x+4}{(x-4)(x+2)} = \frac{4}{x-4} + \frac{1}{x+2}$

This method is called the zeros' method. This method is especially useful with linear factors in the denominator D(x).

Identity: An identity is an equation, which is satisfied by all the values of the variables involved. For example, 2(x + 1) = 2x+2 and $\frac{2x^2}{x} = 2x$ are identities, as these equations are satisfied by all values of x.

Example 2:

Resolve $\frac{1}{3+x-2x^2}$ into partial fractions.

Solution: $\frac{1}{3+x-2x^2}$ can be written as for convenience $\frac{-1}{2x^2-x-2}$

The denominator

$$D(x) = 2x^2 - x - 3$$

$$= 2x^2 - 3x + 2x - 3$$

$$= x(2x - 3) + 1(2x - 3)$$

$$= (x + 1)(2x - 3)$$

Let, $\frac{-1}{2x^2-x-3} = \frac{-1}{(x+1)(2x-3)} = \frac{A}{x+1}+\frac{B}{2x-3}$

Multiplying both sides by (x + 1) (2x − 3) , We get −1 = A (2x − 3) + B(x + 1).........(i)

$$-1 = 2Ax - 3A + Bx + B$$

$$-1 = 2Ax + Bx - 3A + B$$

$$0x - 1 = (2A+B)x - 3A + B$$

Equating coefficients of x and constants on both sides, We get

$2A + B = 0$...(i) $\quad -3A + B = -1$...(ii)

Subtracting (ii) from (i),

$$(2A + B) - (-3A + B) = 0-(-1)$$

$$2A + B + 3A - B = 1$$

$$5A = 1 \Rightarrow \boxed{A = \frac{1}{5}}$$

Putting the value of A in equation (ii)

$$-3\left(\frac{1}{5}\right) + B = -1$$

$$\left(\frac{-3}{5}\right) + B = -1$$

$$B = -1 + \frac{3}{5}$$

$$B = \frac{-5+3}{5} \Rightarrow \boxed{B = \frac{-2}{5}}$$

Thus, $\frac{1}{3+x-2x^2} = \frac{1}{5(x+1)} - \frac{2}{5(2x-3)}$

Note: General method applicable to resolve all rational fractions of the form $\frac{N(x)}{D(x)}$ is as follows:

(i) The numerator N(x) must be of lower degree than the denominator D(x)

(ii) If degree of N(x) is greater than the degree of D(x), then division is used and the remainder fraction R(x) can be broken into partial fractions.

(iii) Make substitution of constants accordingly.

(iv) Multiply both the sides by L.C.M.

(v) Arrange the terms on both sides in descending order.

(vi) Equate the coefficients of like powers of x on both sides, we get as many as equations as there are constants in assumption.

(vii) Solving these equations, we can find the values of constants.

EXERCISE 4.1

Resolve into partial fractions.

Q.1 $\dfrac{7x-9}{(x+1)(x-3)}$

Solution: $\dfrac{7x-9}{(x+1)(x-3)}$

Let $\dfrac{7x-9}{(x+1)(x-3)} = \dfrac{A}{x+1} + \dfrac{B}{x-3}$ ...... (i)

Multiplying equation (i) by (x + 1) (x – 3)

$7x - 9 = A(x-3) + B(x+1)$ ...... (ii)

As equation (ii) is an identity which is true for all values of x.

Put $x - 3 = 0$ i.e $x = 3$ and

Put $x + 1 = 0$ i.e $x = -1$

Putting $x = 3$ and $x = -1$ in (ii) we get

For $x = 3$	For $x = -1$
$7(3)-9 = +B(3+1)$	$7(-1)-9 = A(-1-3)$
$21 - 9 = 4B$	$-7-9 = -4A$
$12 = 4B$	$-16 = -4A$
$\Rightarrow \boxed{B=3}$	$\Rightarrow \boxed{A=4}$

Putting the value of A and B in equation (i)

We get the required partial fractions as.

$\dfrac{4}{x+1} + \dfrac{3}{x-3}$

Thus $\dfrac{7x-9}{(x+1)(x-3)} = \dfrac{4}{x+1} + \dfrac{3}{x-3}$

Q.2 $\dfrac{x-11}{(x-4)(x+3)}$

Solution: $\dfrac{x-11}{(x-4)(x+3)}$

Let $\dfrac{x-11}{(x-4)(x+3)} = \dfrac{A}{x-4} + \dfrac{B}{x+3}$ .....(i)

Multiplying by (x–4)(x+3) on both sides, we get

$x - 11 = A(x+3) + B(x-4)$....... (ii)

As equation (ii) is an identity which is true for all value of x.

Putting $x + 3 = 0$ i.e $x = -3$

and $x - 4 = 0$ i.e $x = 4$ in (ii) we get

For $x = -3$	For $x = 4$
$-3-11 = B(-3-4)$	$4 - 11 = A(4+3)$
$-14 = -7B$	$-7 = 7A$
$\Rightarrow \boxed{B=2}$	$\Rightarrow \boxed{A=-1}$

Hence the required partial fractions are

$$\frac{x-11}{(x-4)(x+3)} = \frac{-1}{x-4} + \frac{2}{x+3}$$

Q.3 $\dfrac{3x-1}{x^2-1}$

Solution: $\dfrac{3x-1}{x^2-1}$

$$\frac{3x-1}{x^2-1} = \frac{3x-1}{(x-1)(x+1)}$$

Let $\dfrac{3x-1}{(x-1)(x+1)} = \dfrac{A}{x-1} + \dfrac{B}{x+1}$ ......(i)

Multiplying both sides by (x–1)(x+1), we get

$3x - 1 = A(x+1) + B(x-1)$ .... (ii)

As equation (ii) is an identity which is true for all values of x.

Let $x + 1 = 0$ i.e $x = -1$ and

$x - 1 = 0$ i.e $x = 1$

Putting $x = -1$ and $x = 1$ in (ii) We get

For $x = 1$	For $x = -1$
$3(1) - 1 = A(1+1)$	$3(-1) - 1 = B(-1-1)$
$3 - 1 = 2A$	$-3 - 1 = -2B$
$2 = 2A$	$-4 = -2B$
$\Rightarrow \boxed{A=1}$	$\Rightarrow \boxed{B=2}$

Hence the required partial fractions are

$$\frac{3x-1}{(x-1)(x+1)} = \frac{1}{x-1} + \frac{2}{x+1}$$

Q.4 $\frac{x-5}{x^2+2x-3}$

Solution: $\frac{x-5}{x^2+2x-3}$

$$\frac{x-5}{x^2+2x-3} = \frac{x-5}{x^2+3x-x-3}$$

$$= \frac{x-5}{x(x+3)-1(x+3)}$$

$$= \frac{x-5}{(x-1)(x+3)}$$

$$\frac{x-5}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3} \ldots\ldots\ (i)$$

Multiplying both sides by (x–1) (x+3), we get

$$x-5 = A(x+3) + B(x-1) \ldots\ldots\ (ii)$$

As equation (ii) is an identity which is true for all values of x.

Let $x+3 = 0 \Rightarrow x = -3$

and $x-1 = 0 \Rightarrow x = 1$

Putting x = – 3 and x=1 in equation (ii) we get

For x = –3	For x = 1
$-3-5 = +B(-3-1)$	$1-5 = A(1+3)$
$-8 = -4B$	$-4 = 4A$
$B = \frac{-8}{-4}$	$A = \frac{-4}{4}$
$\Rightarrow \boxed{B=2}$	$\Rightarrow \boxed{A=-1}$

Hence the required partial fractions are

$$\frac{x-5}{x^2+2x-3} = \frac{-1}{x-1} + \frac{2}{x+3}$$

Q.5 $\frac{3x+3}{(x-1)(x+2)}$

Solution: $\frac{3x+3}{(x-1)(x+2)}$

Let $\frac{3x+3}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} \ldots\ldots\ (i)$

Multiplying both sides by (x–1)(x+2), we get

$$3x+3 = A(x+2) + B(x-1) \ldots\ldots\ (ii)$$

As equation (ii) is an identity which is true for all values of x.

Let $x-1 = 0$ i.e $x = 1$

and $x+2 = 0$ i.e $x = -2$

Putting $x = 1$ and $x = -2$ in equation (ii) we get

For x = 1	For x = –2
$3(1)+3 = A(1+2)$	$3(-2)+3 = B(-2-1)$
$3+3 = 3A$	$-6+3 = -3B$
$6 = 3A$	$-3 = -3B$
$A = \frac{6}{3}$	$B = \frac{-3}{-3}$
$\Rightarrow \boxed{A=2}$	$\Rightarrow \boxed{B=1}$

Hence the required partial fractions are

$$\frac{3x+3}{(x-1)(x+2)} = \frac{2}{x-1} + \frac{1}{x+2}$$

Q.6 $\frac{7x-25}{(x-4)(x-3)}$

Solution: $\frac{7x-25}{(x-4)(x-3)}$

Let $\frac{7x-25}{(x-4)(x-3)} = \frac{A}{x-4} + \frac{B}{x-3}$

Multiplying both sides by(x–4) (x – 3), we get

$$7x-25 = A(x-3) + B(x-4) \ldots\ (ii)$$

As equation (ii) is an identity which is true for all values of x.

Let $x-3 = 0$ i.e $x = 3$

and $x-4 = 0$ i.e $x = 4$

Putting $x = 3$ and $x = 4$ in equation (ii) we get

For x = 3	For x = 4
$7(3)-25 = B(3-4)$	$7(4)-25 = A(4-3)$
$21-25 = -B$	$28-25 = 1A$
$-4 = -B$	$3 = A$
$\Rightarrow \boxed{B=4}$	$\Rightarrow \boxed{A=3}$

Hence the required partial fractions are

$$\frac{7x-25}{(x-4)(x-3)} = \frac{3}{x-4} + \frac{4}{x-3}$$

Q.7 $\dfrac{x^2+2x+1}{(x-2)(x+3)}$

Solution: $\dfrac{x^2+2x+1}{(x-2)(x+3)}$ is an improper fraction. First we resolve it into proper fraction.

By long division we get

$$x^2+x-6\overline{\big)x^2+2x+1}\quad 1$$
$$\underline{\pm x^2 \pm x \mp 6}$$
$$x+7$$

We have $\dfrac{x^2+2x+1}{x^2+x-6} = 1+\dfrac{x+7}{x^2+x-6}$

Let $\dfrac{x+7}{(x-2)(x+3)} = \dfrac{A}{x-2}+\dfrac{B}{x+3}$ ....(i)

Multiplying both sides by (x–2) (x +3), we get

$$x + 7 = A(x+3) + B(x-2) \text{.........} \quad \text{(ii)}$$

As equation (ii) is an identity which is true for all values of x.

Let $x + 3 = 0$ i.e $x = -3$

and $x - 2 = 0$ i.e $x = 2$

Putting $x = -3$ and $x = 2$ in equation (ii) we get

For $x = -3$	For $x = 2$
$-3 + 7 = B(-3-2)$	$2 + 7 = A(2+3)$
$4 = -5B$	$9 = 5A$
$\Rightarrow \boxed{B = -\dfrac{4}{5}}$	$\Rightarrow \boxed{A = \dfrac{9}{5}}$

Hence the required partial fractions are

$$\frac{x^2+2x+1}{(x-2)(x+3)} = 1+\frac{9}{5(x-2)}-\frac{4}{5(x+3)}$$

Q.8 $\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$

Solution: $\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$ is an improper fraction.

First we resolve it into proper fraction.

$$3x^2-2x-1\overline{\big)6x^3+5x^2-7}\quad 2x+3$$
$$\underline{\pm 6x^3 \mp 4x^2 \mp 2x}$$
$$9x^2+2x-7$$
$$\underline{\pm 9x^2 \mp 6x \mp 3}$$
$$8x-4$$

$$\frac{6x^3+5x^2-7}{3x^2-2x-1} = (2x+3)+\frac{8x-4}{(3x+1)(x-1)}$$

Now, Let $\dfrac{8x-4}{(3x+1)(x-1)} = \dfrac{A}{3x+1}+\dfrac{B}{x-1}$ ....(i)

Multiplying both sides by(3x+1) (x–1), we get

$$8x - 4 = A(x-1) + B(3x+1) \text{......} \quad \text{(ii)}$$

As equation (ii) is an identity which is true for all values of x.

Let $x - 1 = 0$ i.e $x = 1$

and $3x + 1 = 0$ i.e $x = -\dfrac{1}{3}$

Putting $x = 1$ and $x = \dfrac{-1}{3}$ in equation (ii) we get

For $x = 1$	For $x = \dfrac{-1}{3}$
$8(1) - 4 = B[3(1)+1]$	$8\left(\dfrac{-1}{3}\right)-4 = A\left(\dfrac{-1}{3}-1\right)$
$-4 = 4B$	$\dfrac{-8}{3}-4 = \dfrac{A(-1-3)}{3}$
$4 = 4B$	$\dfrac{-8-12}{3} = \dfrac{A(-4)}{3}$
$\Rightarrow 4B = 4$	
$B = \dfrac{4}{4}$	$\dfrac{-20}{3} = \dfrac{-4}{3}A$
$\Rightarrow \boxed{B = 1}$	$\Rightarrow \boxed{A = 5}$

Hence the required partial functions are

$$\frac{6x^3+5x^2-7}{3x^2-2x-1} = 2x+3+\frac{5}{3x+1}+\frac{1}{x-1}$$

Rule II:

Resolution of a fraction when D(x) consists of repeated linear factors:

If a linear factor (ax + b) occurs n times as a factor of D(x), then there are n partial fractions of the form.

$$\frac{A_1}{(ax+b)}+\frac{A_2}{(ax+b)^2}+\dots+\frac{A_n}{(ax+b)^n}$$, where

$A_1, A_2, \dots, A_n$ are constants and $n \geq 2$ is a positive integer.

$$\therefore \quad \frac{N(x)}{D(x)}=\frac{A_1}{(ax+b)}+\frac{A_2}{(ax+b)^2}+\dots+\frac{A_n}{(ax+b)^n}$$

Example:

Resolve $\frac{1}{(x-1)^2(x-2)}$ into partial fractions.

Solution: Let,

$$\frac{1}{(x-1)^2(x-2)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x-2}$$

Multiplying both sides by $(x-1)^2(x-2)$, we get

$$1=A(x-1)(x-2)+B(x-2)+C(x-1)^2$$

$$A(x^2-3x+2)+B(x-2)+C(x^2-2x+1)=1 \quad \dots\text{(i)}$$

Since (i) is an identity and is true for all values of x

Put $x-1=0$ or $x=1$ in (i), we get

$$B(1-2)=1$$

$$\Rightarrow -B=1 \text{ or } \boxed{B=-1}$$

Put $x-2=0$ or $x=2$ in (i), we get

$$C(2-1)^2=1$$

$$C(1)=1 \quad \Rightarrow \quad \boxed{C=1}$$

Equating coefficients of x^2 on both the sides of (i)

$$A+C=0$$

$$\Rightarrow \quad A=-C$$

$$A=-(1) \quad \Rightarrow \quad \boxed{A=-1}$$

Hence required partial fractions are

$$\frac{-1}{x-1}-\frac{1}{(x-1)^2}+\frac{1}{(x-2)}$$

Thus, $$\frac{1}{(x-1)^2(x-2)}=\frac{1}{x-2}-\frac{1}{(x-1)}-\frac{1}{(x-1)^2}$$

EXERCISE 4.2

Resolve into partial fractions:

Q.1 $\dfrac{x^2-3x+1}{(x-1)^2(x-2)}$

Solution:

Let $\dfrac{x^2-3x+1}{(x-1)^2(x-2)}=\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x-2}$ ...(i)

Multiplying both sides by $(x-1)^2(x-2)$, we get

$x^2-3x+1= A(x-1)(x-2)+B(x-2)+C(x-1)^2$ ..(ii)

$x^2-3x+1= A(x^2-3x+2)+B(x-2)+C(x^2-2x+1)$

Putting $x-1=0$ i.e $x=1$ in (ii) we get

$$(1)^2-3(1)+1 = (1-2)$$
$$1-3+1 = B(-1)$$
$$-1 = -B$$
$$\Rightarrow B = 1$$

Putting $x-2=0$ i.e $x = 2$ in (ii) we get

$$(2)^2-3(2)+1 = C(2-1)^2$$
$$4-6+1 = C$$
$$-1 = C$$

Equating the coefficient of x^2 in (ii) we get

$$1=A+B$$
$$1 = A-1$$
$$\Rightarrow A = 1+1$$
$$\boxed{A = 2}$$

Hence the required partial fractions are

$$\frac{x^2-3x+1}{(x-1)^2(x-2)} = \frac{2}{x-1}+\frac{1}{(x-1)^2}-\frac{1}{x-2}$$

Q.2 $\dfrac{x^2+7x+11}{(x+2)^2(x+3)}$

Solution:

Let $\dfrac{x^2+7x+11}{(x+2)^2(x+3)}=\dfrac{A}{x+2}+\dfrac{B}{(x+2)^2}+\dfrac{C}{x+3}$ ...(i)

Multiplying both sides by $(x+2)^2(x+3)$

$\Rightarrow x^2+7x+11=A(x+2)(x+3)+B(x+3)+C(x+2)^2$

$x^2+7x+11=A(x^2+5x+6)+B(x+3)+C(x^2+4x+4)$..(ii)

Putting $x+2=0$ i.e $x=-2$ in (ii) we get

$$(-2)^2+7(-2)+11=B(-2+3)$$
$$4-14+11 = B$$
$$\Rightarrow \boxed{B = 1}$$

Putting $x+3=0$ i.e $x=-3$ in (ii) we get

$$(-3)^2+7(-3)+11=C(-3+2)^2$$
$$9-21+11=C(-1)^2$$
$$20-21=C(1)$$
$$-1=C \quad \Rightarrow \quad \boxed{C=-1}$$

Equating coefficient of x^2 in (ii) we get

$$A+C=1$$
$$A-1=1$$
$$A=1+1$$
$$\boxed{A=2}$$

Hence the required partial fractions are:

$$\frac{x^2+7x+11}{(x+2)^2(x+3)}=\frac{2}{x+2}+\frac{1}{(x+2)^2}-\frac{1}{x+3}$$

Q.3 $\frac{9}{(x-1)(x+2)^2}$

Solution:

Let $\frac{9}{(x-1)(x+2)^2}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$ .... (i)

Multiplying both sides by $(x-1)(x+2)^2$, we get

$9=A(x+2)^2+B(x-1)(x+2)+C(x-1)$..(ii)

Putting $x-1=0$ i.e $x=1$ in (ii) we get

$$9=A(1+2)^2$$
$$9=A(3)^2$$
$$9=9A$$
$$\Rightarrow \quad \boxed{A=1}$$

Putting $x+2=0$ i.e $x=-2$ in (ii) we get

$$9=C(-2-1)$$
$$9=-3C$$
$$\Rightarrow \quad \boxed{C=-3}$$

Equating the coefficient of x^2 in (ii) we get

$$A+B=0$$
$$B=-A$$
$$\boxed{B=-1}$$

Hence the partial fractions are

$$\frac{9}{(x-1)(x+2)^2}=\frac{1}{x-1}-\frac{1}{x+2}-\frac{3}{(x+2)^2}$$

Q.4 $\frac{x^4+1}{x^2(x-1)}$

Solution: $\frac{x^4+1}{x^2(x-1)}=\frac{x^4+1}{x^3-x^2}$ is an improper fraction. First we resolve it into proper fraction.

$$\begin{array}{r} x+1 \\ x^3-x^2 \,\big)\, \overline{x^4 \qquad +1} \\ \underline{\pm x^4 \quad \mp x^3} \\ x^3 \quad +1 \\ \underline{\pm x^3 \quad \mp x^2} \\ x^2+1 \end{array}$$

$$\frac{x^4+1}{x^2(x-1)}=(x+1)+\frac{x^2+1}{x^2(x-1)} \quad \text{........ (i)}$$

Let $\frac{x^2+1}{x^2(x-1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}$ .... (ii)

Multiplying both sides by $x^2(x-1)$ we get

$x^2+1=A(x)(x-1)+B(x-1)+Cx^2$......(iii)

Putting $x=0$ in (iii) we get

$$0+1=B(0-1)$$
$$1=-B$$
$$\Rightarrow \quad \boxed{B=-1}$$

Putting $x-1=0$ i.e $x=1$ in (iii) we get

$$(1)^2+1=C(1)^2$$
$$1+1=C(1)$$
$$2=C$$
$$\Rightarrow \quad \boxed{C=2}$$

Equating the coefficient of x^2 in (iii) we get

$$A+C=1$$
$$A+2=1$$
$$A=1-2$$
$$\Rightarrow \quad \boxed{A=-1}$$

Putting the value of A, B and C in equation(ii)

Thus required partial fractions are

$$\frac{x^4+1}{x^2(x-1)}=(x+1)-\frac{1}{x}-\frac{1}{x^2}+\frac{2}{x-1}$$

Q.5 $\frac{7x+4}{(3x+2)(x+1)^2}$

Solution: $\frac{7x+4}{(3x+2)(x+1)^2}$

Let $\frac{7x+4}{(3x+2)(x+1)^2}=\frac{A}{3x+2}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$ .....(i)

Multiplying both sides by $(3x+2)(x+1)^2$ we get

$7x+4=A(x+1)^2+B(3x+2)(x+1)+C(3x+2)$... (ii)

Putting $3x+2=0$ i.e $x=\frac{-2}{3}$ in (ii) we get

$$7\left(\frac{-2}{3}\right)+4 = A\left(\frac{-2}{3}+1\right)^2$$

$$\frac{-14}{3}+4 = A\left(\frac{-2+3}{3}\right)^2$$

$$\frac{-14+12}{3} = A\left(\frac{1}{3}\right)^2$$

$$\frac{-2}{3} = \frac{1}{9}A$$

$$-18 = 3A$$

$$A = \frac{-18}{3}$$

$$\Rightarrow \quad \boxed{A=-6}$$

Putting $x+1 = 0$ i.e $x = -1$ in (ii) we get

$$7(-1)+4 = C(3(-1)+2)$$

$$-7+4 = -C$$

$$\Rightarrow \quad -3 = -C$$

$$\Rightarrow \quad \boxed{C=3}$$

Equating the coefficient of x^2 we get

$$A+3B=0$$

$$-6+3B=0$$

$$3B=6$$

$$B=\frac{6}{3} \quad \Rightarrow \quad \boxed{B=2}$$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{7x+4}{(3x+2)(x+1)^2} = \frac{-6}{3x+2}+\frac{2}{x+1}+\frac{3}{(x+1)^2}$$

Q.6 $\quad \dfrac{1}{(x-1)^2(x+1)}$

Solution: $\dfrac{1}{(x-1)^2(x+1)}$

Let $\dfrac{1}{(x-1)^2(x+1)} = \dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x+1}$ .....(i)

Multiplying both sides by $(x-1)^2(x+1)$ we get

$1 = A(x-1)(x+1)+B(x+1)+C(x-1)^2$...(ii)

Putting $x-1=0$ i.e $x=1$ in (ii) we get

$$1 = B(1+1)$$

$$1 = 2B \quad \Rightarrow \quad \boxed{B=\frac{1}{2}}$$

Putting $x+1=0$ i.e $x=-1$ in (ii) we get

$$1 = C(-1-1)^2$$

$$1 = C(-2)^2$$

$$1 = 4C \quad \Rightarrow \quad \boxed{C=\frac{1}{4}}$$

Equating the coefficient of x^2 in (ii) we get

$$A+C=0$$

$$A=-C$$

$$A=-\left(\frac{1}{4}\right) \Rightarrow \boxed{A=\frac{-1}{4}}$$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{1}{(x-1)^2(x+1)} = \frac{-1}{4(x-1)}+\frac{1}{2(x-1)^2}+\frac{1}{4(x+1)}$$

Q.7 $\quad \dfrac{3x^2+15x+16}{(x+2)^2}$

Solution: $\dfrac{3x^2+15x+16}{(x+2)^2} = \dfrac{3x^2+15x+16}{x^2+4x+4}$

The given fraction is improper fraction. First we resolve it into proper fraction.

By long division,

$$\begin{array}{r|l} & 3 \\ x^2+4x+4 & 3x^2+15x+16 \\ & \pm 3x^2 \pm 12x \pm 12 \\ \hline & 3x+4 \end{array}$$

$$\frac{3x^2+15x+16}{(x+2)^2} = 3+\frac{3x+4}{x^2+4x+4} \quad ....(i)$$

Let $\dfrac{3x+4}{(x+2)^2} = \dfrac{A}{x+2}+\dfrac{B}{(x+2)^2}$ ....(ii)

Multiplying both sides by $(x+2)^2$ we get

$3x+4 = A(x+2)+B$....... (iii)

Putting $x+2=0$ i.e $x=-2$ in (iii) we get

$$3(-2)+4 = B$$

$$-6+4 = B$$

$$\Rightarrow \quad \boxed{B=-2}$$

Equating the coefficient of 'x' we get

$$3 = A$$

$$\Rightarrow \quad \boxed{A=3}$$

Putting the value of A and B in equation (ii) and using equation (i) we get required partial fractions.

$$\frac{3x^2+15x+16}{(x+2)^2} = 3+\frac{3}{x+2}-\frac{2}{(x+2)^2}$$

Q.8 $\frac{1}{(x^2-1)(x+1)}$

Solution: $\frac{1}{(x^2-1)(x+1)}=\frac{1}{(x-1)(x+1)(x+1)}$

$$=\frac{1}{(x-1)(x+1)^2}$$

Let $\frac{1}{(x-1)(x+1)^2}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$ ....(i)

Multiplying both sides by $(x-1)(x+1)^2$ we get

$1= A(x+1)^2 + B(x+1)(x-1)+C(x-1)$ .....(ii)

Putting $x-1=0$ i.e $x=1$ in (ii) we get

$1 = A(1+1)^2$

$1 = A(2)^2$

$1 = 4A$

$\Rightarrow$ $\boxed{A = \frac{1}{4}}$

Putting $x+1=0$ i.e $x=-1$ in (ii) we get

$1 = C(-1-1)$

$1 = -2C$

$\Rightarrow$ $\boxed{C = \frac{-1}{2}}$

Equating the coefficient of x^2 in equation (ii) we get $A+B=0$

$B = -A$

$B = -\left(\frac{1}{4}\right)$

$\boxed{B = -\frac{1}{4}}$

Putting the value of A and B in equation (ii) we get required partial fractions.

$$\frac{1}{(x-1)(x+1)^2}=\frac{1}{4(x-1)}-\frac{1}{4(x+1)}-\frac{1}{2(x+1)^2}$$

Rule III:

Resolution of fraction when D(x) consists of non-repeated irreducible quadratic factors:

If a quadratic factor (ax^2+bx+c) with $a \neq 0$ occurs once as a factor of D(x), the partial fraction is of the form $\frac{Ax+B}{(ax^2+bx+c)}$, where A and B are constants to be found.

Example:

Resolve $\frac{11x+3}{(x-3)(x^2+9)}$ into partial fractions.

Solution:

Let, $\frac{11x+3}{(x-3)(x^2+9)}=\frac{A}{(x-3)}+\frac{Bx+C}{x^2+9}$

Multiplying both sides by $(x-3)(x^2+9)$

$\Rightarrow$ $11x+3 = A(x^2+9)+(Bx+C)(x-3)$

$\Rightarrow$ $11x+3 = A(x^2+9)+B(x^2-3x)+C(x-3)$ ......(i)

Since (i) is an identity, we have on substituting

$x-3=0 \Rightarrow x=3$

Put $x=3$ in equation (i)

$33+3 = A(9+9)$

$36 = A(18)$

$\Rightarrow$ $18A = 36$

$\Rightarrow$ $\boxed{A=2}$

Comparing the coefficients of x^2 and x on both the sides of (i), we get

$A+B=0$

$B=-A$

$B=-(2)$

$\Rightarrow$ $\boxed{B=-2}$

$-3B+C=11$

$\Rightarrow$ $-3(-2)+C=11$

$6+C=11$

$C=11-6$

$\Rightarrow$ $\boxed{C=5}$

Putting the value of A, B and C, we get required partial fractions.

$$\frac{11x+3}{(x-3)(x^2+9)}=\frac{2}{x-3}+\frac{-2x+5}{x^2+9}$$

EXERCISE 4.3

Resolve into partial fractions.

Q.1 $\dfrac{3x-11}{(x+3)(x^2+1)}$

Solution: $\dfrac{3x-11}{(x+3)(x^2+1)}$

Let $\dfrac{3x-11}{(x+3)(x^2+1)} = \dfrac{A}{x+3} + \dfrac{Bx+C}{x^2+1}$ ... (i)

Multiplying both sides (x+3) (x^2+1), we get

$3x - 11 = A(x^2+1) + (Bx+C)(x+3)$... (ii)

$3x - 11 = A(x^2+1) + Bx(x+3) + C(x+3)$... (iii)

Putting x + 3 = 0 i.e x = –3 in (ii), we get

$$3(-3) - 11 = A[(-3)^2+1]$$
$$-9-11 = A(9+1)$$
$$-20 = 10A$$
$$A = \frac{-20}{10}$$
$$\Rightarrow \boxed{A = -2}$$

Now equating the coefficients of x^2 and x we get from equation (iii)

$A + B = 0$
$-2 + B = 0$
$B = 2$
$\Rightarrow \boxed{B = 2}$

$3B + C = 3$
$3(2) + C = 3$
$6 + C = 3$
$C = 3 - 6$
$\Rightarrow \boxed{C = -3}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{3x-11}{(x+3)(x^2+1)} = \frac{-2}{x+3} + \frac{2x-3}{x^2+1}$$

Q.2 $\dfrac{3x+7}{(x^2+1)(x+3)}$

Solution: $\dfrac{3x+7}{(x^2+1)(x+3)}$

Let $\dfrac{3x+7}{(x^2+1)(x+3)} = \dfrac{Ax+B}{x^2+1} + \dfrac{C}{x+3}$......(i)

Multiplying both sides by (x^2+1) (x + 3)

$3x + 7 = (Ax + B)(x + 3) + C(x^2+1)$

$3x+7 = Ax(x+3) + B(x+3) + C(x^2+1)$ .....(ii)

Putting x + 3 = 0 i.e x = – 3 in (ii), we get

$$3(-3)+7 = C[(-3)^2+1]$$
$$-9+7 = C(9+1)$$
$$-2 = 10C$$
$$\Rightarrow C = \frac{-2}{10}$$
$$\boxed{C = \frac{-1}{5}}$$

Now equating the coefficients of x^2 and x in equation (iii) we get

$A + C = 0$
$A + \left(\dfrac{-1}{5}\right) = 0$
$A - \dfrac{1}{5} = 0$
$\Rightarrow \boxed{A = \dfrac{1}{5}}$

$3A + B = 3$
$3\left(\dfrac{1}{5}\right) + B = 3$
$B = 3 - \dfrac{3}{5}$
$B = \dfrac{15-3}{5}$
$\Rightarrow \boxed{B = \dfrac{12}{5}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{3x+7}{(x^2+1)(x+3)} = \frac{x+12}{5(x^2+1)} - \frac{1}{5(x+3)}$$

Q.3 $\dfrac{1}{(x+1)(x^2+1)}$

Solution: $\dfrac{1}{(x+1)(x^2+1)}$

Let $\dfrac{1}{(x+1)(x^2+1)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+1}$ ....(i)

Multiplying both sides by $(x+1)(x^2+1)$, we get

$$1 = A(x^2+1) + (Bx+C)(x+1)$$

$$1 = A(x^2+1) + Bx(x+1) + C(x+1) \ldots \text{(ii)}$$

Putting $x + 1 = 0$ i.e $x = -1$ in (ii), we get

$$1 = A[(-1)^2 + 1]$$

$$1 = A(1+1)$$

$$1 = 2A$$

$$\boxed{A = \frac{1}{2}}$$

Equating the coefficients of x^2 and x in equation (ii) we get

$A + B = 0$	$B + C = 0$
$\frac{1}{2} + B = 0$	$-\frac{1}{2} + C = 0$
$\Rightarrow \boxed{B = -\frac{1}{2}}$	$\Rightarrow \boxed{C = \frac{1}{2}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{1}{(x+1)(x^2+1)} = \frac{1}{2(x+1)} - \frac{x-1}{2(1+x^2)}$$

Q.4 $\dfrac{9x-7}{(x+3)(x^2+1)}$

Solution: $\dfrac{9x-7}{(x+3)(x^2+1)}$

Let $\dfrac{9x-7}{(x+3)(x^2+1)} = \dfrac{A}{x+3} + \dfrac{Bx+C}{x^2+1}$ .....(i)

Multiplying both sides by $(x+3)(x^2+1)$ we get

$$9x - 7 = A(x^2+1) + (Bx+C)(x+3)$$

$$9x - 7 = A(x^2+1) + Bx(x+3) + C(x+3) \ldots \text{(ii)}$$

Putting $x+3 = 0$ i.e $x = -3$ in (ii), we get

$$9(-3) - 7 = A[(-3)^2 + 1]$$

$$-27 - 7 = A(9+1)$$

$$-34 = 10A$$

$$\Rightarrow A = \frac{-34}{10} \Rightarrow \boxed{A = \frac{-17}{5}}$$

Equating coefficients of x^2 and x in equation (ii) we get

$A + B = 0$	$3B + C = 9$
$\frac{-17}{5} + B = 0$	$3\left(\frac{17}{5}\right) + C = 9$
$\Rightarrow \boxed{B = \frac{17}{5}}$	$\frac{51}{5} + C = 9$
	$C = 9 - \frac{51}{5}$
	$C = \frac{45-51}{5}$
	$\Rightarrow \boxed{C = \frac{-6}{5}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{-17}{5(x+3)} + \frac{17x-6}{5(x^2+1)}$$

Q.5 $\dfrac{3x+7}{(x+3)(x^2+4)}$

Solution: $\dfrac{3x+7}{(x+3)(x^2+4)}$

Let $\dfrac{3x+7}{(x+3)(x^2+4)} = \dfrac{A}{x+3} + \dfrac{Bx+C}{x^2+4}$ ... (i)

Multiplying both sides by $(x+3)(x^2+4)$ we get

$3x + 7 = A(x^2+4) + (Bx+C)(x+3)$

$3x + 7 = A(x^2+4) + Bx(x+3) + C(x+3)$ ...(ii)

Putting $x + 3 = 0$ i.e $x = -3$ in (ii) we get

$$3(-3)+7 = A((-3)^2+4)$$
$$-9+7 = A(9+4)$$
$$-2 = 13A$$

$\Rightarrow$ $\boxed{A = \dfrac{-2}{13}}$

Equating the coefficients of x^2 and x in equation (ii) we get

$A + B = 0$

$\dfrac{-2}{13} + B = 0$

$B = \dfrac{2}{13}$

$\Rightarrow$ $\boxed{B = \dfrac{2}{13}}$

$3B + C = 3$

$3\left(\dfrac{2}{13}\right) + C = 3$

$\dfrac{6}{13} + C = 3$

$C = 3 - \dfrac{6}{13}$

$C = \dfrac{39-6}{13}$

$\Rightarrow$ $\boxed{C = \dfrac{33}{13}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{3x+7}{(x+3)(x^2+4)} = \frac{-2}{13(x+3)} + \frac{2x+33}{13(x^2+4)}$$

Q.6 $\dfrac{x^2}{(x+2)(x^2+4)}$

Solution: $\dfrac{x^2}{(x+2)(x^2+4)}$

Let $\dfrac{x^2}{(x+2)(x^2+4)} = \dfrac{A}{x+2} + \dfrac{Bx+C}{x^2+4}$ ....(i)

Multiplying both sides by $(x+2)(x^2+4)$ we get

$x^2 = A(x^2+4) + (Bx+C)(x+2)$

$x^2 = A(x^2+4) + Bx(x+2) + C(x+2)$ .... (ii)

Putting $x + 2 = 0$ i.e $x = -2$ in (ii) we get

$$(-2)^2 = A[(-2)^2+4]$$
$$4 = A(4+4)$$
$$4 = 8A$$

$\Rightarrow A = \dfrac{4}{8}$

$\boxed{A = \dfrac{1}{2}}$

Equating the coefficients of x^2 and x in equation (ii) we get

$A + B = 1$

$\dfrac{1}{2} + B = 1$

$B = 1 - \dfrac{1}{2}$

$\Rightarrow$ $\boxed{B = \dfrac{1}{2}}$

$2B + C = 0$

$2\left(\dfrac{1}{2}\right) + C = 0$

$1 + C = 0$

$\Rightarrow$ $\boxed{C = -1}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{1}{2(x+2)} + \frac{x-2}{2(x^2+4)}$$

Q.7 $\frac{1}{x^3+1}$

Solution: $\frac{1}{x^3+1}$

$$\frac{1}{x^3+1} = \frac{1}{(x+1)(x^2-x+1)}$$

Let $\frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$ .....(i)

Multiplying both sides by $(x+1)(x^2-x+1)$, we get

$1 = A(x^2-x+1) + (Bx+C)(x+1)$

$1 = A(x^2-x+1) + Bx(x+1) + C(x+1)$...(ii)

Putting $x+1=0$ i.e $x=-1$ in (ii) we get

$1 = A[(-1)^2-(-1)+1]$

$1 = A(1+1+1)$

$1 = 3A$

$\Rightarrow$ $\boxed{A = \frac{1}{3}}$

Comparing the coefficients of x^2 and x in equation (ii) we get

$A + B = 0$	$-A + B + C = 0$
$\frac{1}{3} + B = 0$	$-\frac{1}{3} - \frac{1}{3} + C = 0$
$\Rightarrow B = \frac{-1}{3}$	$\frac{-2}{3} + C = 0$
$\boxed{B = \frac{-1}{3}}$	$\Rightarrow \boxed{C = \frac{2}{3}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{1}{3(x+1)} - \frac{x-2}{3(x^2-x+1)}$$

Q.8 $\frac{x^2+1}{x^3+1}$

Solution: $\frac{x^2+1}{x^3+1}$

$$\frac{x^2+1}{x^3+1} = \frac{x^2+1}{(x+1)(x^2-x+1)}$$

Let $\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$ .....(i)

Multiplying both sides by $(x+1)(x^2-x+1)$, we get

$x^2+1 = A(x^2-x+1) + (Bx+C)(x+1)$

$x^2+1 = A(x^2-x+1) + Bx(x+1) + C(x+1)$...(ii)

Putting $x+1=0$ i.e $x=-1$ in (ii) we get

$(-1)^2+1 = A[(-1)^2-(-1)+1]$

$1+1 = A(1+1+1)$

$2 = 3A$

$\Rightarrow$ $\boxed{A = \frac{2}{3}}$

Equating the coefficients of x^2 and x in equation (ii) we get

$A + B = 1$	$-A + B + C = 0$
$\frac{2}{3} + B = 1$	$\frac{-2}{3} + \frac{1}{3} + C = 0$
$B = 1 - \frac{2}{3}$	$\frac{-1}{3} + C = 0$
$\Rightarrow \boxed{B = \frac{1}{3}}$	$\Rightarrow \boxed{C = \frac{1}{3}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{x^2+1}{x^3+1} = \frac{2}{3(x+1)} + \frac{x+1}{3(x^2-x+1)}$$

Rule IV:

Resolution of a fraction when D(x) has repeated irreducible quadratic factors:

If a quadratic factor $(ax^2 + bx + c)$ with $a \neq 0$, occurs twice in the denominator, the corresponding partial fractions are

$$\frac{Ax+B}{(ax^2+bx+c)}+\frac{Cx+D}{(ax^2+bx+c)^2}$$

The constants A, B, C and D are found in the usual way.

Example 1:

Resolve $\frac{x^3-2x^2-2}{(x^2+1)^2}$ **into partial fractions.**

Solution: $\frac{x^3-2x^2-2}{(x^2+1)^2}$ is a proper fraction as degree of numerator is less than the degree of denominator.

Let, $$\frac{x^3-2x^2-2}{(x^2+1)^2} = \frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}$$

Multiplying both the sides by $(x^2+1)^2$, we get

$$x^3 - 2x^2 - 2 = (Ax + B)(x^2+1) + Cx + D$$

$$x^3-2x^2 - 2 = A(x^3 + x)+B(x^2 + 1)+Cx + D \dots \text{(i)}$$

Equating the coefficients of x^3, x^2, x and constant on both the sides of (i)

Coefficients of x^3: $A = 1$

Coefficients of x^2: $B = -2$

Coefficients of x: $A + C = 0$

$C = -A$

$\Rightarrow$ $C = -1$

$\boxed{C=-1}$

Constants: $B + D = -2$

$D = -2 - B$

$D = -2-(-2)$

$D = -2 + 2$

$\boxed{D=0}$

Putting the value of A, B C and D, we get required partial fractions.

Thus $$\frac{x^3-2x^2-2}{(x^2+1)^2} = \frac{x-2}{x^2+1}+\frac{-x+0}{(x^2+1)^2}$$

$$= \frac{x-2}{x^2+1}-\frac{x}{(x^2+1)^2}$$

Example 2:

Resolve $\frac{2x+1}{(x-1)(x^2+1)^2}$ **into partial fractions.**

Solution: Assume that

$$\frac{2x+1}{(x-1)(x^2+1)^2} = \frac{A}{x-1}+\frac{Bx+C}{x^2+1}+\frac{Dx+E}{(x^2+1)^2}$$

Multiplying both sides by $(x-1)(x^2+1)^2$

we get

$$2x + 1 = A(x^2+1)^2 + (Bx+ C)(x - 1)(x^2 + 1) + (Dx + E)(x - 1) \dots\dots\text{(i)}$$

Now we use zeros, method. Put $x-1 = 0$ or $x= 1$ in (i), we get

$2(1) +1 = [(1)^2 + 1]^2$

$3 = A(1 + 1)^2$

$3 = A(2)^2$

$3 = 4A$

$\Rightarrow \boxed{A = \frac{3}{4}}$

Now writing terms of (i) in descending order.

$$2x + 1 = A(x^4+2x^2+1) + Bx(x^3- x^2 + x - 1) + C(x^3- x^2 + x -1)+D(x^2 - x) + E(x - 1)$$

or $$2x + 1= A(x^4+2x^2+1) +B(x^4-x^3+x^2 - x) + C(x^3 - x^2+x-1)+D(x^2 - x)+E(x - 1)$$

Equating coefficients of x^4, x^3, x^2, and x on both the sides.

Coefficients of x^4: $A + B = 0$

$$B = -A$$

$$\Rightarrow \quad B = \frac{-3}{4}$$

Coefficients of x^3: $-B + C = 0$

$$C = B$$

$$\Rightarrow \quad C = \frac{-3}{4}$$

Coefficients of x^2 : $2A+B-C+D = 0$

$$2\left(\frac{3}{4_2}\right)+\left(\frac{-3}{4}\right)-\left(\frac{-3}{4}\right)+D=0$$

$$\left(\frac{3}{2}\right)-\frac{3}{4}+\frac{3}{4}+D=0$$

$$\Rightarrow \quad D = \frac{-3}{2}$$

Coefficients of x: $-B + C - D + E = 2$

$$\frac{3}{4}-\frac{3}{4}+\frac{3}{2}+E=2$$

$$E = 2-\frac{3}{2}=\frac{4-3}{2}$$

$$\Rightarrow \quad \boxed{E=\frac{1}{2}}$$

Thus required partial fraction are

$$\frac{3}{4(x-1)}+\frac{\frac{-3}{4}x-\frac{3}{4}}{x^2+1}+\frac{\frac{-3}{2}x+\frac{1}{2}}{(x^2+1)^2}$$

$$\therefore \frac{2x+1}{(x-1)(x^2+1)^2}=\frac{3}{4(x-1)}-\frac{3(x+1)}{4(x^2+1)}-\frac{(3x-1)}{2(x^2+1)^2}$$

Equating coefficients of x^4, x^3, x^2, and x on both the sides.

Coefficients of x^4: $A + B = 0$

$$B = -A$$

$$\Rightarrow \quad B = \frac{-3}{4}$$

Coefficients of x^3: $-B + C = 0$

$$C = B$$

$$\Rightarrow \quad C = \frac{-3}{4}$$

Coefficients of x^2 : $2A+B-C+D = 0$

$$\cancel{2}\left(\frac{3}{\cancel{4}_2}\right)+\left(\frac{-3}{4}\right)-\left(\frac{-3}{4}\right)+D=0$$

$$\left(\frac{3}{2}\right)-\frac{\cancel{3}}{\cancel{4}}+\frac{\cancel{3}}{\cancel{4}}+D=0$$

$$\Rightarrow \quad D=\frac{-3}{2}$$

Coefficients of x: $-B + C - D + E = 2$

$$\frac{\cancel{3}}{\cancel{4}}-\frac{\cancel{3}}{\cancel{4}}+\frac{3}{2}+E=2$$

$$E=2-\frac{3}{2}=\frac{4-3}{2}$$

$$\Rightarrow \quad \boxed{E=\frac{1}{2}}$$

Thus required partial fraction are

$$\frac{3}{4(x-1)}+\frac{\frac{-3}{4}x-\frac{3}{4}}{x^2+1}+\frac{\frac{-3}{2}x+\frac{1}{2}}{(x^2+1)^2}$$

$$\therefore \frac{2x+1}{(x-1)(x^2+1)^2}=\frac{3}{4(x-1)}-\frac{3(x+1)}{4(x^2+1)}-\frac{(3x-1)}{2(x^2+1)^2}$$

EXERCISE 4.4

Q.1 $\dfrac{x^3}{(x^2+4)^2}$

Solution: $\dfrac{x^3}{(x^2+4)^2}$

Let $\dfrac{x^3}{(x^2+4)^2} = \dfrac{Ax+B}{x^2+4} + \dfrac{Cx+D}{(x^2+4)^2}$ .... (i)

Multiplying both sides by $(x^2 + 4)^2$, we get

$x^3 = (Ax+B)(x^2+4)+(Cx+D)$

$x^3 = Ax(x^2+4) + B(x^2+4) + (Cx+D)$ ....(ii)

Equating the coefficients of x^3, x^2, x and constants, we get

Coefficients of x^3: $A = 1$

Coefficients of x^2: $B = 0$

Coefficients of x: $4A + C = 0$

$4(1) + C = 0$

$\Rightarrow C = -4$

Constants: $4B + D = 0$

$4(0)+D = 0$

$\Rightarrow D = 0$

Putting the value of A,B,C and D in equation(i) we get required partial fractions.

$$\frac{x^3}{(x^2+4)^2} = \frac{x}{x^2+4} - \frac{4x}{(x^2+4)^2}$$

Q.2 $\dfrac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}$

Solution: $\dfrac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}$

Let $\dfrac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+1} + \dfrac{Dx+E}{(x^2+1)^2}$ ...(i)

Multiplying both sides by $(x+1)(x^2+1)^2$ we get

$x^4+3x^2+x+1 = A(x^2+1)^2 + (Bx+C)(x+1)(x^2+1)$

$$+(Dx+E)(x+1).....(ii)$$

$$x^4+3x^2+x+1 = A(x^4+2x^2+1)+Bx(x^3+x^2+x+1)$$
$$+C(x^3+x^2+x+1)+Dx(x+1)+E(x+1)$$
$$x^4+3x^2+x+1 = A(x^4+2x^2+1)+B(x^4+x^3+x^2+x)$$
$$+C(x^3+x^2+x+1)+D(x^2+x)+E(x+1).. \quad (iii)$$

Putting $x+1=0$ i.e $x=-1$ in eq.(ii), we get

$$(-1)^4+3(-1)^2+(-1)+1 = A[(-1)^2+1]^2$$
$$1+3(1)-1+1 = A(1+1)^2$$
$$4 = 4A$$
$$\Rightarrow \quad \boxed{A = 1}$$

Now equating the coefficients of x^4, x^3, x^2, x and constants, we get from equation (iii)

Coefficients of x^4: $A+B=1$

$$1+B=1$$
$$B=1-1$$
$$\Rightarrow \quad \boxed{B = 0}$$

Coefficients of x^3: $B+C=0$

$$0+C=0$$
$$\Rightarrow \quad \boxed{C = 0}$$

Coefficients of x^2: $2A+B+C+D=3$

$$2(1)+0+0+D=3$$
$$D=3-2$$
$$\boxed{D = 1}$$

Coefficients of x: $B+C+D+E=1$

$$0+0+1+E=1$$
$$E=1-1$$
$$\Rightarrow \quad \boxed{E = 0}$$

Putting the value of A, B, C and D in equation(i) we get required partial fractions.

$$\frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2} = \frac{1}{x+1}+\frac{x}{(x^2+1)^2}$$

Q.3 $\dfrac{x^2}{(x+1)(x^2+1)^2}$

Solution: $\dfrac{x^2}{(x+1)(x^2+1)^2}$

Let $\dfrac{x^2}{(x+1)(x^2+1)^2} = \dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+1}+\dfrac{Dx+E}{(x^2+1)^2}$ .. (i)

Multiply both sides by $(x+1)(x^2+1)^2$ we get

$$x^2 = A(x^2+1)^2+(Bx+C)(x+1)(x^2+1)$$
$$+(Dx+E)(x+1).......(ii)$$
$$x^2 = A(x^4+2x^2+1)+Bx(x^3+x^2+x+1)$$
$$+C(x^3+x^2+x+1)+Dx(x+1)+E(x+1)$$
$$x^2 = A(x^4+2x^2+1)+B(x^4+x^3+x^2+x)$$
$$+C(x^3+x^2+x+1)+D(x^2+x)+E(x+1)...(iii)$$

Putting $x+1=0$ i.e $x=-1$ in equation (ii) we get

$$(-1)^2 = A[(-1)^2+1]^2$$
$$1 = A(1+1)^2$$
$$1 = 4A \Rightarrow \boxed{A = \frac{1}{4}}$$

Now equating the coefficients of x^4, x^3, x^2, x and constants we get from equation (iii)

Coefficients of x^4: $A+B=0$

$$\frac{1}{4}+B=0 \Rightarrow \boxed{B = -\frac{1}{4}}$$

Coefficients of x^3 : $B+C=0$

$$-\frac{1}{4}+C=0 \Rightarrow \boxed{C = \frac{1}{4}}$$

Coefficients of x^2: $2A+B+C+D=1$

$$2\left(\frac{1}{4}\right)-\frac{1}{4}+\frac{1}{4}+D = 1$$
$$\frac{1}{2}+D = 1$$
$$D = 1-\frac{1}{2}$$

$$\boxed{D = \frac{1}{2}}$$

Coefficients of x: B + C + D + E = 0

$$-\frac{1}{4}+\frac{1}{4}+\frac{1}{2}+E = 0$$

$$\frac{1}{2}+E = 0$$

$$\boxed{E = -\frac{1}{2}}$$

Putting the value of A, B, C, D and E in equation(i) we get required partial fractions.

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{1}{4(x+1)} - \frac{x-1}{4(x^2+1)} + \frac{x-1}{2(x^2+1)^2}$$

Q.4 $\frac{x^2}{(x-1)(x^2+1)^2}$

Solution: $\frac{x^2}{(x-1)(x^2+1)^2}$

Let $\frac{x^2}{(x-1)(x^2+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$ ..(i)

Multiplying both sides by $(x-1)(x^2+1)^2$, we get

$$x^2 = A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) + (Dx+E)(x-1) \quad ....(ii)$$

$$x^2 = A(x^4+2x^2+1) + Bx(x-1)(x^2+1) + C(x-1)(x^2+1) + Dx(x-1) + E(x-1)$$

$$x^2 = A(x^4+2x^2+1) + B(x^4-x^3+x^2-x) + C(x^3-x^2+x-1) + D(x^2-x) + E(x-1) \quad ... (iii)$$

Putting x – 1 = 0 i.e x = 1 in equation (ii) we get

$$(1)^2 = A[(1)^2+1]^2$$

$$1 = A(1+1)^2$$

$$1 = 4A \Rightarrow \boxed{A = \frac{1}{4}}$$

Now equating the coefficients of x^4, x^3, x^2 and x in equation (iii) we get

Coefficients of x^4: A + B = 0

$$\frac{1}{4}+B = 0$$

$$\Rightarrow \boxed{B = -\frac{1}{4}}$$

Coefficients of x^3: – B + C = 0

$$-\left(-\frac{1}{4}\right)+C = 0$$

$$\Rightarrow C = -\frac{1}{4}$$

Coefficients of x^2: 2A + B – C + D = 1

$$2\left(\frac{1}{4}\right) - \frac{1}{4} - \left(\frac{-1}{4}\right) + D = 1$$

$$\frac{1}{2} - \frac{1}{4} + \frac{1}{4} + D = 1$$

$$D = 1 - \frac{1}{2}$$

$$D = \frac{2-1}{2}$$

$$\boxed{D = \frac{1}{2}}$$

Coefficients of x: – B + C – D + E = 0

$$-\left(-\frac{1}{4}\right) - \frac{1}{4} - \frac{1}{2} + E = 0$$

$$\frac{1}{4} - \frac{1}{4} - \frac{1}{2} + E = 0$$

$$\frac{-1}{2} + E = 0$$

$$\boxed{E = \frac{1}{2}}$$

Putting the value of A, B, C, D and E in equation(i) we get required partial fractions.

$$\frac{x^2}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} - \frac{x+1}{4(x^2+1)} + \frac{x+1}{2(x^2+1)^2}$$

Q.5 $\dfrac{x^4}{(x^2+2)^2}$

Solution: $\dfrac{x^4}{(x^2+2)^2}$

$\dfrac{x^4}{(x^2+2)^2} = \dfrac{x^4}{x^4+4x^2+4}$ is an improper fraction. First we resolve it into proper fraction.

$$\begin{array}{r|l} & \quad 1 \\ x^4+4x^2+4 & \quad x^4 \\ & \pm x^4 \pm 4x^2 \pm 4 \\ \hline & \quad -4x^2-4 \end{array}$$

$$\frac{x^4}{(x^2+2)^2} = 1 + \frac{-4x^2-4}{(x^2+2)^2}$$

Let $\dfrac{-4x^2-4}{(x^2+2)^2} = \dfrac{Ax+B}{x^2+2} + \dfrac{Cx+D}{(x^2+2)^2}$ .....(i)

Multiplying both sides by $(x^2+2)^2$ we get

$-4x^2-4 = (Ax+B)(x^2+2) + (Cx+D)$

$-4x^2-4 = A(x^3+2x) + B(x^2+2) + Cx + D$......(ii)

Equating the coefficients of x^3, x^2, x and constants in equation (ii) we get

Coefficients of x^3: $A = 0$

Coefficients of x^2: $B = -4$

Coefficients of x: $2A + C = 0$

$2(0) + C = 0$

$\Rightarrow \boxed{C = 0}$

Constants : $2B + D = -4$

$2(-4) + D = -4$

$-8 + D = -4$

$D = 8 - 4$

$\boxed{D = 4}$

Putting the value of A, B, C and D in equation(i) we get required partial fractions.

$$\frac{x^4}{(x^2+2)^2} = 1 + \frac{-4}{x^2+2} + \frac{4}{(x^2+2)^2}$$

$$\frac{x^4}{(x^2+2)^2} = 1 - \frac{4}{x^2+2} + \frac{4}{(x^2+2)^2}$$

Q.6 $\dfrac{x^5}{(x^2+1)^2}$

Solution: $\dfrac{x^5}{(x^2+1)^2}$

$\dfrac{x^5}{(x^2+1)^2} = \dfrac{x^5}{x^4+2x^2+1}$ is an improper fraction.

First we resolve it into proper fraction.

$$\begin{array}{r|l} & \quad x \\ x^4+2x^2+1 & \quad x^5 \\ & \pm x^5 \pm 2x^3 \pm x \\ \hline & \quad -2x^3-x \end{array}$$

$$\frac{x^5}{(x^2+1)^2} = x + \frac{-2x^3-x}{(x^2+1)^2}$$

Let $\dfrac{-2x^3-x}{(x^2+1)^2} = \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{(x^2+1)^2}$ ....(i)

Multiplying both sides by $(x^2+1)^2$ we get

$-2x^3-x = (Ax+B)(x^2+1) + (Cx+D)$

$-2x^3-x = A(x^3+x) + B(x^2+1) + Cx + D$

Equating the coefficients of x^3, x^2, x and constants we get

Coefficients of x^3 : $A = -2$

Coefficients of x^2 : $B = 0$

Coefficients of x : $A + C = -1$

$-2 + C = -1$

$C = -1 + 2$

$\Rightarrow \boxed{C = 1}$

Constants : $B + D = 0$

$0 + D = 0$

$\Rightarrow D = 0$

Hence the required partial fractions are

$$\frac{x^5}{(x^2+1)^2} = x + \frac{-2x}{x^2+1} + \frac{x}{(x^2+1)^2}$$

$$\Rightarrow \frac{x^5}{(x^2+1)^2} = x - \frac{2x}{x^2+1} + \frac{x}{(x^2+1)^2}$$

MISCELLANEOUS EXERCISE – 4

Q. 1 Multiple Choice Questions:

Four possible answers are given for the following questions. Tick (✓) the correct answer.

1. $(x+3)^2 = x^2 + 6x + 9$ is
   (a) a linear equation
   (b) an equation
   (c) an identity
   (d) none of these

2. $\dfrac{2x+1}{(x+1)(x-1)}$ is
   (a) an improper fraction
   (b) an equation
   (c) a proper fraction
   (d) none of these

3. $\dfrac{x^3+1}{(x-1)(x+2)}$ is
   (a) a proper fraction
   (b) an improper fraction
   (c) an identity
   (d) a constant term

4. **A fraction in which the degree of numerator is less than the degree of the denominator is called**
   (a) an equation
   (b) an improper fraction
   (c) an identity
   (d) a proper fraction

5. **A function of the form** $f(x) = \dfrac{N(x)}{D(x)}$, **with** $D(x) \neq 0$, **where** $N(x)$ **and** $D(x)$ **are polynomials in x is called**
   (a) an identity
   (b) an equation
   (c) a fraction
   (d) none of these

6. **The identity** $(5x+4)^2 = 25x^2 + 40x + 16$ **is true for**
   (a) one value of x
   (b) two values of x
   (c) all values of x
   (d) none of these

7. **A fraction in which the degree of the numerator is greater or equal to the degree of denominator is called**
   (a) a proper fraction
   (b) an improper fraction
   (c) an equation
   (d) algebraic relation

8. **Partial fractions of** $\dfrac{x-2}{(x-1)(x+2)}$ **are of the form**
   (a) $\dfrac{A}{x-1}+\dfrac{B}{x+2}$ (b) $\dfrac{Ax}{x-1}+\dfrac{B}{x+2}$
   (c) $\dfrac{A}{x-1}+\dfrac{Bx+C}{x+2}$ (d) $\dfrac{Ax+B}{x-1}+\dfrac{C}{x+2}$

9. **Partial fractions of** $\dfrac{x+2}{(x+1)(x^2+2)}$ **are of the form**
   (a) $\dfrac{A}{x+1}+\dfrac{B}{x^2+2}$
   (b) $\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+2}$
   (c) $\dfrac{Ax+B}{x+1}+\dfrac{C}{x^2+2}$
   (d) $\dfrac{A}{x+1}+\dfrac{Bx}{x^2+2}$

10. **Partial fractions of** $\dfrac{x^2+1}{(x+1)(x-1)}$ **are of the form**
   (a) $\dfrac{A}{x+1}+\dfrac{B}{x-1}$
   (b) $1+\dfrac{A}{x+1}+\dfrac{Bx+C}{x-1}$
   (c) $1+\dfrac{A}{x+1}+\dfrac{B}{x-1}$
   (d) $\dfrac{Ax+B}{(x+1)}+\dfrac{C}{x-1}$

ANSWER KEY

1.	c	2.	c	3.	b	4.	d	5.	c
6.	c	7.	b	8.	a	9.	b	10.	c

Q. 2 **Write short answers of the following questions:**

(i) **Define a rational fraction.**

An expression of the form $\frac{N(x)}{D(x)}$ with $D(x) \neq 0$ and N(x) and D(x) are polynomials in x with real coefficients, is called a rational fraction. Every fractional expression can be expressed as a quotient of two polynomials.

(ii) **What is a proper fraction?**

A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called a proper fraction if degree of the polynomial N(x) in the numerator is less than the degree of the polynomial D(x) in the denominator.

(iii) **What is an improper fraction?**

A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called an improper fraction if degree of the polynomial N(x) is greater or equal to the degree of the polynomial D(x) e.g $\frac{x^2+1}{x-1}$

(iv) **What are partial fractions?**

Every proper fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ can be resolved into an algebraic sum of components fractions. These components fractions of a resultant fraction are called its partial fractions.

(v) **How can we make partial fractions of $\frac{x-2}{(x+2)(x+3)}$?**

Solution: $\frac{x-2}{(x+2)(x+3)}$

Let $\frac{x-2}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$ ......(i)

Multiplying both sides by (x+2)(x+3), we get

$x - 2 = A(x+3) + B(x+2)$ .... (ii)

As both sides of the identity are equal for all values of x,

Put x + 2 = 0 i.e x = – 2 in equation (ii), we get

$-2 - 2 = A(-2+3)$

$-4 = A$

$\Rightarrow \boxed{A = -4}$

Now put x+3=0 i.e x= –3 in equation (ii) we get

$-3 - 2 = B(-3+2)$

$-5 = -B$

$\Rightarrow \boxed{B = 5}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{x-2}{(x+2)(x+3)} = \frac{-4}{x+2} + \frac{5}{x+3}$$

(vi) **Resolve $\frac{1}{x^2-1}$ into partial fractions.**

Solution: $\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)}$

Let $\frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$ .... (i)

Multiplying both sides By (x – 1) (x + 1), we get

$1 = A(x+1) + B(x-1)$ ....(ii)

As both sides of identity are equal for all values of x

Putting x– 1=0 i.e x = 1in equation (ii) we get

$1 = A(1+1)$

$1 = 2A$

$\Rightarrow \boxed{A = \frac{1}{2}}$

Putting x+1 = 0 i.e x = – 1 in equation (ii) we get

$1 = B(-1-1)$

$1 = -2B$

$\Rightarrow \boxed{B = -\frac{1}{2}}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{1}{x^2-1} = \frac{1}{2(x-1)} - \frac{1}{2(x+1)}$$

(vii) Find partial fractions of $\frac{3}{(x+1)(x-1)}$

Solution: $\frac{3}{(x+1)(x-1)}$

Let $\frac{3}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$ ....(i)

Multiplying both sides by (x+1) (x – 1), we get

$3 = A(x-1) + B(x+1)$............(ii)

As both sides of the identity are equal for all values of x.

Put x + 1= 0 i.e x = –1put in equation (ii) we get

$3 = A(-1-1)$

$3 = -2A \Rightarrow \boxed{A = \frac{-3}{2}}$

Now put x – 1 = 0 i.e x = 1 in equation (ii) we get

$\Rightarrow 3 = B(1+1)$

$3 = 2B \Rightarrow \boxed{B = \frac{3}{2}}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{3}{(x+1)(x-1)} = \frac{-3}{2(x+1)} + \frac{3}{2(x-1)} = \frac{3}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

(viii) Resolve $\frac{x}{(x-3)^2}$ into partial fractions.

Solution: $\frac{x}{(x-3)^2}$

Let $\frac{x}{(x-3)^2} = \frac{A}{x-3} + \frac{B}{(x-3)^2}$ ....(i)

Multiplying both sides by $(x-3)^2$, we get

$x = A(x-3) + B$ ....(ii)

As both sides of the identity are equal for all values of x,

Put x – 3 = 0 i.e x =3 in equation (ii) we get

$3 = B$

$\Rightarrow \boxed{B = 3}$

Now comparing the coefficients of x, we have

$\Rightarrow \boxed{A = 1}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{x}{(x-3)^2} = \frac{1}{x-3} + \frac{3}{(x-3)^2}$$

(ix) How we can make the partial fractions of $\frac{x}{(x+a)(x-a)}$?

Solution: $\frac{x}{(x+a)(x-a)}$

Let $\frac{x}{(x+a)(x-a)} = \frac{A}{x+a} + \frac{B}{x-a}$ ....(i)

Multiplying both sides by (x + a) (x – a), we get

$x = A(x-a) + B(x+a)$ .... (ii)

As both sides of the identity are equal for all values of x,

Put x + a = 0 i.e x = –a put in equation (ii) we get

$-a = A(-a-a)$

$-a = -2aA$

$\Rightarrow A = \frac{-a}{-2a}$

$\Rightarrow \boxed{A = \frac{1}{2}}$

Now put x – a = 0 i.e x = a in equation (ii) we get

$a = B(a+a)$

$a = 2aB$

$\Rightarrow B = \frac{a}{2a}$

$\Rightarrow \boxed{B = \frac{1}{2}}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{x}{(x+a)(x-a)} = \frac{1}{2(x+a)} + \frac{1}{2(x-a)}$$

$$= \frac{1}{2}\left(\frac{1}{x+a} + \frac{1}{x-a}\right)$$

(x) Whether $(x+3)^2 = x^2 + 6x + 9$ is an identity?

Answer:

Yes $(x+3)^2 = x^2 + 6x + 9$ is an identity because it is true for all the values of x.