Unit 05

SETS AND FUNCTIONS

SET

A collection of well defined distinct objects is called set. It is denoted by capital letters A, B, C etc.

Some Important Sets:

The following are commonly used notation for sets on the real line, which will be used subsequently. In set theory, we usually deal with the following sets of numbers denoted by standard symbols:

The set of natural numbers

$N = \{1, 2, 3, 4, \ldots\}$

The set of whole numbers

$W = \{0, 1, 2, 3, 4, \ldots\}$

The set of all integers

$Z = \{0, \pm1, \pm2, \pm3, \ldots\}$

The set of all even integers

$E = \{0, \pm2, \pm4, \ldots\}$

The set of all odd integers

$O = \{\pm1, \pm3, \pm5, \ldots\}$

The set of prime numbers

$P = \{2, 3, 5, 7, 11, 13, 17, \ldots\}$

The set of all rational numbers

$Q = \{x \mid x = \frac{m}{n}, \text{ where } m, n \in Z \text{ and } n \neq 0\}$

The set of all irrational numbers

$Q' = \{x \mid x \neq \frac{m}{n}, \text{ where } m, n \in Z \text{ and } n \neq 0\}$

The set of all real numbers

$R = Q \cup Q'$

Recognize Operations On Sets ($\cap, \cup, \backslash, \ldots$)

(a) Union of sets:

The union of two sets A and B written as $A \cup B$ (read as A union B) is the set consisting of all the elements which are either in A or in B or in both. Thus

$A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ or } x \in A \text{ and } B \text{ both}\}$

For example, if $A = \{1, 2, 3, 4\}$ and $B = \{4,5,6,7\}$, then $A \cup B = \{1,2,3,4,5,6,7\}$

(b) Intersection of sets:

The intersection of two sets A and B, written as $A \cap B$ (read as 'A intersection B') is the set consisting of all the common elements of A and B. thus

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Clearly $x \in A \cap B \Rightarrow x \in A$ and $x \in B$

For example, if $A = \{a, b, c, d\}$ and $B = \{c,d,e,f\}$, then $A \cap B = \{c,d\}$

(c) Difference of sets:

If A and B are two sets, then their difference $A - B$ or $A \backslash B$ is defined as:

$A - B = \{x \mid x \in A \text{ and } x \notin B\}$

Similarly $B - A = \{x \mid x \in B \text{ and } x \notin A\}$

For example, if

$A = \{1, 2, 3, 4, 5\}$ and $B = \{2,4,5,6,8\}$, then

$A - B = \{1, 2, 3, 4, 5\} - \{2, 4, 5, 6, 8\}$
$= \{1, 3\}$

Also $B - A = \{2, 4, 5, 6, 8\} - \{1, 2, 3, 4, 5\}$
$= \{6, 8\}$

(d) **Complement of a set**:

If U is a universal set and A is a subset of U, then the complement of A is the set of those elements of U, which are not contained in A and is denoted by A' or A^c.

$$A' = U - A = \{x \mid x \in U \text{ and } x \notin A\}$$

For example, if $U = \{1, 2, 3, \ldots, 10\}$ and $A = \{2, 4, 6, 8\}$, then

$A' = U - A$

$= \{1, 2, 3, \ldots, 10\} - \{2, 4, 6, 8\}$

$= \{1, 3, 5, 7, 9, 10\}$

Example:

If $U = \{1, 2, 3, \ldots, 10\}$,

$A = \{2, 3, 5, 7\}, B = \{3, 5, 8\}$, then

Find (i) $A \cup B$ (ii) $A \cap B$

(iii) $A - B$ (iv) A' and B'

Solution:

(i) $A \cup B = \{2, 3, 5, 7\} \cup \{3, 5, 8\}$

$= \{2, 3, 5, 7, 8\}$

(ii) $A \cap B = \{2, 3, 5, 7\} \cap \{3, 5, 8\}$

$= \{3, 5\}$

(iii) $A - B = \{2, 3, 5, 7\} - \{3, 5, 8\}$

$= \{2, 7\}$

(iv) $A' = U - A$

$= \{1, 2, 3, \ldots, 10\} - \{2, 3, 5, 7\}$

$= \{1, 4, 6, 8, 9, 10\}$

$B' = U - B$

$= \{1, 2, 3, \ldots, 10\} - \{3, 5, 8\}$

$= \{1, 2, 4, 6, 7, 9, 10\}$

EXERCISE 5.1

Q.1 If $X = \{1, 4, 7, 9\}$ and $Y = \{2, 4, 5, 9\}$ then find:

(i) $X \cup Y$ (ii) $X \cap Y$

(iii) $Y \cup X$ (iv) $Y \cap X$

Solution:

(i) $X \cup Y = \{1, 4, 7, 9\} \cup \{2, 4, 5, 9\}$

$= \{1, 2, 4, 5, 7, 9\}$

(ii) $X \cap Y = \{1, 4, 7, 9\} \cap \{2, 4, 5, 9\}$

$= \{4, 9\}$

(iii) $Y \cup X = \{2, 4, 5, 9\} \cup \{1, 4, 7, 9\}$

$= \{1, 2, 4, 5, 7, 9\}$

(iv) $Y \cap X = \{2, 4, 5, 9\} \cap \{1, 4, 7, 9\}$

$= \{4, 9\}$

Q.2 If X = Set of Prime numbers less than or equal to 17.
Y = Set of first 12 natural numbers, then find.

(i) $X \cup Y$ (ii) $X \cap Y$

(iii) $Y \cup X$ (iv) $Y \cap X$

Solution:

$X = \{2, 3, 5, 7, 11, 13, 17\}$

$Y = \{1, 2, 3, 4, \ldots, 12\}$

(i) $X \cup Y$

$= \{2, 3, 5, 7, 11, 13, 17\} \cup \{1, 2, 3, 4, \ldots, 12\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17\}$

(ii) $Y \cup X$

$= \{1, 2, 3, 4, \ldots, 12\} \cup \{2, 3, 5, 7, 11, 13, 17\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17\}$

(iii) $X \cap Y$

$= \{2, 3, 5, 7, 11, 13, 17\} \cap \{1, 2, 3, 4, 5, \ldots, 12\}$

$= \{2, 3, 5, 7, 11\}$

(iv) $Y \cap X$

$= \{1, 2, 3, 5, \ldots, 12\} \cap \{2, 3, 5, 7, 11, 13, 17\}$

$= \{2, 3, 5, 7, 11\}$

Q.3 If $X = \phi, Y = Z^+, T = O^+$, then find.

(i) $X \cup Y$ (ii) $X \cup T$ (iii) $Y \cup T$

(iv) $X \cap Y$ (v) $X \cap T$ (vi) $Y \cap T$

Solution:

(i) $X \cup Y = \phi \cup Z^+$

$= Z^+ = Y$

(ii) $X \cup T = \phi \cup O^+$

$= O^+ = T$

(iii) $Y \cup T = Z^+ \cup O^+$

$= \{1, 2, 3, 4, 5 \ldots\} \cup \{1, 3, 5, 7, \ldots\}$

$= \{1, 2, 3, 4, 5, \ldots\} = Z^+ = Y$

(iv) $X \cap Y = \phi \cap Z^+$

$= \phi = X$

(v) $X \cap T = \phi \cap O^+$

$= \phi = X$

(vi) $Y \cap T = Z^+ \cap O^+$

$= \{1, 2, 3, 4, 5, \ldots\} \cap \{1, 3, 5, 7, \ldots\}$

$= \{1, 3, 5, 7, \ldots\} = O^+ = T$

Q.4 If $U = \{x | x \in N \wedge 3 < x \leq 25\}$
$X = \{x | x \text{ is Prime} \wedge 8 < x < 25\}$
$Y = \{x | x \in W \wedge 4 \leq x \leq 17\}$
then find the value of:

(i) $(X \cup Y)'$ (ii) $X' \cap Y'$

(iii) $(X \cap Y)'$ (iv) $X' \cup Y'$

Solution:

$U = \{4, 5, 6, 7, \ldots, 25\}$

$X = \{11, 13, 17, 19, 23\}$

$Y = \{4, 5, 6, 7, \ldots, 17\}$

(i) $(X \cup Y)'$

$X \cup Y = \{11, 13, 17, 19, 23\} \cup \{4, 5, 6, 7, \ldots, 17\}$

$= \{4,5,6,7,8,9,10,11,12,13,14,15,16,17,19,23\}$

$(X \cup Y)' = U - (X \cup Y)$

$= \{4,5,6,7,\ldots,25\} - \{4,5,6,7,8,9,10,11,12,$
$13,14,15,16,17,19,23\}$

$= \{18, 20, 21, 22, 24, 25\}$

(ii) $X' \cap Y'$

$X' = U - X$

$= \{4, 5, 6, 7, \ldots, 25\} - \{11,13,17,19,23\}$

$= \{4,5,6,7,8,9,10,12,14,15,16,18,20,21,22,24,25\}$

$Y' = U - Y$

$= \{4,5,6,7, \ldots, 25\} - \{4,5,6,7,\ldots,17\}$

$= \{18, 19, 20, 21, 22, 23, 24, 25\}$

$X' \cap Y' = \{4,5,6,7,8,9,10,12,14,15,16,18,20,21,$
$22,24,25\} \cap \{18,19,20,21,22,23,24,25\}$

$= \{18, 20, 21, 22, 24, 25\}$

(iii) $(X \cap Y)'$

$X \cap Y = \{11,13,17,19,23\} \cap \{4,5,6,7,\ldots,17\}$

$= \{11, 13, 17\}$

$(X \cap Y)' = U - (X \cap Y)$

$= \{4, 5, 6, 7, \ldots, 25\} - \{11,13,17\}$

$= \{4,5,6,7,8,9,10,12,14,15,16,$
$18,19,20,21,22,23,24,25\}$

(iv) $X' \cup Y'$

$X' = U - X = \{4,5,6,7,\ldots,25\} - \{11,13,17,19,23\}$

$= \{4,5,6,7,8,9,10,12,14,15,16,$
$18,20,21,22,24,25\}$

$Y' = U - Y$

$= \{4,5,6,7,\ldots,25\} - \{4,5,6,7,\ldots,17\}$

$= \{18, 19, 20, 21, 22, 23, 24, 25\}$

$X' \cup Y' = \{4, 5, 6, 7, 8, 9, 10, 12, 14, 15,$
$16, 18, 20, 21, 22, 24, 25\} \cup$
$\{18, 19, 20, 21, 22, 23, 24, 25\}$

$= \{4, 5, 6, 7, 8, 9, 10, 12, 14, 15,$
$16,18, 19, 20, 21, 22,23, 24,25\}$

Q.5 **If $X = \{2, 4, 6, \ldots, 20\}$ and $Y=\{4, 8, 12, \ldots 24\}$ then find the following: (i) $X - Y$ (ii) $Y - X$**

Solution:

(i) $X-Y=\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} - \{4, 8, 12, 16, 20, 24\}$

$= \{2, 6, 10, 14, 18\}$

(ii) $Y-X= \{4, 8, 12, 16, 20, 24\} - \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$= \{24\}$

Q.6 **If $A = N$ and $B = W$ then find the value of**

(i) $A-B$ (ii) $B-A$

Solution:

(i) $A-B = N - W$

$= \{1, 2, 3, \ldots\} - \{0, 1, 2, 3, \ldots\}$

$= \{\ \}$

(ii) $B-A = W - N$

$= \{0, 1, 2, 3, \ldots\} - \{1, 2, 3, \ldots\}$

$= \{0\}$

Properties of Union and Intersection:

(a) Commutative property of union:

For any two sets A and B, $A \cup B = B \cup A$ is called commutative property of union .

Proof:

Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$ (by definition of union of sets)

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in B \cup A$

$\Rightarrow A \cup B \subseteq B \cup A$ ....(i)

Now let $y \in B \cup A$

$\Rightarrow y \in B$ or $y \in A$ (by definition of union of sets)

$\Rightarrow y \in A$ or $y \in B$

$\Rightarrow y \in A \cup B$

$\Rightarrow B \cup A \subseteq A \cup B$ .... (ii)

From (i) and (ii), we have $A \cup B = B \cup A$ (by definition of equal sets)

(b) Commutative property of intersection:

For any two sets A and B, $A \cap B = B \cap A$ is called commutative property of intersection.

Proof: Let $x \in A \cap B$

$\Rightarrow x \in A$ and $x \in B$ (by definition intersection of sets)

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in B \cap A$

$A \cap B \subseteq B \cap A$ ....(i)

Now let $y \in B \cap A$

$\Rightarrow y \in B$ and $y \in A$ (by definition intersection of sets)

$\Rightarrow y \in A$ and $y \in B$

$\Rightarrow y \in A \cap B$

Therefore, $B \cap A \subseteq A \cap B$ ....(ii)

From (i) and (ii), we have $A \cap B = B \cap A$ (by definition of equal sets)

(c) Associative property of union:

For any three sets A,B and C, $(A \cup B) \cup C = A \cup (B \cup C)$ is called associative property of union.

Proof: Let $x \in (A \cup B) \cup C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow x \in A$ or $x \in B \cup C$

$\Rightarrow x \in A \cup (B \cup C)$

$(A \cup B) \cup C \subseteq A \cup (B \cup C)$...............(i)

Similarly $A \cup (B \cup C) \subseteq (A \cup B) \cup C$ ......(ii)

From (i) and (ii), we have

$(A \cup B) \cup C = A \cup (B \cup C)$

(d) Associative property of intersection:

For any three sets A, B and C, $(A \cap B) \cap C = A \cap (B \cap C)$ is called associative property of intersection.

Proof: Let $x \in (A \cap B) \cap C$

$\Rightarrow x \in (A \cap B)$ and $x \in C$

$\Rightarrow (x \in A$ and $x \in B)$ and $x \in C$

$\Rightarrow x \in A$ and $(x \in B$ and $\in C)$

$\Rightarrow x \in A$ and $x \in B \cap C$

$\Rightarrow x \in A \cap (B \cap C)$

$\therefore (A \cap B) \cap C \subseteq A \cap (B \cap C)$......(i)

Similarly $A \cap (B \cap C) \subseteq (A \cap B) \cap C$ ... (ii)

From (i) and (ii), we have

$(A \cap B) \cap C = A \cap (B \cap C)$

(e) **Distributive property of union over intersection**:

If A,B and C are three sets then

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ is called distributive property of union over intersection

Proof: Let $x \in A \cup (B \cap C)$

$\Rightarrow x \in A$ or $x \in B \cap C$

$\Rightarrow x \in A$ or $(x \in B$ and $x \in C)$

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$\Rightarrow x \in A\cup B$ and $x \in A \cup C$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

$\therefore A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.......(i)

Similarly, now let $y \in (A \cup B) \cap (A \cup C)$

$\Rightarrow y \in (A \cup B)$ and $y \in (A \cup C)$

$\Rightarrow (y \in A$ or $y \in B)$ and $(y\in A$ or $y\in C)$

$\Rightarrow y \in A$ or $(y \in B$ and $y \in C)$

$\Rightarrow y \in A$ or $y \in B \cap C$

$\Rightarrow y \in A$ or $(B \cap C)$

$\Rightarrow y \in A \cup (B \cap C)$

$\Rightarrow (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$......(ii)

From (i) and (ii), we have

$A\cup (B\cap C) = (A \cup C) \cap (A \cup C)$

(f) **Distributive property of intersection over union**:

If A, B and C are three sets then

$A \cap (B \cup C) = (A\cap B)\cup(A\cap C)$ is called distributive property of intersection over union.

Proof: Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in B \cup C$

$\Rightarrow x \in A$ and $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ and $x\in B)$ or $(x\in A$ and $x\in C)$

$\Rightarrow (x\in A\cap B)$ or $(x \in A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

Hence by definition of subsets

$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$..... (i)

Similarly

$(A \cap B) \cup (A \cap C) \subseteq A \cap (B\cup C)$..... (ii)

From (i) and (ii), we have

$A\cap(B\cup C) = (A \cap B) \cup (A \cap C)$

(g) **De-Morgan's laws**:

If two sets A and B are the sub sets of U then De-Morgan's laws are expressed as

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Proof:

(i) $(A \cup B)' = A' \cap B'$

Let $x \in (A \cup B)'$

$\Rightarrow x \notin A \cup B$ (by definition of complement of set)

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A'\cap B'$ (by definition of intersection of sets)

$\Rightarrow (A \cup B)' \subseteq A' \cap B'$ ....(i)

Similarly $A' \cap B' \subseteq (A \cup B)'$............(ii)

Using (i) and (ii), we have

$(A\cup B)' = A'\cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Let $x \in (A \cap B)'$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \in A'\cup B'$

$\Rightarrow (A \cap B)' \subseteq A' \cup B'$...............(i)

Let $y \in A' \cup B'$

$\Rightarrow y \in A'$ or $y \in B'$

$\Rightarrow y \notin A$ or $y \notin B$

$\Rightarrow y \notin A\cap B$

$\Rightarrow y \in (A \cap B)'$

$\Rightarrow A' \cup B' \subseteq (A \cap B)'$ ....(ii)

From (i) and (ii) we have proved that

$(A \cap B)' = A' \cup B'$

EXERCISE 5.2

Q.1 If $X = \{1, 3, 5, 7, ..., 19\}$
$Y = \{0, 2, 4, 6, 8, ..., 20\}$
$Z = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$, then find the following:

(i) $X\cup(Y\cup Z)$ (ii) $(X\cup Y)\cup Z$
(iii) $X\cap(Y\cap Z)$ (iv) $(X\cap Y)\cap Z$
(v) $X\cup\cap Y\cap Z)$ (vi) $(X\cup Y)\cap(X\cup Z)$
(vii) $X\cap(Y\cup Z)$ (viii) $(X\cap Y)\cup(X\cap Z)$

Solution:

(i) $X\cup(Y\cup Z)$

$= X \cup (\{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$

$= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\} \cup \{0,2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 23\}$

$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,13, 14,15,16,17,18,19,20,23\}$

(ii) $(X\cup Y)\cup Z$

$= (\{1,3,5,7,...,19\}\cup\{0,2,4,6,8,....,20\})\cup Z$

$= \{0, 1, 2, 3, 4, 5, 6, 7, ..., 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$= \{0, 1, 2, 3, 4, 5, 6, 7, ..., 20, 23\}$

(iii) $X\cap (Y\cap Z)$

$= X \cap (\{0,2,4,6, 8, ..., 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$

$= \{1, 3, 5, 7, ..., 19\} \cap \{2\}$

$= \phi$

(iv) $(X\cap Y)\cap Z$

$= (\{1,3,5,7, ..,19\}\cap\{0,2,4,6, 8,..,20\}) \cap Z$

$= \{\ \} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$= \phi$

(v) $X\cup(Y\cap Z)$

$= X \cup (\{0,2,4,6,8, ..., 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$

$= \{1, 3, 5, 7, ..., 19\} \cup \{2\}$

$= \{1, 2, 3, 5, 7, ..., 19\}$

(vi) $(X\cup Y)\cap(X\cup Z)$

$X\cup Y = \{1,3,5, 7,..., 19\}\cup\{0,2,4,6,8, ..., 20\}$

$= \{0, 1, 2, 3, 4, 5, ..., 20\}$

$X\cup Z = \{1,3,5,7, ..., 19\}\cup\{2,3,5,7,11,13,17, 19, 23\}$

$= \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 23\}$

$(X\cup Y)\cap(X\cup Z)$

$= \{0,1,2,3,4, ..., 20\}\cap\{1,2,3,5,7,9,11,13,15,17,19,23\}$

$= \{1,2, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

(vii) $X\cap(Y\cup Z)$

$X\cap(Y\cup Z)$

$= X\cap(\{0,2,4,6,8, ...,20\}\cup\{2,3,5,7,11,13, 17,19,23\})$

$= \{1,3,5,7, ..., 19\}\cap\{0,2,3,4,5,6,7,8, 10,11, 12,13,14,16,17,18,19,20,23\}$

$= \{3, 5, 7, 11, 13, 17, 19\}$

(viii) $(X\cap Y)\cup(X\cap Z)$

$X\cap Y = \{1,3,5,7, ..., 19\}\cap\{0,2,4,6,8, ..., 20\}$

$= \{\ \}$

$X\cap Z=\{1,3,5,7, ..., 19\}\cap\{2,3,5,7,11,13,17, 19,23\}$

$= \{3, 5, 7, 11, 13, 17, 19\}$

$(X\cap Y) \cup(X\cap Z)=\{\ \}\cup \{3,5,7,11,13, 17,19\}$

$= \{3, 5, 7, 11, 13, 17, 19\}$

Q. 2. If $A = \{1, 2, 3, 4, 5, 6\}$
$B = \{2, 4, 6, 8\}$ $C = \{1, 4, 8\}$ Prove the following identities:

(i) $A\cap B = B\cap A$
(ii) $A\cup B = B\cup A$
(iii) $A\cap (B\cap C) = (A\cap B) \cup (A\cap C)$
(iv) $A\cup (B\cap C) = (A\cup B) \cap (A\cup C)$

Solution:

(i) $A \cap B = B \cap A$

L.H.S $= A \cap B$

$= \{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}$

$= \{2, 4, 6\}$

R.H.S $= B \cap A$

$= \{2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6\}$

$= \{2, 4, 6\}$

L.H.S = R.H.S, so

$A \cap B = B \cap A$

(ii) $A \cup B = B \cup A$

L.H.S $= A \cup B$

$= \{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

R.H.S $= B \cup A$

$= \{2, 4, 6, 8\} \cup \{1, 2, 3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

L.H.S = R.H.S,

So, $A \cup B = B \cup A$

(iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S $= A \cap (B \cup C)$

$= A \cap (\{2, 4, 6, 8\} \cup \{1, 4, 8\})$

$= \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 4, 6, 8\}$

$= \{1, 2, 4, 6\}$

R.H.S $= (A \cap B) \cup (A \cap C)$

$A \cap B = \{1,2,3,4,5,6\} \cap \{2,4,6,8\}$

$= \{2, 4, 6\}$

$A \cap C = \{1, 2, 3, 4, 5, 6\} \cap \{1, 4, 8\}$

$= \{1, 4\}$

$(A \cap B) \cup (A \cap C) = \{2, 4, 6\} \cup \{1, 4\}$

$= \{1, 2, 4, 6\}$

L.H.S = R.H.S

So, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iv) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $= A \cap (B \cap C)$

$= A \cup (\{2, 4, 6, 8\} \cap \{1, 4, 8\})$

$= \{1, 2, 3, 4, 5, 6\} \cup \{4, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

R.H.S $= (A \cup B) \cap (A \cup C)$

$A \cup B = \{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

$A \cup C = \{1, 2, 3, 4, 5, 6\} \cup \{1, 4, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

$(A \cup B) \cap (A \cup C)$

$= \{1,2,3,4,5,6,8\} \cap \{1,2,3,4,5,6,8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

L.H.S = R.H.S,

So, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Q.3 If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 5, 7\}$ then verify the De Morgan's laws i.e., $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$

Solution:

(i) $(A \cup B)' = A' \cap B'$

L.H.S $= (A \cup B)'$

$A \cup B = \{1, 3, 5, 7, 9\} \cup \{2, 3, 5, 7\}$

$= \{1, 2, 3, 5, 7, 9\}$

$(A \cup B)' = U - (A \cup B)$

$= \{1,2,3,4, \ldots,10\} - \{1,2,3,5,7,9\}$

$= \{4, 6, 8, 10\}$............(i)

R.H.S $= A' \cap B'$

$A' = U - A$

$= \{1, 2, 3, 4, \ldots, 10\} - \{1, 3, 5, 7, 9\}$

$= \{2, 4, 6, 8, 10\}$

$B' = U - B$

$= \{1, 2, 3, 4, 5, \ldots, 10\} - \{2, 3, 5, 7\}$

$= \{1, 4, 6, 8, 9, 10\}$

$A' \cap B' = \{2, 4, 6, 8, 10\} \cap \{1, 4, 6, 8, 9, 10\}$

$= \{4, 6, 8, 10\}$..........(ii)

From (i) and (ii)

L.H.S = R.H.S

$(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

L.H.S = $(A \cap B)'$

$A \cap B = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\}$

$= \{3, 5, 7\}$

$(A \cap B)' = U - (A \cap B)$

$= \{1, 2, 3, 4, \ldots, 10\} - \{3, 5, 7\}$

$= \{1, 2, 4, 6, 8, 9, 10\}$.........(i)

R.H.S $= A' \cup B'$

$A' = U - A$

$= \{1,2,3,4,5,\ldots,10\} - \{1,3,5,7,9\}$

$= \{2, 4, 6, 8, 10\}$

$B' = U - B$

$= \{1, 2, 3, 4, 5, \ldots, 10\} - \{2, 3,5, 7\}$

$= \{1, 4, 6, 8, 9, 10\}$

$A' \cup B' = \{2, 4, 6, 8, 10\} \cup \{1, 4,6,8,9,10\}$

$= \{1, 2, 4, 6, 8, 9, 10\}$.........(ii)

From (i) and (ii)

L.H.S = R.H.S

So $(A \cap B)' = A' \cup B'$

Q.4 If $U = \{1, 2, 3, \ldots, 20\}$

$X = \{1, 3, 7, 9, 15, 18, 20\}$

$Y = \{1, 3, 5, \ldots, 17\}$ then show that,

(i) $X - Y = X \cap Y'$ (ii) $Y - X = Y \cap X'$

Solution:

(i) $X - Y = X \cap Y'$

L.H.S = X–Y

$= \{1, 3, 7, 9, 15, 18, 20\} -$
$\{1, 3, 5, 7, 9, 11, 13, 15, 17\}$

$= \{18, 20\}$..........(i)

R.H.S $= X \cap Y'$

$Y' = U - Y$

$= \{1,2,3,\ldots, 20\} - \{1,3,5,\ldots,17\}$

$= \{2,4,6,8,10,12,14,16,18,19,20\}$

$X \cap Y' = \{1, 3, 7, 9, 15, 18, 20\} \cap$
$\{2, 4, 6, 8, 10, 12, 14, 16, 18, 19, 20\}$

$= \{18, 20\}$..........(ii)

From (i) and (ii)

L.H.S = R.H.S,

So, $X - Y = X \cap Y'$

(ii) $Y - X = Y \cap X'$

L.H.S = Y–X

$= \{1,3,5,\ldots,17\} - \{1,3,7,9,15,18,20\}$

$= \{5, 11, 13, 17\}$..........(i)

R.H.S $= Y \cap X'$

$X' = U - X$

$= \{1,2,3, \ldots,20\} - \{1,3,7,9,15,18,20\}$

$= \{2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16, 17, 19\}$

$Y \cap X' = \{1, 3, 5, 7, 9, 11, 13, 15, 17\} \cap$
$\{2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16, 17, 19\}$

$= \{5, 11, 13, 17\}$..........(ii)

From (i) and (ii)

L.H.S = R.H.S

So, $Y - X = Y \cap X'$

Verification of Fundamental Properties of Sets

(a) Commutative Property of Union i.e.,

$A \cup B = B \cup A$

For example $A = \{1, 3, 5, 7\}$ and

$B = \{2, 3, 5, 7\}$

Then $A \cup B = \{1, 3, 5, 7\} \cup \{2, 3, 5, 7\}$

$= \{1, 2, 3, 5, 7\}$

and $B \cup A = \{2, 3, 5, 7\} \cup \{1, 3, 5, 7\}$

$= \{1, 2, 3, 5, 7\}$

Hence, verified that $A \cup B = B \cup B$

(b) Commutative property of intersection

i.e., $A \cap B = B \cap A$

For example $A = \{1, 3, 5, 7\}$ and

$B = \{2, 3, 5, 7\}$

Then $A \cap B = \{1, 3, 5, 7\} \cap \{2, 3, 5, 7\}$

$= \{3, 5, 7\}$

and $B \cap A = \{2, 3, 5, 7\} \cap \{1, 3, 5, 7\}$

$= \{3, 5, 7)$

Hence, verified that $A \cap B = B \cap A$

(c) Associative Property of Union

i.e., $(A \cup B) \cup C = A \cup (B \cup C)$.

Suppose $A = \{1, 2, 4, 8\}$, $B = \{2, 4, 6\}$

and $C = \{3, 4, 5, 6\}$ then

L.H.S $= (A \cup B) \cup C$

$= (\{1, 2, 4, 8\} \cup \{2, 4, 6\}) \cup \{3, 4, 5, 6\}$

$= \{1, 2, 4, 6, 8\} \cup \{3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

and R.H.S $= A \cup (B \cup C)$

$= \{1, 2, 4, 8\} \cup (\{2, 4, 6\} \cup \{3, 4, 5, 6\})$

$= \{1, 2, 4, 8\} \cup \{2, 3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

L.H.S = R.H.S

Hence, union of sets is associative

(d) Associative Property of intersection

i.e., $(A \cap B) \cap C = A \cap (B \cap C)$

Suppose $A = \{1, 2, 4, 8\}$

$B = \{2, 4, 6\}$

and $C = \{3, 4, 5, 6\}$

Then L.H.S $= (A \cap B) \cap C$

$= (\{1,2,4,8\} \cap \{2,4,6\}) \cap \{3,4,5,6\}$

$= \{2, 4\} \cap \{3, 4, 5, 6\}$

$= \{4\}$

and R.H.S $= A \cap (B \cap C)$

$= \{1,2,4,8\} \cap (\{2,4,6\} \cap \{3,4,5,6\})$

$= \{1, 2, 4, 8\} \cap \{4, 6\}$

$= \{4\}$

L.H.S = R.H.S

Hence, intersection of sets is associative law.

(e) Distributive Property of Union over Intersection

i.e., $A \cup (B \cap C) = (A \cup B) \cap (A \cup B)$:

Suppose $A = \{1, 2, 4, 8\}$

$B = \{2, 4, 6\}$ and

$C = \{3, 4, 5, 6\}$

Let L.H.S $= A \cup (B \cap C)$

$= \{1, 2, 4, 8\} \cup (\{2, 4, 6\} \cap \{3, 4, 5, 6\})$

$= \{1, 2, 4, 8\} \cup \{4, 6\}$

$= \{1, 2, 4, 6, 8\}$

R.H.S $= (A \cup B) \cap (A \cup C)$

$(A \cup B) = (\{1, 2, 4, 8\} \cup \{2, 4, 6\})$

$= \{1, 2, 4, 6, 8\}$

$(A \cup C) = (\{1, 2, 4, 8\} \cup \{3, 4, 5, 6\})$

$= \{1, 2, 3, 4, 5, 6, 8\}$

$(A \cup B) \cap (A \cup C)$

$= \{1,2,4,6,8\} \cap \{1, 2, 3, 4, 5, 6, 8\}$

$= \{1, 2, 4, 6, 8\}$

L.H.S = R.H.S

(f) Distributive Property of Intersection over Union

i.e., $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$

Suppose

$A = \{1, 2, 3, 4, 5, \ldots, 20\}$

$B = \{5, 10, 15, 20, 25, 30\}$

$C = \{3, 9, 15, 21, 27, 33\}$

L.H.S $= A\cap(B\cup C)$

$= A\cap(\{5,10,15,20,25,30\}\cup\{3,9,15,21,27,33\})$

$= \{1,2,3,4,5, \ldots 20\}\cap$

$\{3, 5, 9, 10, 15, 20, 21, 25, 27, 30, 33\}$

L. H.S $=\{3, 5, 9, 10, 15, 20\}$

R.H.S $= (A\cap B)\cup(A\cap C)$

$(A\cap B)=\{1,2,3,4, .., 20\}\cap\{5,10,15,20,25,30\}$

$= \{5, 10, 15, 20\}$

$(A\cap C)=\{1,2,3,4,5,\ldots,20)\cap\{3,9,15,21,27,33\}$

$= \{3, 9, 15\}$

$(A\cap B)\cup(A\cap C)$

$= \{5, 10, 15, 20\}\cup\{3, 9, 15\}$

$= \{3, 5, 9, 10, 15, 20\}$

L.H.S = R.H.S

(g) De Morgan's laws

If set A and B are the subsets of universal set U then De Morgan's laws are expressed as.

(i) $(A\cup B)' = A'\cap B'$

(ii) $(A\cap B)' = A'\cup B'$

Proof:

(i) $(A\cup B)' = A'\cap B'$

Suppose

$U = \{1, 2, 3, 4, \ldots, 10\}$

$A = \{2, 4, 6, 8, 10\} \Rightarrow A'=\{1, 3, 5, 7, 9\}$

$B = \{1,2, 3, 4, 5, 6\}\Rightarrow B'=\{7, 8, 9, 10\}$

L.H.S $= (A\cup B)'$

$(A\cup B)=\{2, 4, 6, 8, 10\}\cup\{1, 2, 3, 4, 5, 6\}$

$=\{1, 2,3, 4, 5, 6, 8, 10\}$

L.H.S $= (A\cup B)'$

$= U-(A\cup B)$

$= \{1,2,3,4, \ldots, 10\}-\{1,2,3,4,5,6,8,10\}$

L.H.S$= \{7, 9\}$ .... (i)

R.H.S $= A'\cap B'$

$= \{1, 3, 5, 7, 9\}\cap\{7, 8, 9, 10\}$

$= \{7, 9\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

(ii) $(A\cap B)' = A'\cup B'$

Suppose

$U = \{1, 2, 3, 4, \ldots, 10\}$

$A = \{2, 4, 6, 8, 10\} \Rightarrow A'= \{1, 3, 5, 7, 9\}$

$B = \{1, 2, 3, 4, 5, 6\}\Rightarrow B'= \{7, 8, 9, 10\}$

Let L.H.S $= (A\cap B)'$

$A\cap B=\{2, 4, 6, 8, 10\}\cap\{1, 2, 3, 4, 5, 6\}$

$= \{2, 4, 6\}$

L.H.S $= (A\cap B)'$

$= U-(A\cap B)$

$= \{1, 2, 3, 4, .., 10\}-\{2, 4, 6\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$...............(i)

R.H.S $= A'\cup B'$

$= \{1, 3, 5, 7, 9\}\cup\{7, 8, 9, 10\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$...............(ii)

From (i) and (ii)

L.H.S = R.H.S

Venn Diagram

British mathematician John Venn (1834–1923) introduced rectangle for a universal set U and its subsets A and B as closed figures inside this rectangle.

Use Venn diagrams to represent:

(a) Union and intersection of sets

Disjoint sets | Overlapping sets | Subsets $(A \subseteq B)$

$A \cup B$

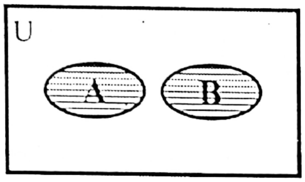


*Fig.*1

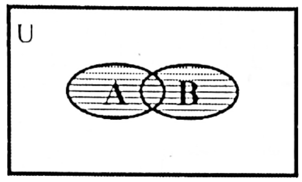


*Fig.*2

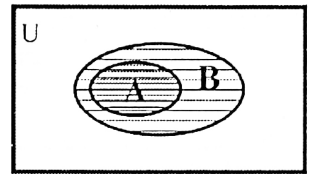


*Fig.*3

$A \cap B$

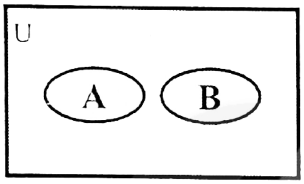


*Fig.*4

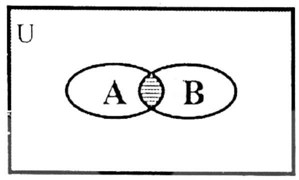


*Fig.*5

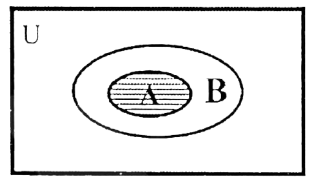
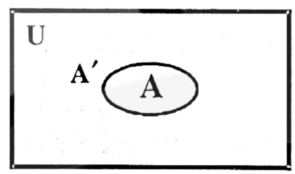


*Fig.*6

Regions shown by horizontal line segments in figures 1 to 6 shows $A \cup B$ and $A \cap B$

(b) Complement of a set

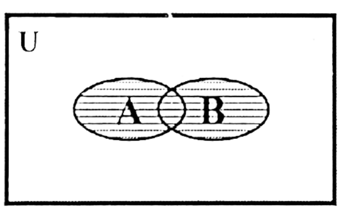
$U - A = A'$ is shown by Shaded area.



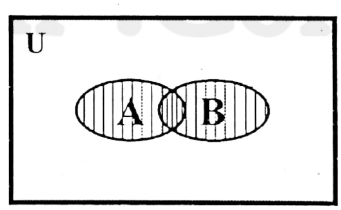
Use Venn diagram to verify:

Commutative law for union and intersection of sets.

- **Commutative Law for Union:**



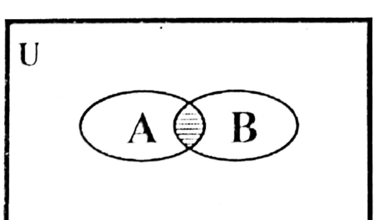
$A \cup B$ is shown by horizontal line segments



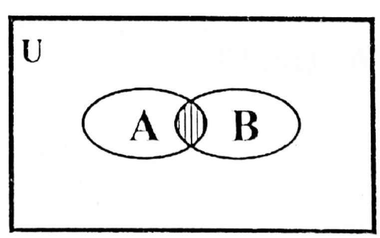
$B \cup A$ is shown by vertical line segments

The region shown in both cases are equal. Thus $A \cup B = B \cup A$.

- **Commutative Law for Intersection:**



$A \cap B$ is shown by horizontal line segments



$B \cap A$ is shown by vertical line segments

The regions shown in both cases are equal. Thus $A \cap B = B \cap A$

(c) **De Morgan's laws**

(i) $(A\cup B)' = A'\cap B'$ (ii) $(A\cap B)' = A'\cup B'$

(i) $(A\cup B)' = A'\cap B'$

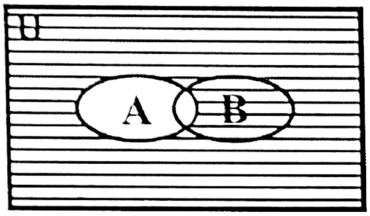


Fig. 1 : A′ is shown by horizontal line segments

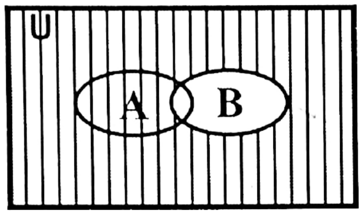


Fig. 2 : B′ is shown by vertical line segments

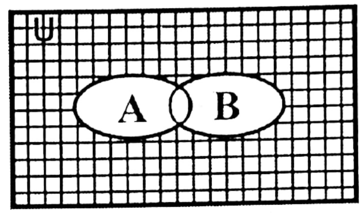


Fig. 3 : $A'\cap B'$ is shown by squares

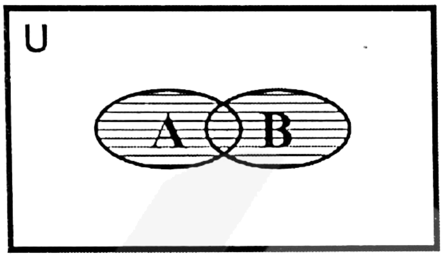


Fig. 4 : $A\cup B$ is shown by horizontal line segments

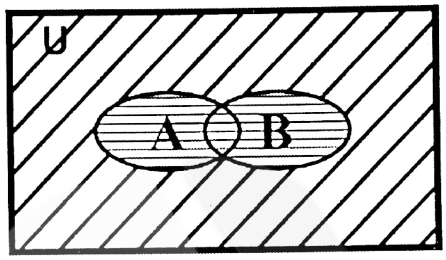
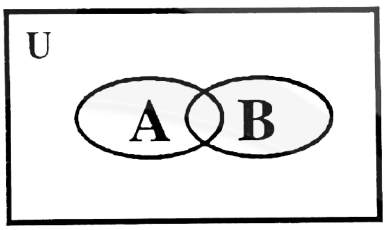


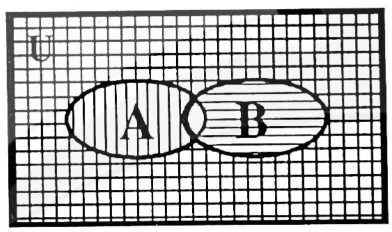
Fig. 5 : $(A\cup B)'$ is shown by slanting line segments

Regions shown in figure 3 and 5 are equal thus $(A\cup B)' = A'\cap B'$.

(ii) $(A\cap B)' = A'\cup B'$



U−)A∩B(=)A∩B(is shown by shading



$A'\cup B'$ is shown by squares, horizontal and vertical line segments.

Regions shown in fig. 6 and fig. 7 are equal.

Thus $(A\cap B)' = A'\cup B'$

(d) **Associative law of Union and Intersection:**

(i) **Associative law of Union: $(A\cup B)\cup C = A\cup(B\cup C)$**

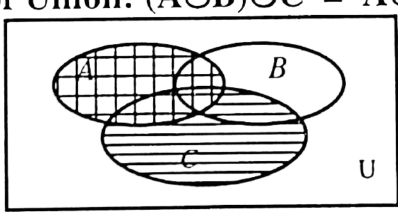


Fig-1

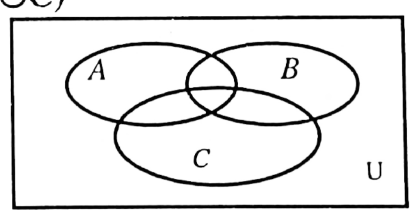


Fig-2

$(A\cup B)\cup C$ is shown in the above Fig-1. $A\cup(B\cup C)$ is shown in the above Fig-2. Regions shown in Fig. 1 and Fig. 2 by different ways are equal.

Thus $(A\cup B)\cup C = A\cup(B\cup C)$

(ii) **Associative law of Intersection: (A∩B)∩C = A∩(B∩C)**

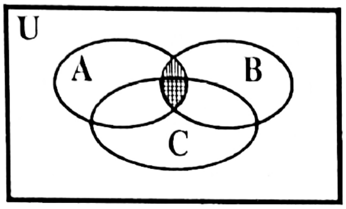


Fig. 3

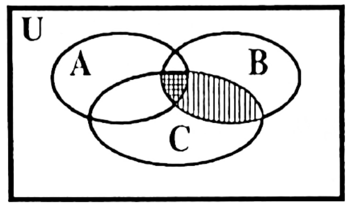


Fig. 4

(A∩B)∩C is shown in figure 3 by double crossing line segments.

A∩(B∩C) is shown in figure 4 by double crossing line segments.

Regions shown in Fig 3 and Fig. 4 are equal.

Thus (A∩B)∩C = A∩(B∩C)

(e) **Distributive law:**

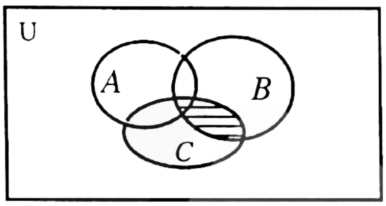


Fig. 1

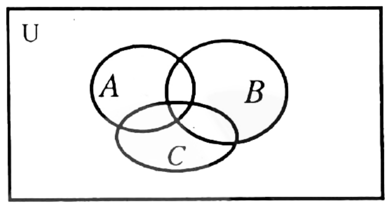


Fig. 2

Fig. 1: A∪(B∩C) is shown by horizontal line segments in the above figure 1.

Fig. 2: A∪B is shown by horizontal line segments in the fig. 2.

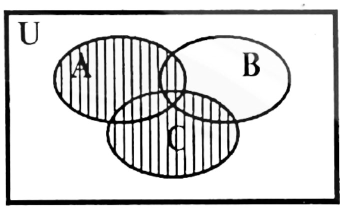


Fig. 3

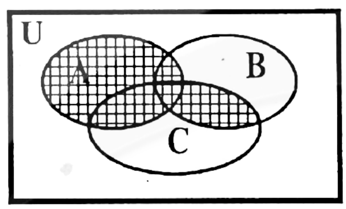


Fig. 4

Fig. 3: A∪C is shown by vertical line segments in fig.3

Fig. 4: (A∪B)∩(A∪C) is shown by double crossing line segments in fig. 4.

Regions shown in fig 1 and Fig.4 are equal. Thus A∪(B∩C) = (A∪B)∩(A∪C)

Distributive Law of Intersection over Union:

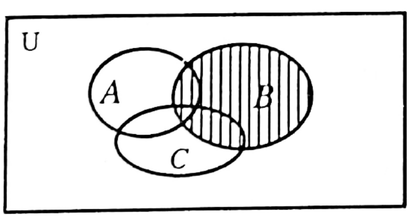


Fig.5

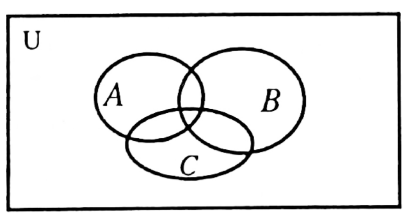


Fig.6

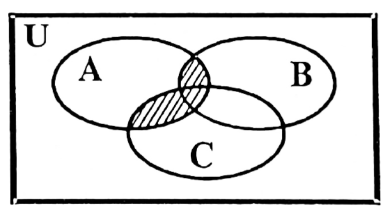


Fig. 7

Fig. 5: B∪C is shown by vertical line segments in Fig 5.

Fig. 6: A∩(B∪C) is shown in Fig.6 by vertical line segments.

Fig. 7: (A∩B)∪(A∩C) is shown in fig. 7 by slanting line segments.

Regions displayed in Fig. 6 and Fig. 7 are equal.

Thus A∩(B∪C) = (A∩B)∪(A∩C)

EXERCISE 5.3

Q.1 If $U = \{1, 2, 3, 4, \ldots, 10\}$
$A = \{1, 3, 5, 7, 9\}$, $B = \{1, 4, 7, 10\}$
then verify the following questions:
Solution:

(i) **$A - B = A \cap B'$**

L.H.S $= A - B = \{1,3,5,7,9\} - \{1,4,7,10\}$
$= \{3, 5, 9\}$ .... (i)

R.H.S $= A \cap B'$

$B' = U - B = \{1, 2, 3, \ldots, 10\} - \{1, 4, 7, 10\}$
$= \{2, 3, 5, 6, 8, 9\}$

$A \cap B' = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 6, 8, 9\}$
$= \{3, 5, 9\}$ .... (ii)

From (i) and (ii)
L.H.S = R.H.S
$A - B = A \cap B'$

(ii) **$B - A = B \cap A'$**

L.H.S $= B - A = \{1,4,7,10\} - \{1, 3, 5, 7, 9\}$
$= \{4, 10\}$ .... (i)

R.H.S $= B \cap A'$

$A' = U - A = \{1,2,3, \ldots, 10\} - \{1,3,5,7,9\}$
$= \{2, 4, 6, 8, 10\}$

$B \cap A' = \{1, 4, 7, 10\} \cap \{2, 4, 6, 8, 10\}$
$= \{4, 10\}$ .... (ii)

From (i) and (ii)
L.H.S = R.H.S
$B - A = B \cap A'$

(iii) **$(A \cup B)' = A' \cap B'$**

L.H.S $= (A \cup B)'$

$A \cup B = \{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\}$
$= \{1, 3, 4, 5, 7, 9, 10\}$

$(A \cup B)' = U - (A \cup B)$
$= \{1,2,3, \ldots, 10\} - \{1,3,4,5,7,9,10\}$
$= \{2, 6, 8\}$ .... (i)

R.H.S $= A' \cap B'$

$A' = U - A = \{1,2,3, \ldots, 10\} - \{1,3,5,7,9\}$
$= \{2, 4, 6, 8, 10\}$

$B' = U - B = \{1,2,3, \ldots, 10\} - \{1,4,7,10\}$
$= \{2, 3, 5, 6, 8, 9\}$

$A' \cap B' = \{2, 4, 6, 8, 10\} \cap \{2, 3, 5, 6, 8, 9\}$
$= \{2, 6, 8\}$ .... (ii)

From (i) and (ii)
L.H.S = R.H.S

(iv) **$(A \cap B)' = A' \cup B'$**

L.H.S $= (A \cap B)'$

$A \cap B = \{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\}$
$= \{1, 7\}$

$(A \cap B)' = U - (A \cap B)$
$= \{1, 2, 3, \ldots, 10\} - \{1, 7\}$
$= \{2, 3, 4, 5, 6, 8, 9, 10\}$ ...... (i)

R.H.S $= A' \cup B'$

$A' = U - A = \{1,2,3, \ldots, 10\} - \{1,3,5,7,9\}$
$= \{2, 4, 6, 8, 10\}$

$B' = U - B = \{1,2,3, \ldots, 10\} - \{1, 4,7,10\}$
$= \{2, 3, 5, 6, 8, 9\}$

$A' \cup B' = \{2, 4, 6, 8, 10\} \cup \{2, 3, 5, 6, 8, 9\}$
$= \{2, 3, 4, 5, 6, 8, 9, 10\}$....... (ii)

From (i) and (ii)
L.H.S = R.H.S

(v) **$(A - B)' = A' \cup B$**

L.H.S $= (A - B)'$

$A - B = \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$
$= \{3, 5, 9\}$

$(A - B)' = U - (A - B)$
$= \{1, 2, 3, \ldots, 10\} - \{3, 5, 9\}$
$= \{1, 2, 4, 6, 7, 8, 10\}$....... (i)

R.H.S $= A' \cup B$

$A' = U - A = \{1,2,3, \ldots, 10\} - \{1,3,5,7,9\}$
$= \{2, 4, 6, 8, 10\}$

$A' \cup B = \{2, 4, 6, 8, 10\} \cup \{1, 4, 7, 10\}$
$= \{1, 2, 4, 6, 7, 8, 10\}$....... (ii)

From (i) and (ii)
L.H.S = R.H.S

(vi) **$(B - A)' = B' \cup A$**

L.H.S $= (B - A)'$

$B - A = \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\}$
$= \{4, 10\}$

$(B - A)' = U - (B - A)$
$= \{1, 2, 3, \ldots, 10\} - \{4, 10\}$
$= \{1, 2, 3, 5, 6, 7, 8, 9\}$....... (i)

R.H.S $= B' \cup A$

$B' = U - B = \{1,2,3,4, \ldots, 10\} - \{1, 4, 7, 10\}$
$= \{2, 3, 5, 6, 8, 9\}$

$B' \cup A = \{2,3,5,6,8,9\} \cup \{1, 3, 5, 7, 9\}$
$= \{1, 2, 3, 5, 6, 7, 8, 9\}$....... (ii)

From (i) and (ii)
L.H.S = R.H.S

Q.2 If $A = \{1, 3, 5, 7, 9\}$ $B = \{1, 4, 7, 10\}$ $C = \{1, 5, 8, 10\}$

Then verify the following:

Solution:

(i) $(A\cup B)\cup C = A\cup(B\cup C)$

L.H.S $= (A\cup B)\cup C$

$= (\{1,3,5,7,9\}\cup\{1,4,7,10\})\cup\{1,5,8,10\}$

$= \{1, 3, 4, 5, 7, 9, 10\}\cup\{1, 5, 8, 10\}$

$= \{1, 3, 4, 5, 7, 8, 9, 10\}$ .... (i)

R.H.S $= A\cup(B\cup C)$

$= \{1,3,5,7,9\}\cup(\{1,4,7,10\}\cup\{1,5,8,10\})$

$= \{1,3,5,7,9\}\cup\{1,4,5,7,8,10\}$

$= \{1, 3, 4, 5, 7, 8, 9, 10\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

(ii) $(A\cap B)\cap C = A\cap(B\cap C)$

L.H.S $= (A\cap B)\cap C$

$= (\{1,3,5,7,9\}\cap\{1,4,7,10\})\cap\{1,5,8,10\}$

$= \{1, 7\}\cap\{1, 5, 8, 10\}$

$= \{1\}$ .... (i)

R.H.S $= A\cap(B\cap C)$

$= \{1,3,5,7,9\}\cap(\{1,4,7,10\}\cap\{1,5,8,10\})$

$= \{1, 3, 5, 7, 9\}\cap\{1, 10\}$

$= \{1\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

(iii) $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

L.H.S $= A\cup(B\cap C)$

$= \{1,3,5,7,9\}\cup(\{1,4,7,10\}\cap\{1,5,8,10\})$

$= \{1, 3, 5, 7, 9\}\cup\{1, 10\}$

$= \{1, 3, 5, 7, 9, 10\}$ .... (i)

R.H.S $= (A\cup B)\cap(A\cup C)$

$A\cup B = \{1, 3, 5, 7, 9\}\cup\{1, 4, 7, 10\}$

$= \{1, 3, 4, 5, 7, 9, 10\}$

$A\cup C = \{1, 3, 5, 7, 9\}\cup\{1, 5, 8, 10\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$

Now $(A\cup B)\cap(A\cup C)$

$= \{1,3,4,5,7,9,10\}\cap\{1,3,5,7,8,9,10\}$

$= \{1, 3, 5, 7, 9, 10\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

(iv) $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$

L.H.S $= A\cap(B\cup C)$

$= \{1,3,5,7,9\}\cap(\{1,4,7,10\}\cup\{1, 5, 8, 10\})$

$= \{1, 3, 5, 7, 9\}\cap\{1, 4, 5, 7, 8, 10\}$

$= \{1, 5, 7\}$ .... (i)

R.H.S $= (A\cap B)\cup(A\cap C)$

$A\cap B = \{1,3,5,7,9\}\cap\{1, 4, 7, 10\}$

$= \{1, 7\}$

$A\cap C = \{1,3, 5, 7, 9\}\cap\{1, 5, 8, 10\}$

$= \{1, 5\}$

Now $(A\cap B)\cup(A\cap C) = \{1,7\}\cup\{1,5\}$

$= \{1,5,7\}$....... (ii)

From (i) and (ii)

L.H.S = R.H.S

Q.3 If $U = N$, then verify De-Morgan's laws by using:

$A = \phi$, $B = P$

Solution:

$A = \{\ \}$

$B = \{2, 3, 5, 7, \ldots\}$

$U = \{1, 2, 3, 4, 5, 6, 7\ldots\}$

(i) $(A\cup B)' = A'\cap B'$

L.H.S $= (A\cup B)'$

$A\cup B = \{\ \}\cup\{2, 3, 5, 7, \ldots\}$

$= \{2, 3, 5, 7, \ldots\}$

$(A\cup B)' = U - (A\cup B)$

$= \{1, 2, 3, 4,5,6,7\ \ldots\} - \{2, 3, 5, 7, \ldots$

$= \{1, 4, 6, \ldots\}$ .... (i)

R.H.S $= A'\cap B'$

$A' = U - A = \{1, 2, 3, 4, 5, 6, 7\ldots\} - \{\ \}$

$= \{1, 2, 3, 4, 5, 6, 7\ldots\}$

$B' = U - B = \{1,2,3,4,5,6,7\ldots\} - \{2,3,5,7,\ldots\}$

$= \{1, 4, 6, \ldots\}$

$A'\cap B' = \{1,2,3,4,5,6,7\ \ldots\}\cap\{1, 4, 6, \ldots\}$

$= \{1, 4, 6, \ldots\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

(ii) $(A\cap B)' = A'\cup B'$

L.H.S $= (A\cap B)'$

$(A\cap B) = \{\ \} \cap \{2, 3, 5, 7, \ldots\}$

$= \{\ \}$

$(A\cap B)' = U - (A\cap B)$

$= \{1, 2, 3, 4, 5, 6, 7 \ldots.\} - \{\ \}$

$= \{1, 2, 3, 4, 5, 6, 7\ldots\}$....... (i)

R.H.S $= A'\cup B'$

$A' = U - A = \{1, 2, 3, 4, \ldots\} - \{\ \}$

$= \{1, 2, 3, 4, \ldots\}$

$B' = U - B = \{1,2,3,4,5,6,7\ldots.\} - \{2,3,5,7,\ldots\}$

$= \{1, 4, 6, \ldots\}$

$A'\cup B' = \{1,2,3,4,5,6,7 \ldots\} \cup \{1, 4, 6, \ldots\}$

$= \{1, 2, 3, 4, \ldots\}$....... (ii)

From (i) and (ii)

L.H.S = R.H.S

Q.4 If $\cup = \{1, 2, 3, 4, \ldots.10\}$, $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 3, 4, 5, 8\}$ then prove the following questions by Venn Diagram.

Solution: $A \cap B = \{1, 3, 5, 7, 9\}\cap\{2, 3, 4, 5, 8\} = \{3, 5\}$

So given sets A and B are overlapping sets

(i) **$A - B = A\cap B'$**

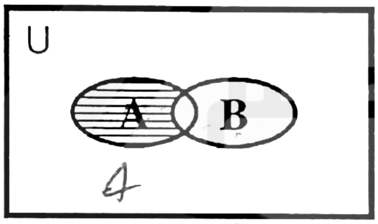


Fig: 1 (A-B)

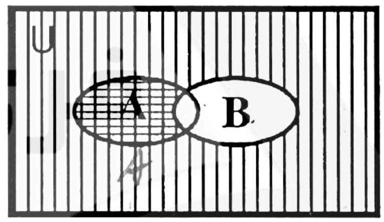


Fig: 2)A∩B′(

- $A - B$ is shown by horizontal line segments in fig. 1.
- B' is shown by vertical line segments and squares in fig. 2.
- $A\cap B'$ is shown by squares in fig. 2.

Regions shown in fig. 1 and fig. 2 are equal, thus $A - B = A\cap B'$

(ii) **$B - A = B\cap A'$**

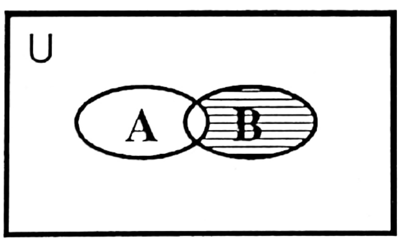


Fig. 1 (B - A)

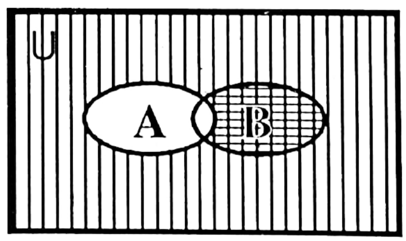


Fig: 2 (B∩A′)

- $B - A$ is shown by horizontal line segments in fig. 1.
- A' is shown by vertical line segments and squares in fig. 2.
- $B\cap A'$ is shown by squares in fig. 2.

Regions shown in fig. 1 and fig. 2 are equal, thus $B - A = B\cap A'$.

(iii) $(A \cup B)' = A' \cap B'$ **(De-Morgan's Law)**

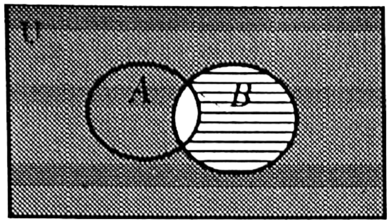


Fig.1

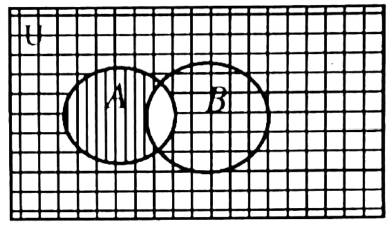


Fig.2

- $A \cup B$ is shown by horizontal line segments in Fig. 1.
- $(A \cup B)'$ is shown by shaded area in Fig. 1.
- A' is shown by horizontal line segments and squares in Fig. 2.
- B' is shown by vertical line segments and squares in Fig. 2.
- $A' \cap B'$ is shown by squares in Fig. 2.

Shaded area shown in Fig. 1 and square area shown in Fig. 2 are equal.

thus $(A \cup B)' = A' \cap B'$

(iv) $(A \cap B)' = A' \cup B'$

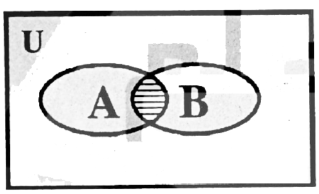


Fig. 1

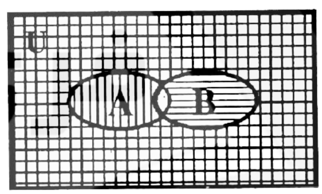


Fig. 2

- $A \cap B$ is shown by horizontal line segments in Fig. 1.
- $(A \cap B)'$ is shown by shaded area in Fig. 1.
- A' is shown by horizontal line segments and squares in Fig. 2.
- B' is shown by vertical line segments and squares in Fig. 2.
- $A' \cup B'$ is shown by squares, horizontal and vertical line segments in Fig. 2.

Shaded area shown in Fig. 1 and area of squares, vertical and horizontal line segments shown in Fig. 2 are equal. thus $(A \cap B)' = A' \cup B'$

(v) $(A - B)' = A' \cup B$

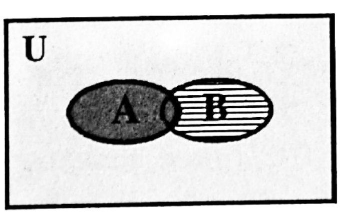


Fig. 1

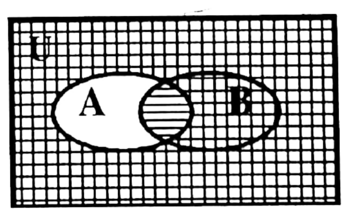


Fig. 2

- $A - B$ is shown by horizontal line segments in Fig. 1.
- $(A - B)'$ is shown by shaded area in Fig.1.
- A' is shown by squares Fig. 2.
- $A' \cup B$ is shown by squares and horizontal line segments in Fig. 2.

Shaded area in Fig. 1 and area of squares & horizontal line segments in Fig. 2 are equal.

thus $(A - B)' = A' \cup B$

(vi) $(B-A)' = B'\cup A$

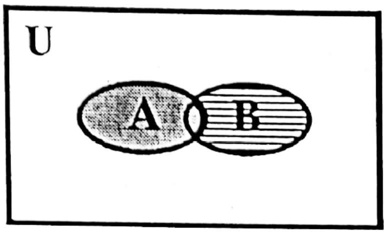


Fig. 1

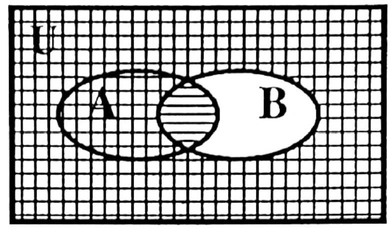


Fig. 2

- $B-A$ is shown by horizontal line segments in Fig. 1.
- $(B-A)'$ is shown by shaded area in Fig.1.
- B' is shown by squares in Fig. 2.
- $B'\cup A$ is shown by squares and horizontal line segments in Fig. 2.

Shaded area in Fig. 1 and area of squares & vertical line segments in Fig. 2 are equal.
thus $(B-A)' = B'\cup A$

Ordered Pairs and Cartesian Product:

(a) Ordered pairs

Any two numbers x and y, written in the form (x, y) is called an ordered pair. In an ordered pair (x, y) the order of numbers is important, that is, x is the first co-ordinate and y is the second co-ordinate. For example, (3, 2) is different from (2, 3).

It is obvious that $(x, y) \neq (y, x)$ unless $x = y$.

Note that $(x, y) = (s, t)$, iff $x=s$ and $y = t$

(b) Cartesian product:

Cartesian product of two non-empty sets A and B denoted by $A \times B$ consists of all ordered pairs (x, y) such that $x \in A$ and $y \in B$.

$$A\times B = \{(x, y) \mid x \in A \wedge y \in B\}$$

Example: If A = {1, 2, 3} and B = {2, 5}, then find A × B and B × A

Solution:

$A\times B = \{1,2,3\} \times \{2,5\}$

$A\times B=\{(1,2),(1,5),(2,2),(2,5),(3,2)(3, 5)\}$

Since set A has 3 elements and set B has 2 elements.

Hence product set $A\times B$ has $3\times2=6$ ordered pairs. We note that

$B\times A =\{(2,1),(2,2),(2,3),(5,1),(5,2), (5,3)\}$

Evidently $A \times B \neq B \times A$

EXERCISE 5.4

Q.1 If $A = \{a, b\}$ and $B = \{c, d\}$, then find $A \times B$ and $B \times A$

Solution:

$$A \times B = \{a, b\} \times \{c, d\}$$
$$= \{(a, c), (a, d), (b, c), (b, d)\}$$
$$B \times A = \{c, d\} \times \{a, b\}$$
$$= \{(c, a), (c, b), (d, a), (d, b)\}$$

Q.2 If $A = \{0, 2, 4\}$, $B = \{-1,3\}$, then find $A\times B$, $B \times A$, $A \times A$, $B \times B$

Solution:

$$A \times B = \{0, 2, 4\} \times \{-1, 3\}$$
$$=\{(0,-1),(0,3),(2,-1),(2,3),(4,-1),(4,3)\}$$
$$B \times A = \{-1, 3\} \times \{0, 2, 4\}$$
$$= \{(-1,0),(-1,2),(-1,4),(3,0),(3,2),(3,4)\}$$
$$A\times A = \{0, 2, 4\} \times \{0, 2, 4\}$$
$$=\{(0,0),(0,2),(0,4),(2,0),(2,2),(2,4),(4,0)(4,2),(4,4)\}$$
$$B \times B = \{-1, 3\} \times \{-1, 3\}$$
$$= \{(-1, -1), (-1, 3), (3, -1), (3, 3)\}$$

Q.3 Find a and b if

Solution:

(i) $(a - 4, b - 2) = (2, 1)$

$a - 4 = 2$, $b - 2 = 1$

$a = 2+ 4$, $b = 1 + 2$

$\boxed{a = 6}$, $\boxed{b = 3}$

(ii) $(2a + 5, 3) = (7, b - 4)$

$2a + 5 = 7$, $3 = b - 4$

$2a = 7 - 5$, $3 + 4 = b$

$2a = 2 \quad , \quad 7 = b$

$a = \frac{2}{2} = 1 \quad , \quad \boxed{b = 7}$

$\boxed{a = 1}$

(iii) (3 – 2a, b – 1) = (a – 7, 2b + 5)

$3 - 2a = a - 7 \quad , \quad b - 1 = 2b + 5$

$3 + 7 = a + 2a \quad , \quad -1 - 5 = 2b - b$

$10 = 3a \quad , \quad -6 = b$

$\frac{10}{3} = a \qquad \boxed{b = -6}$

$\boxed{a = \frac{10}{3}}$

Q.4 Find the sets X and Y if

$X \times Y = \{(a, a), (b, a), (c, a), (d, a)\}$

Solution:

$X \times Y = \{(a, a), (b, a), (c, a), (d, a)\}$

$X \times Y = \{a, b, c, d\} \times \{a\}$

$X = \{a, b, c, d\}$

$Y = \{a\}$

Q.5 If X = {a, b, c} and Y = {d, e}, then find the number of elements in

(i) X × Y (ii) Y × X (iii) X × X

Solution:

No. of elements in X = 3

No. of elements in Y = 2

(i) No. of Elements in $X \times Y = 3 \times 2 = 6$

(ii) No. of Elements in $Y \times X = 2 \times 3 = 6$

(iii) No. of Elements in $X \times X = 3 \times 3 = 9$

Binary Relation:

If A and B are any two non-empty sets, then a subset $R \subseteq A \times B$ is called binary relation from set A into set B, because there exists some relationship between first and second element of each ordered pair in R.

Domain of relation denoted by Dom R is the set consisting of all the first elements of each ordered pair in the relation.

Range of relation denoted by Rang R is the set consisting of all the second elements of each ordered pair in the relation.

Example 1: Suppose

$A = \{2, 3, 4, 5\}$ and $B = \{2, 4, 6, 8\}$

Form a relation

$R: A \to B = \{x\ R\ y \mid y = 2x \text{ for } x \in A, y \in B\}$

$R = \{(2, 4), (3, 6), (4, 8)\}$

Dom $R = \{2, 3, 4\} \subseteq A$ and

Rang $R = \{4, 6, 8\} \subseteq B$

Example 2: Suppose

$A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 5\}$

Form a relation

$R: A \to B = \{x\ R y \mid x + y = 6 \text{ for } x \in A, y \in B\}$

$R = \{(1, 5), (3, 3), (4, 2)\}$

Dom $R = \{1, 3, 4\} \subseteq A$ and

Rang $R = \{2, 3, 5\} \subseteq B$

Function or Mapping:

(i) Suppose A and B are two non-empty sets, then relation $f : A \to B$ is called a function if (i) Dom f = A (ii) every $x \in A$ appears in one and only one ordered pair in f.

Alternate Definition:

Suppose A and B are two non-empty sets, then relation $f : A \to B$ is called a function if (i) Dom f = A (ii) $\forall x \in A$ we can associate some unique image element $y = f(x) \in B$.

Domain, Co-domain and Range of Function:

If $f : A \to B$ is a function then A is called the domain of f and B is called the co-domain of f.

Domain f is the set consisting of all first elements of each ordered pair in f and range f is the set consisting of all second elements of each ordered pair in f.

Example:

Suppose $A = \{0,1,2,3\}$ and $B = \{1,2,3,4,5\}$

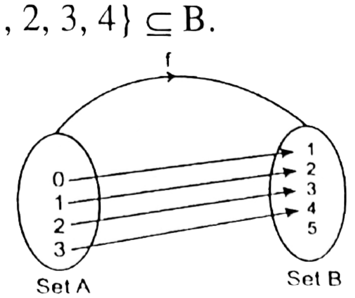
Define a function $f : A \to B$

$f = \{(x, y) \mid y = x + 1\ \forall x \in A, y \in B\}$

$f = \{(0, 1), (1, 2), (2, 3), (3, 4)\}$

Dom $f = \{0, 1, 2, 3\} = A$

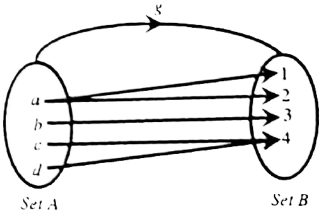
Rang $f = \{1, 2, 3, 4\} \subseteq B$.

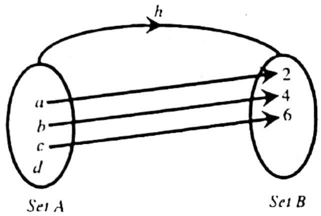


The following are the examples of relations but not functions.

g is not a function, because an element $a \in A$ has two images in set B.

And h is not a function because an element $d \in A$ has no images in set B.

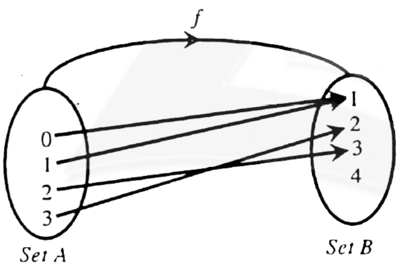




Demonstrate the following:

(a) Into function:

A function $f : A \rightarrow B$ is called an into function, if at least one element in B is not an image of some element of set A i.e, Range of $f \subset$ set B.



For example, we define a function $f: A \rightarrow B$ such that.

$f : A \rightarrow B$ such that

$f = \{(0, 1), (1, 1), (2, 3), (3, 2)$

where $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$

f is an into function.

(b) One- one function:

A function $f : A \rightarrow B$ is called one – one function, if all distinct elements of A have distinct images in B, i.e:

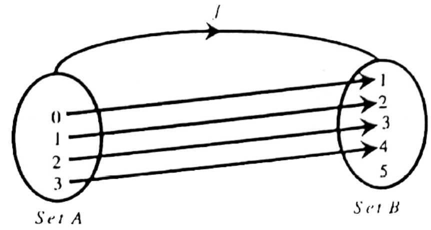
$f(x_1) = f(x_2) \quad x_1 = x_2 \in A$ or

$\forall\, x_1 \neq x_2 \in A \quad f(x_1) \neq f(x_2)$

For example, if $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$, then we define a function $f : A \rightarrow B$ such that

$f = \{(x, y) \mid y = x + 1, \forall\, x \in A, y \in B\}$

$= \{(0, 1), (1, 2), (2, 3), (3, 4)\}$



f is one-one function

(c) Into and one - one function: (Injective function)

The function f discussed in (b) is also an into function. Thus f is an into and one-one function.

(d) An onto or surjective function:

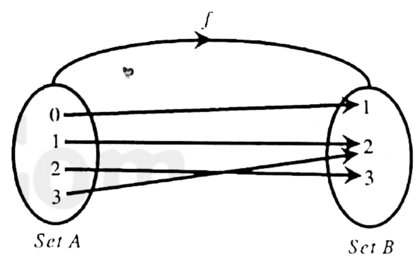
A function $f : A \rightarrow B$ is called an onto function, if every element of set B is an image of at least one element of set A i.e, Range of $f = B$

For example, if $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3\}$, then $f : A \rightarrow B$ such that

$f = \{(0, 1), (1, 2), (2, 3), (3, 2)\}$

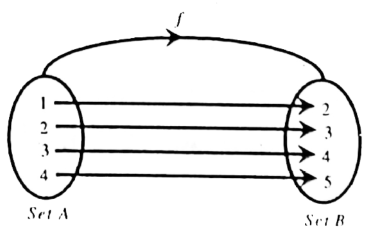
Here Rang $f = \{1, 2, 3\} = B$

Thus f so defined is an onto function.



(e) Bijective function or one to one correspondence:

A function $f : A \rightarrow B$ is called bijective function if function f is one- one and onto. e.g., if $A = \{1, 2, 3, 4\}$ and $B = \{2, 3, 4, 5\}$



We define a function $f : A \rightarrow B$ such that

$f = \{(x, y) \mid y = x + 1, \ \forall\, x \in A, \ y \in B\}$

Then $f = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$

Evidently this function is one-one because distinct elements of A have distinct images in B. This is an onto function also because every element of B is the image of at least one element of A.

Note: (1) Every function is a relation but converse may not be true.

(2) Every function may not be one – one.

(3) Every function may not be onto.

Example: Suppose A = {1, 2, 3}

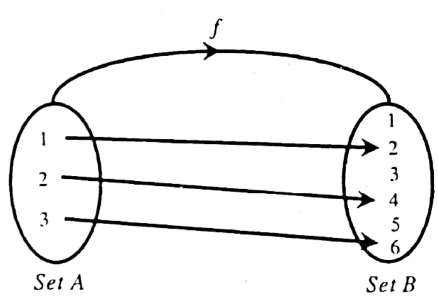
B = {1, 2, 3, 4, 5, 6}

We define a function

$f : A \to B = \{(x, y) \mid y=2x, \forall\, x \in A,\; y \in B\}$

Then f = {(1, 2), (2, 4), (3, 6)}

Evidently this function is one-one but not an onto.



Examine whether a given relation is a function:

A relation in which each x ∈ its domain, has a unique image in its range, is a function.

Differentiate between one-to-one correspondence and one-one function:

A function f from set A to set B is one-one if distinct elements of A has distinct images in B. The domain of f is A and its range is contained in B.

In one-to-one correspondence between two sets A and B, each element of either set is assigned with exactly one element of the other set. If the sets A and B are finite, then these sets have the same number of elements, that is, n(A) = n(B).

EXERCISE 5.5

Q.1 If L = {a, b, c}, M = {3, 4}, then find two binary relations of L×M and M×L.

Solution: $L = \{a, b, c\}$, $M = \{3, 4\}$

$L\times M = \{a, b, c\} \times \{3, 4\}$

$= \{(a, 3), (a, 4), (b, 3), (b, 4), (c, 3), (c, 4)\}$

Two binary Relations:

$R_1 = \{(a, 3), (a, 4)\}$

$R_2 = \{(b, 4), (c, 3), (c, 4)\}$

$M\times L = \{3, 4\} \times \{a, b, c\}$

$= \{(3, a), (3, b), (3, c), (4, a), (4, b), (4, c)\}$

Two binary Relations:

$R_1 = \{(3, a), (3, b)\}$

$R_2 = \{(4, b), (4, c)\}$

Q.2 If Y = {-2, 1, 2}, then make two binary relations for Y × Y. Also find their domain and range.

Solution: $Y = \{-2, 1, 2\}$

$Y\times Y = \{-2, 1, 2\}\times\{-2, 1, 2\}$

$= \{(-2,-2), (-2,1), (-2,2), (1,-2), (1,1), (1,2),$
$(2,-2,)\ (2,1), (2,2)\}$

$R_1 = \{(-2, 1), (-2, 2), (1, -2)\}$

Domain $R_1 = \{-2, 1\}$

Range $R_1 = \{1, 2, -2\}$

$R_2 = \{(1, 1), (2, -2), (2, 2)\}$

Domain $R_2 = \{1, 2\}$

Range $R_2 = \{1, -2, 2\}$

Q.3 If L = {a, b, c} and M = {d, e, f, g}, then find two binary relations in each.

(i) L×L (ii) L×M (iii) M×M

Solution: $L = \{a, b, c\}$, $M = \{d, e, f, g\}$

(i) $L\times L = \{a, b, c\} \times \{a, b, c\}$

$L\times L=\{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),$
$(c,a), (c, b),(c, c)\}$

Two binary Relations:

$R_1 = \{(a, a), (a, b), (a, c)\}$

$R_2 = \{(b, c), (c, a), (c, b)\}$

(ii) L×M

$L\times M = \{a, b, c\} \times \{d, e, f, g\}$

$= \{(a,d),(a,e),(a,f),(a,g),(b,d),(b,e),(b,f),$
$(b,g), (c, d), (c, e), (c, f), (c, g)\}$

Two binary Relations:

$R_1 = \{(a, d), (a, e), (a, f)\}$

$R_2 = \{(b, d), (b, e), (b, f)\}$

(iii) M×M

$M\times M = \{d, e, f, g\} \times \{d, e, f, g\}$

$=\{(d,d),(d,e),(d,f),(d,g),(e,d),(e,e),(e,f),(e,g),$
$(f,d),(f,e),(f,f),(f,g),(g,d),(g,e),(g,f),(g,g)\}$

Two binary Relations:

$R_1= \{(d, e), (d, f), (d, g)\}$

$R_2 = \{(f, d), (g, d)\}$

Q.4 If set M has 5 elements then find the number of binary relations in M.

No. of binary relations in $M=2^{5\times 5}=2^{25}$

Q.5 If $L = \{x|x \in N \wedge x \leq 5\}$,

$M = \{y|y \in P \wedge y < 10\}$, then make the following relations from L to M. Also write Domain and Range of each Relation.

(i) $R_1 = \{(x, y) \mid y < x\}$,

(ii) $R_2 = \{(x, y) \mid y = x\}$

(iii) $R_3 = \{(x, y) \mid x + y = 6\}$

(iv) $R_4 = \{(x, y) \mid y - x = 2\}$

Solution

$L = \{1, 2, 3, 4, 5\}$,

$M = \{2, 3, 5, 7\}$

$L\times M = \{1, 2, 3, 4, 5\} \times \{2, 3, 5, 7\}$

$= \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),$
$(2,7),(3,2),(3,3),(3,5),(3,7),(4,2),(4,3),(4,5),$
$(4,7),(5,2),(5,3),(5,5),(5,7)\}$

(i) $R_1 = \{(x, y) \mid y < x\}$

$R_1 = \{(3, 2), (4, 2), (4, 3), (5, 2), (5, 3)\}$

Domain $R_1 = \{3, 4, 5\}$

Range $R_1 = \{2, 3\}$

(ii) $R_2 = (x, y) \mid y = x\}$

$R_2 = \{(2, 2), (3, 3), (5, 5)\}$

Domain $R_2 = \{2, 3, 5\}$

Range $R_2 = \{2, 3, 5\}$

(iii) $R_3 = \{(x, y) \mid x + y = 6\}$

$R_3 = \{(1, 5), (3, 3), (4, 2)\}$

Domain $R_3 = \{1, 3, 4\}$

Range $R_3 = \{5, 3, 2\}$

(iv) $R_4 = \{(x, y) \mid y - x = 2\}$

$R_4 = \{(1, 3), (3, 5), (5, 7)\}$

Domain $R_4 = \{1, 3, 5\}$

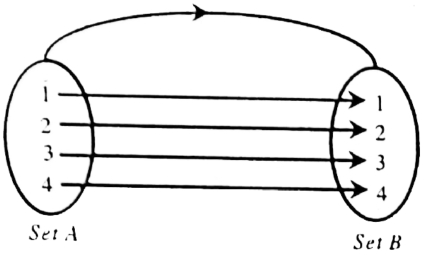
Range $R_4 = \{3, 5, 7\}$

Q.6 Indicate relations, into function, one-one function, onto function, and bijective function from the following. Also find their domain and the range.

$R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

Domain $R_1 = \{1, 2, 3, 4\}$

Range $R_1 = \{1, 2, 3, 4\}$

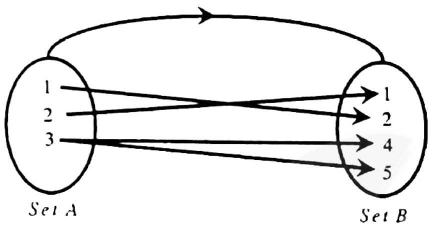


It is a bijective function.

(ii) $R_2 = \{(1, 2), (2, 1), (3, 4), (3, 5)\}$

Domain $R_2 = \{1, 2, 3\}$

Range $R_2 = \{1, 2, 4, 5\}$

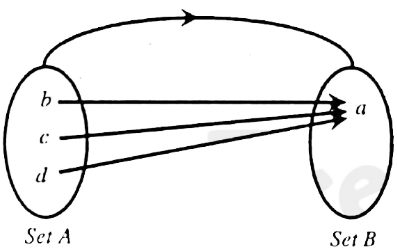


It is a relation. As 3 has no distinct image.

(iii) $R_3 = \{(b, a), (c, a), (d, a)\}$

Domain $R_3 = \{b, c, d\}$

Range $R_3 = \{a\}$

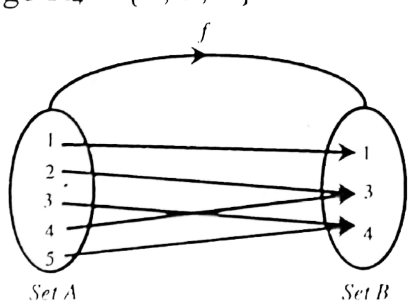


It is an onto function.

(iv) $R_4 = \{(1, 1), (2, 3), (3, 4), (4, 3), (5, 4)\}$

Domain $R_4 = \{1, 2, 3, 4, 5\}$

Range $R_4 = \{1, 3, 4\}$



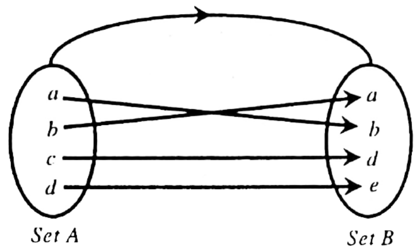
It is an onto function.

(v) $R_5 = \{(a, b), (b, a), (c, d), (d, e)\}$

Domain $R_5 = \{a, b, c, d\}$

Range $R_5 = \{a, b, d, e\}$

It is a bijective function.

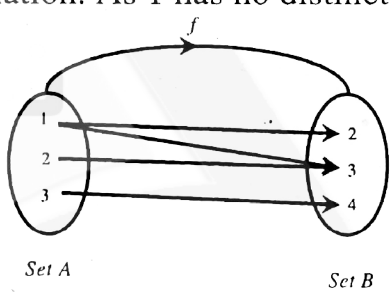


(vi) $R_6 = \{(1, 2), (2, 3), (1, 3), (3, 4)\}$

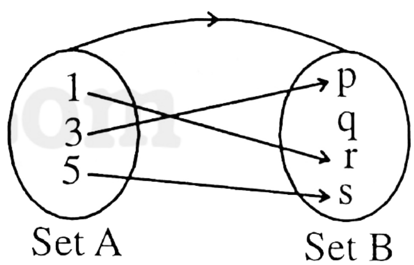
Domain $R_6 = \{1, 2, 3\}$

Range $R_6 = \{2, 3, 4\}$

It is a relation. As 1 has no distinct image.



(vii) $R_7 =$

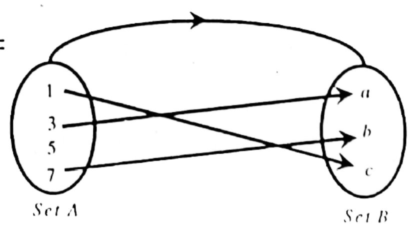


Domain $R_7 = \{1, 3, 5\}$

Range $R_7 = \{p, r, s\}$

R_7 is one-one function.

(viii) $R_8 =$



Domain $R_8 = \{1, 3, 7\}$

Range $R_8 = \{a, b, c\}$

It is a relation. As 5 has no distinct image.

Miscellaneous Exercises - 5

Q.1 Multiple choice questions. Four possible answers are given for the following questions. Tick mark ( ) the correct answer.

1. The different number of ways to describe a set are
   (a) 1 (b) 2
   (c) 3 (d) 4
2. The set $\{x / x \in W \wedge x \leq 101\}$ is
   (a) infinite set (b) subset
   (c) Null set (d) finite set
3. A collection of well-defined distinct objects is called
   (a) subset (b) power set
   (c) set (d) none of these
4. If $A \subseteq B$ then $A \cup B$ is equal to
   (a) A (b) B
   ϕ
5. A set $Q = \frac{a}{b} | a, b \in Z \wedge b \neq 0$ is called a set of
   (a) Whole numbers
   (b) Natural numbers
   (c) Irrational numbers
   (d) Rational numbers
6. If $A \subseteq B$ then $A - B$ is equal to
   (a) A (b) B
   (c) ϕ (d) None of these
7. If A and B are disjoint sets, then $A \cup B$ is equal to
   (a) A (b) B
   (c) ϕ (d) $B \cup A$
8. The number of elements in power set $\{1, 2, 3\}$ is
   (a) 4 (b) 6
   (c) 8 (d) 9
9. A set with no element is called
   (a) subset (b) empty set
   (c) singleton set (d) super set
10. If $A \subseteq B$ then $A \cap B$ is equal to
    (a) A (b) B
    (c) ϕ (d) None of these
11. The set having only one element is called
    (a) Null set (b) power set
    (c) singleton set (d) subset
12. The relation $\{(1,2),(2,3),(3,3)(3,4)\}$ is
    (a) onto function
    (b) into function
    (c) not a function
    (d) one-one function
13. If $A \subseteq B$ and $B \subseteq A$, then
    (a) A = B (b) A ≠ B
    (c) $A \cap B = \phi$ (d) $A \cup B = \phi$
14. Power set of an empty set is
    (a) ϕ (b) $\{a\}$
    (c) $\{\phi, \{a\}\}$ (d) $\{\phi\}$
15. If number of elements in set A is 3 and in set B is 4, then number of elements in A×B is
    (a) 3 (b) 4
    (c) 12 (d) 7
16. Point (-1,4) lies in the quadrant
    (a) I (b) II
17. The domain of R $=\{(0,2),(2,3),(3,3)(3,4)\}$ is
    (a) $\{0,3,4\}$ (b) $\{0,2,3\}$
    (c) $\{0,2,4\}$ (d) $\{2,3,4\}$
18. The point $(-5, -7)$ lies in … quadrant
    (a) I (b) II
    (c) III (d) IV

19. If number of elements in se A is 3 and in set B is 2, then number of binary relations in A×B is

(a) 2^3 (b) 2^6

(c) 2^8 (d) 2^2

20. $(A \cup B) \cup C$ is equal to

(a) $A \cap (B \cup C)$ (b) $(A \cup B) \cap C)$

(c) $A \cup (B \cup C)$ (d) $A \cap (B \cap C)$

21. If $A \cap B = \phi$, then set A and B are ....sets.

(a) sub (b) over lapping

(c) disjoint (d) power

22. $A \cup (B \cap C)$ is equal to

(a) $(A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cap C)$

(c) $(A \cap B) \cap (A \cap C)$

(d) $A \cup (B \cup C)$

23. The range of {(a,a), (b,b), (c,c)} is......

(a) {a,b} (b) {a,b,c}

(c) {a} (d) ϕ

24. The complement of ϕ is ......

(a) U (b) ϕ

(c) impossible (d) union

25. $A \cap A^c = ...$

(a) U (b) A

(c) A^c (d) ϕ

26. Venn diagram was first used by.......

(a) John Venn

(b) Netwon

(c) Arthur Cayley

(d) John Napier

27. The set { x | x ∈ A and x ∉ B } is ......

(a) $A \cup B$ (b) $A \cap B$

(c) A – B (d) B – A

28. The Range of R ={(1,3),(2,2),(3,1)(4,4)} is

(a) {1,2,4} (b) {3,2,4}

(c) {1,2,3,4} (d) {1,3,4}

29. $N \cup W = ...$

(a) ϕ (b) {0}

(c) N (d) W

30. y co-ordinate of every pint on x – axis is..

(a) +ve (b) –ve

(c) zero (d) 1

31. By definition, which of the following is a set?

(a) {a, b, c, a} (b) {1, 2, 3, 2}

{ ,m, n, o} {0, 1, 2, 3, 1}

32. The domain of {(a,b), (b,c), (c,d)} is...

(a) {a,b,c} (b) {b,c,d}

(c) {a,b} (d) {a,b,c,d,}

33. The complement of U is ....

(a) U (b) ϕ

(c) impossible (d) union

34. $A \cup A^c = ...$

(a) U (b) A

(c) A^c (d) ϕ

35. Which of the following is true?

(a) $P \subseteq N \subseteq Z \subseteq W$

(b) $P \subseteq N \subseteq W \subseteq Z$

(c) $P \subseteq W \subseteq N \subseteq Z$

(d) $P \subseteq Z \subseteq N \subseteq W$

36. W – N = ...

(a) ϕ (b) {0}

(c) N (d) W

37. N – W = ...

(a) ϕ (b) {0}

(c) N (d) W

38. The relation {(a,b),(b,c),(a,d),} is....

(a) a function (b) not a function

(c) range (d) domain 05(091)

39. x co-ordinate of every pint on y – axis is..

(a) +ve (b) –ve

(c) zero (d) 1

40. Which of the following is true?

(a) $W \subseteq N$

(b) $Z \subseteq W$

(c) $N \subseteq P$

(d) $P \subseteq W$

41. A subset of $A \times A$ is called......in A.

(a) set (b) relation

(c) function (d) into function

42. Which of the following is true?

(a) N and $W \subseteq Z$

(b) P and $O \subseteq W$

(c) O and $E \subseteq W$

(d) P and $E \subseteq N$

43. If $x \in A$ and $x \in B$, then $\{x\}$ is equal to

(a) $A - B$ (b) A^c

(c) $A \cap B$ (d) B^c

44. The point (4, –6) lies in .... Quadrant

(a) I (b) II

(c) III (d) IV

45. If $f: A \to B$ and range of $f \neq B$, then f is an ....

(a) into function

(b) onto function

(c) bijective function

(d) function

46. If $f: A \to B$ and range of $f = B$, then f is an......

(a) into function

(b) onto function

(c) bijective function

(d) function

47. If set A has all its elements common with set B then set A is called....set.

(a) sub (b) overlapping

(c) disjoint (d) super

48. $O \cup E =$ ....

(a) ϕ (b) O

(c) E (d) Z

49. Which of the following is commutative law?

(a) $A \cup (B \cup C) = (A \cup B) \cup C$

(b) $A \cap (B \cap C) = (A \cap B) \cap C$

(c) $A \cap B = B \cap A$

(d) $)A \cup B(^C = A^C \cap B^C$

50. Which of the following is associative law of intersection?

(a) $A \cup (B \cup C) = (A \cup B) \cup C$

(b) $A \cap (B \cap C) = (A \cap B) \cap C$

(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

51. Which of the following is distributive property of intersection over union?

(a) $A \cup (B \cup C) = A \cup (B \cup C)$

(b) $A \cap (B \cap C) = (A \cap B) \cap C$

(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

52. $N \cap W =$ ....

(a) ϕ (b) $\{0\}$

(c) N (d) W

53. If A is subset of U, then $(A^C)^C =$ ......

(a) A (b) A^c

(c) ϕ^C (d) U^c

54. If two sets have some elements common but not all are called....sets.

(a) sub (b) overlapping

(c) disjoint (d) super

55. Which of the following is De-Morgan's law?

(a) $(A \cup B) \cup C = A \cup (B \cup C)$

(b) $)A \cap B(^C = A^C \cup B^C$

(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

56. Which of the following is associative law of union?

(a) $A \cup (B \cup C) = (A \cup B) \cup C$

(b) $A \cap (B \cap C) = (A \cap B) \cap C$

(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

57. $O - E =$ ....

(a) ϕ (b) O

(c) E (d) Z

58. If union and intersection of two sets are equal then sets are ......sets.

(a) disjoint (b) overlapping

(c) equal (d) super

59. Which of the following is complete description of Real numbers?

(a) $N \cup W = R$ (b) $O \cup E = R$

(c) $P \cup Q = R$ (d) $Q \cup Q' = R$

60. $E - O =$ ....

(a) ϕ (b) O

(c) E (d) Z

61. Two sets having no common element, are called .... sets.

(a) subet (b) overlapping

(c) disjoint (d) super

62. If $x \in U$ and $x \notin A$, then $\{x\}$ is equal to.............

(a) U^C (b) A^C

(c) ϕ^C (d) $A - U$

63. $O \cap E =$ ....

(a) ϕ (b) O

(c) E (d) Z

64. A and A^C are ....sets.

(a) universal (b) overlapping

(c) disjoint (d) super

65. Which of the following is distributive property of union over intersection?

(a) $A \cup (B \cup C) = A \cup (B \cup C)$

(b) $A \cap (B \cap C) = (A \cap B) \cap C$

(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

66. If $x \in A$ and $x \notin B$, then $\{ x \}$ is equal to..........

(a) $A - B$ (b) $B - A$

(c) $A \cap B$ (d) A^c

ANSWER KEY

1	c	**2**	d	**3**	c	**4**	b	**5**	d	**6**	c	**7**	d	**8**	c
9	b	**10**	a	**11**	c	**12**	c	**13**	a	**14**	d	**15**	c	**16**	b
17	b	**18**	c	**19**	b	**20**	c	**21**	c	**22**	a	**23**	b	**24**	a
25	d	**26**	a	**27**	c	**28**	c	**29**	d	**30**	c	**31**	c	**32**	a
33	b	**34**	a	**35**	b	**36**	b	**37**	a	**38**	b	**39**	c	**40**	d
41	b	**42**	a	**43**	c	**44**	d	**45**	a	**46**	b	**47**	a	**48**	d
49	c	**50**	b	**51**	d	**52**	c	**53**	a	**54**	b	**55**	b	**56**	a
57	b	**58**	c	**59**	d	**60**	c	**61**	c	**62**	b	**63**	a	**64**	c
65	c	**66**	a												

Q.2 Write short answers of the following questions.

(i) Define a subset and give one example.

Subset: If A and B are two sets and every element of A is an element of B then set A is called subset of set B. It is denoted by $A \subseteq B$.

Example: $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$

As all elements of Set A are also present in Set B. Therefore $A \subseteq B$.

(ii) Write all subsets of the set {a, b}

Solution:

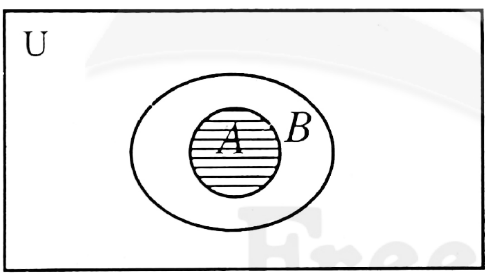
All subsets ($2^n = 2^2 = 4$)

{ }, {a}, {b}, {a, b}

(iii) Show $A \cap B$ by Venn Diagram when $A \subseteq B$.

Solution:

If $A \subseteq B$ then $A \cap B = A$



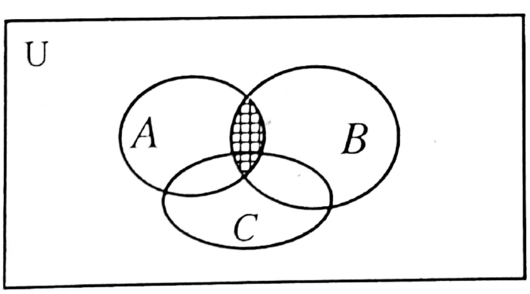
$A \cap B$

Horizontal line segments show $A \cap B$

(iv) Show by Venn Diagram $A \cap (B \cup C)$

Solution:

$A \cap (B \cup C)$ by Venn diagram



- Horizontal line segments and squares show $B \cup C$.
- $A \cap (B \cup C)$ is shown by squares.

(v) Define intersection of two sets.

Intersection of two sets:

The intersection of two sets A and B, written as $A \cap B$ (read as A intersection B) is the set consisting of all the common elements of A and B.

(vi) Define a function.

Function: Suppose A and B are two non-empty sets, then relation $f : A \rightarrow B$ is called a function if

(i) Dom f = A

(ii) Every $x \in A$ appears in one and only and ordered pair in f.

(vii) Define an one – one function.

One – one function:

A function $f : A \rightarrow B$ is called one – one function, if all distinct elements of A have distinct images in B, i.e:

$f(x_1) = f(x_2)$ $\quad x_1 = x_2 \in A$ $\quad$ or

$\forall\, x_1 \neq x_2 \in A$ $\quad f(x_1) \neq f(x_2)$

(viii) Define an onto function.

Onto function:

A function $f : A \rightarrow B$ is called an onto function, if every element of set B is an image of at least one element of set A.

i.e. Range of f = B

(ix) Define a Bijective function.

Bijective function:

A function $f : A \rightarrow B$ is called bijective function iff function f is one-one and onto.

(x) Write De Morgan's law.

De Morgan's Law:

If two sets A and B are the sub sets of U then De-Morgan's laws are expressed as

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Q.3 Fill in the Blanks

i. If $A \subseteq B$ then $A \cup B =$ _______.

ii. If $A \cap B = \phi$ then A and B are _______.

iii. If $A \subseteq B$ and $B \subseteq A$ then_________.

iv. $A \cap (B \cup C) =$ _______ .

v. $A \cup (B \cap C) =$ _______ .

vi. The complement of U is __________.

vii. The complement of ϕ is__________.

viii. $A \cap A^c =$ _______.

ix. $A \cup A^c =$ _______.

x. The set $\{x | x \in A \; and \; x \notin B\} =$ _____.

xi. The point (-5, -7) lies in _______ quadrant.

xii. The point (4,-6) lies in _______quadrant.

xiii. The y co-ordinate of every point is ___________on x-axis.

xiv. The x co-ordinate of every point is ___________on y-axis.

xv. The domain of {(a,b), (b,c), (c,d)} is _______.

xvi. The range of {(a,a), (b,b), (c,c)} is _______.

xvii. Venn-diagram was first used by________.

xviii. A subset of A x A is called the ______in A.

xix. If $f: A \longrightarrow B$ and range of f=B, then f is an _______function.

xx. The relation of {(a,b), (b,c), (a,d)} is ________ function.

Answers:

(i)	B	(ii)	Disjoint sets
(iii)	A = B	(iv)	$(A \cap B) \cup (A \cap C)$
(v)	$(A \cup B) \cap (A \cup C)$		
(vi)	ϕ	(vii)	U
(viii)	ϕ	(ix)	U
(x)	$A \setminus B$	(xi)	IIIrd
(xii)	IVth		
(xiii)	Zero	(xiv)	Zero
(xv)	{a, b, c}		
(xvi)	{a, b, c}	(xvii)	John venn
(xviii)	Binary relation		
(xix)	onto	(xx)	not