Unit 07 INTRODUCTION TO TRIGONOMETRY

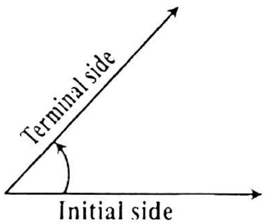
Measurement of an Angle

Angle:

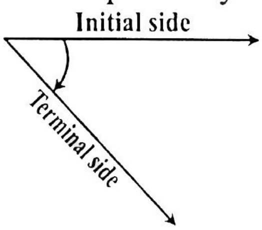
An angle is defined as the union of two non–collinear rays with some common end point. The rays are called arms of the angle and the common end point is known as vertex of the angle.

Clockwise or Anti–clockwise angle:

It is easy if we make an angle by rotating a ray from one position to another. When we form an angle in this way, the original position of the ray is called initial side and final position of the ray is called the terminal side of the angle. If the rotation of the ray is anti–clockwise or clockwise, the angle has positive or negative measure respectively.



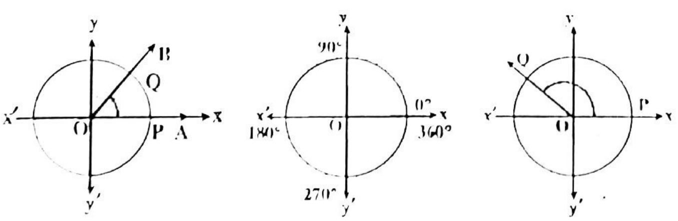
Anti - clockwise rotation
positive measure



Clockwise rotation
Negative measure

Measurement of an angle in sexagesimal system (degree, minute and second)

Degree: We divide the circumference of a circle into 360 equal arcs. The angle subtended at the centre of the circle by one arc is called one degree and is denoted by $1°$.



The symbols $1°$, $1'$ and $1''$ are used to denote a degree, a minute and a second respectively.

Thus 60 seconds $(60'')$ make one minute $(1')$

60 minutes $(60')$ make one degree $(1°)$

90 degrees $(90°)$ make one right angle.

360 degrees $(360°)$ make 4 right angles.

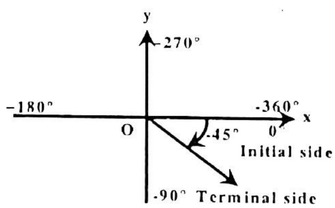
An angle of $360°$ denotes a complete circle or one revolution. We use coordinate system to locate any angle to a standard position, where its initial side is the positive x–axis and its vertex is the origin.

Example: Locate

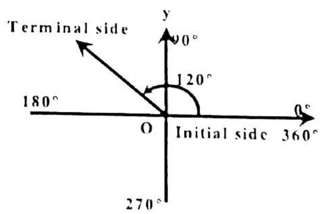
(a) $-45°$ (b) $120°$

(c) $45°$ (d) $-270°$

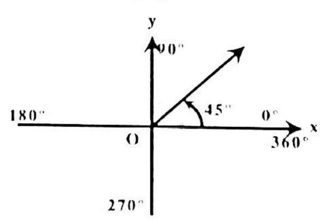
Solution: The angles are shown in figure.



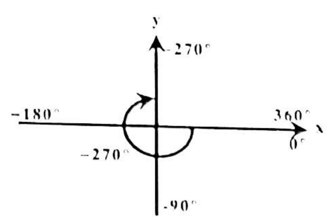
(a)



(b)



(c)



(d)

Conversion of an angle given in D°M′S″ form into decimal form and vice versa.

Example 1:

(i). Convert 25°30′ to decimal degrees.

(ii) Convert 32.25° to D°M′S″ form.

Solution:

(i) $25°30' = 25° + \left(\frac{30}{60}\right)^{o} = 25° + 0.5° = 25.5°$

(ii) $32.25° = 32° + 0.25° = 32° + \left(\frac{25}{100}\right)^{o}$

$= 32° + \frac{1°}{4} = 32° + \left(\frac{1}{4} \times 60\right)'$

$= 32°15'$

Example 2:

Convert 12°23′35″ to decimal degrees correct to three decimal places.

Solution:

$12°23'35'' = 12° + \frac{23°}{60} + \frac{35°}{60 \times 60}$

$= 12° + \frac{23°}{60} + \frac{35°}{3600}$

$= 12° + 0.3833° + 0.00972°$

$= 12.3930°$

$= 12.393°$

Example 3:

Convert 45.36° to D°M′S″ form.

Solution: $(45.36)°$

$= 45° + (0.36)°$

$= 45° + \left(\frac{36}{100} \times 60\right)'$

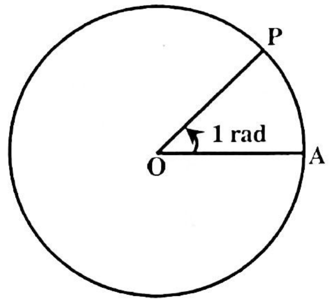
$= 45° + 21.6'$

$= 45° + 21' + (0.6 \times 60)''$

$= 45°21'36''$

Radian measure of an angle (circular system)

Radian: The angle subtended at the centre of the circle by an arc, whose length is equal to the radius of the circle is called one Radian.

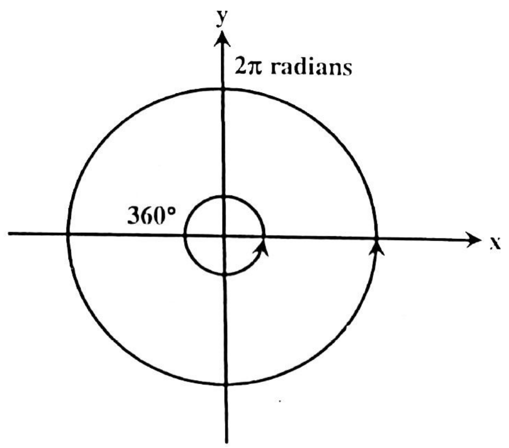


Consider a circle of radius r whose centre is O. From any point A on the circle cut off an arc AP whose length is equal to the radius of the circle. Join O with A and O with P. The ∠AOP is one radian. This means that when

Length of arc AP = length of radius $\overline{OA}$ then $m\angle AOP = 1$ radian

Relationship between radians and degrees

We know that circumference of a circle is $2\pi r$ where r is the radius of the circle. Since a circle is an arc whose length is $2\pi r$. The radian measure of an angle that form a complete circle is $\frac{2\pi r}{r} = 2\pi$



From this we see that $360° = 2\pi$ radians

or $180° = \pi$ radians............ (i)

Using this relation we can convert degrees into radians and radians into degrees as follows:

$$180^\circ = \pi \text{ radian}$$

$$1^\circ = \frac{\pi}{180} \text{ radian,}$$

$$x^\circ = x \,.\, 1^\circ$$

$$x^\circ = x \frac{\pi}{180} \text{ radian.......(ii)}$$

$$1 \text{ radian} = \frac{180}{\pi}^\circ$$

$$y \text{ radian} = y \frac{180}{\pi} \text{ degrees.....(iii)}$$

Some special angles in degree and radians

$$180^\circ = 1(180^\circ) = \pi \text{ radians}$$

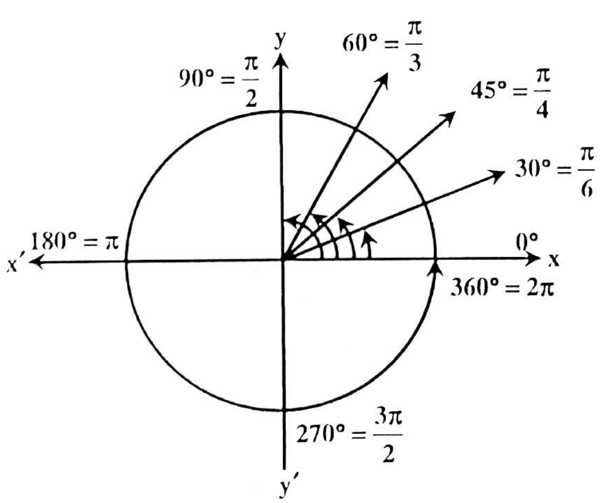
$$90^\circ = \frac{1}{2}(180^\circ) = \frac{\pi}{2} \text{radians}$$

$$60^\circ = \frac{1}{3}(180^\circ) = \frac{\pi}{3} \text{radians}$$

$$45^\circ = \frac{1}{4}(180^\circ) = \frac{\pi}{4} \text{radians}$$

$$30^\circ = \frac{1}{6}(180^\circ) = \frac{\pi}{6} \text{radians}$$

$$270^\circ = \frac{3}{2}(180^\circ) = \frac{3\pi}{2} \text{radians}$$



Example 4:

Convert the following angles into radian measure:

(a) 15° (b) 124°22′

Solution:

(a) $15^\circ = 15 \frac{\pi}{180}$ rads by using (i)

$$15^\circ = \frac{\pi}{12} \text{radians}$$

(b) $124^\circ 22' = \left(124 + \frac{22}{60}\right)^\circ$

$$124^\circ 22' = (124.3666)\frac{\pi}{180} \text{ radians}$$

$$124^\circ 22' \approx 2.171 \text{ radians}$$

Example 5:

Express the following into degree.

(a) $\frac{2\pi}{3}$ radians (b) 6.1 radians

Solution:

(a) $\frac{2\pi}{3}$ radians $= \frac{2\pi}{3}\frac{180}{\pi}$ degrees

$$= 120^\circ$$

(b) 6.1 radians $= (6.1)\frac{180}{\pi}$ degrees

$$= 6.1\,(57.295779)$$

$$= 349.5043 \text{ degrees}$$

Remember that:

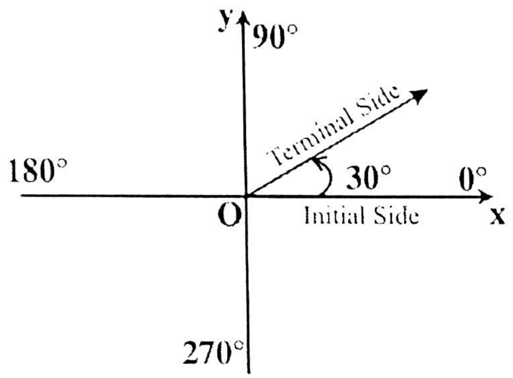
$$1 \text{ radian} \approx \frac{180}{3.1416}^\circ \approx 57.295795^\circ \approx 57^\circ 17' 45''$$

$$1^\circ \approx \frac{3.1416}{180} \approx 0.0175 \text{ radians}$$

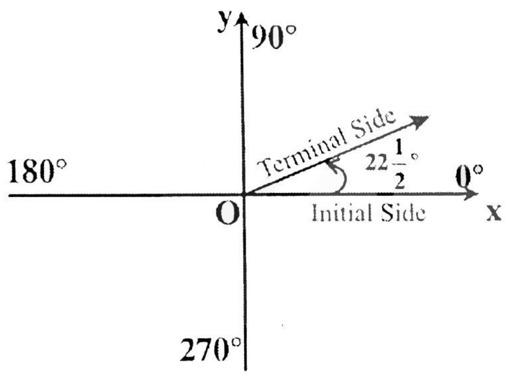
EXERCISE 7.1

Q.1. Locate the following angles:

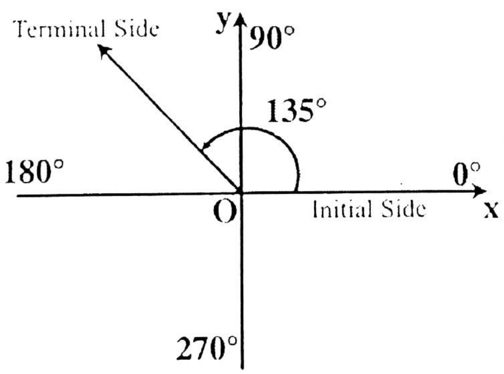
(i) 30°



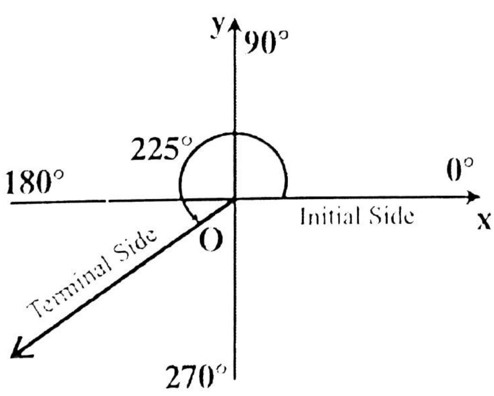
(ii) $22\frac{1}{2}^\circ$



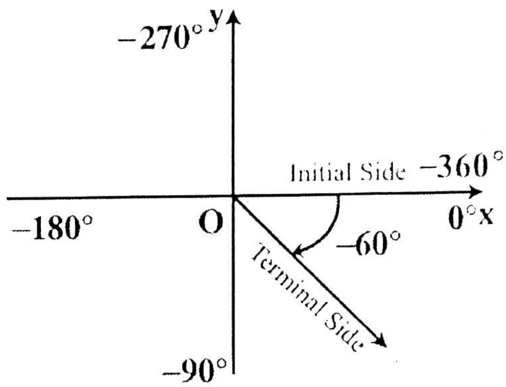
(iii) 135°



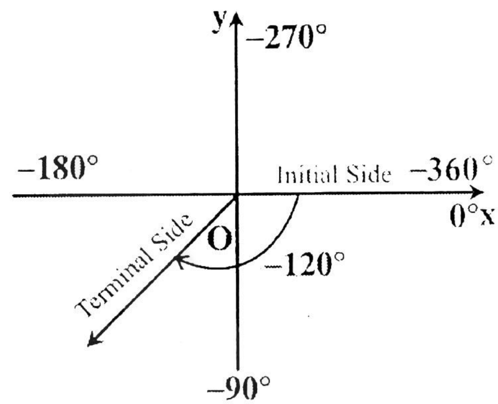
(iv) 225°



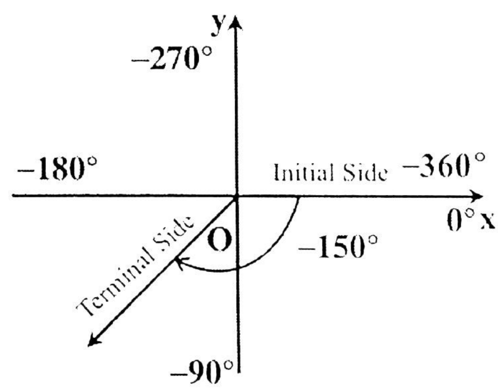
(v) -60°



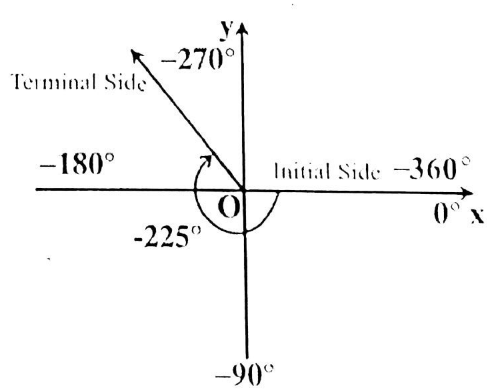
(vi) -120°



(vii) -150°



(viii) -225°



Q.2. Express the following sexagesiml measures of angles in decimal form.

(i) 45°30′

Solution: 45°30′

$$= 45° + \left(\frac{30}{60}\right)^{\circ}$$
$$= 45° + 0.5°$$
$$= 45.5°$$

(ii) 60°30′30″

Solution: 60°30′30″

$$= 60° + \left(\frac{30}{60}\right)^{\circ} + \left(\frac{30}{60\times 60}\right)^{\circ}$$
$$= 60° + 0.5° + 0.008°$$
$$= 60.508°$$

(iii) 125°22′50″

Solution: 125°22′50″

$$= 125° + \left(\frac{22}{60}\right)^{\circ} + \left(\frac{50}{60\times 60}\right)^{\circ}$$
$$= 125° + 0.367° + 0.0139°$$
$$= 125.3808°$$

Q.3. Express the following in D°M′S″:

(i) 47.36°

Solution: 47.36°

$$= 47° + 0.36°$$
$$= 47° + (0.36 \times 60)'$$
$$= 47° + 21.6'$$
$$= 47° + 21' + (0.6 \times 60)''$$
$$= 47° + 21' + 36''$$
$$= 47°\,21'36''$$

(ii) 125.45°

Solution: 125.45°

$$= 125° + 0.45°$$
$$= 125° + (0.45 \times 60)'$$
$$= 125° + 27'$$
$$= 125°27'0''$$

(iii) 225.75°

Solution: 225.75°

$$= 225° + 0.75°$$
$$= 125° + (0.75 \times 60)'$$
$$= 225° + 45'$$
$$= 225°45'0''$$

(iv) – 22.5°

Solution: – 22.5°

$$= -\left[22° + 0.5°\right]$$
$$= -[22° + (0.5 \times 60)']$$
$$= -[22° + 30']$$
$$= -22°30'$$

(v) – 67.58°

Solution: – (67° + 0.58°)

$$= -[67° + (0.58 \times 60)']$$
$$= -[67° + 34.8']$$
$$= -[67° + 34' + 0.8']$$
$$= -[67° + 34' + (0.8 \times 60)'']$$
$$= -[67° + 34' + 48'']$$
$$= -67°34'48''$$

(vi) 315.18°

Solution: 315.18°

$$= 315° + 0.18°$$
$$= 315° + (0.18 \times 60)'$$
$$= 315° + 10.8'$$
$$= 315° + 10' + 0.8'$$
$$= 315° + 10' + (0.8 \times 60)''$$
$$= 315° + 10' + 48''$$
$$= 315°\,10'\,48''$$

Q.4. Express the following angles into radians.

(i) 30°

Solution: 30°

$$= 30\left(\frac{\pi}{180}\right) radians$$
$$= \cancel{30}\left(\frac{\pi}{\cancel{30}\times 6}\right) radians$$
$$= \frac{\pi}{6} \text{ radians}$$

(ii) 60°

Solution: 60°

$$= 60\left(\frac{\pi}{180}\right) radians$$
$$= \cancel{60}\left(\frac{\pi}{\cancel{60}\times 3}\right) radians$$
$$= \frac{\pi}{3} \text{ radians}$$

(iii) **135°**

Solution: 135°

$$= 135 \frac{\pi}{180} \textit{radians}$$

$$= \cancel{45} \times 3 \frac{\pi}{\cancel{45} \times 4} \textit{radians}$$

$$= \frac{3\pi}{4} \text{ radians}$$

(iv) **225°**

Solution: 225°

$$= 225 \frac{\pi}{180} \textit{radians}$$

$$= \cancel{45} \times 5 \frac{\pi}{\cancel{45} \times 4} \textit{radians}$$

$$= \frac{5\pi}{4} \textit{radains}$$

(v) **– 150°**

Solution: – 150°

$$= -150 \frac{\pi}{180} \textit{radians}$$

$$= -5 \times \cancel{30} \frac{\pi}{\cancel{30} \times 6} \textit{radians}$$

$$= \frac{-5\pi}{6} \text{ radians}$$

(vi) **– 225°**

Solution: – 225°

$$= -225 \frac{\pi}{180} \text{radians}$$

$$= -5 \times \cancel{45} \frac{\pi}{\cancel{45} \times 4} \text{radians}$$

$$= \frac{-5\pi}{4} \text{ radians}$$

(vii) **300°**

Solution: 300°

$$= 300 \frac{\pi}{180} \textit{radians}$$

$$= \cancel{60} \times 5 \frac{\pi}{\cancel{60} \times 3} \textit{radians}$$

$$= \frac{5\pi}{3} \text{ radians}$$

(viii) **315°**

Solution: 315°

$$= 315 \frac{\pi}{180} \textit{radians}$$

$$= \cancel{45} \times 7 \frac{\pi}{\cancel{45} \times 4} \textit{radians}$$

$$= \frac{7\pi}{4} \text{ radians}$$

Q.5. Convert each of the following to degrees.

(i) $\frac{3\pi}{4}$

Solution: $\frac{3\pi}{4}$ radians

$$= \frac{3\pi}{4} \frac{180}{\pi} \text{ degrees}$$

$$= \frac{3\cancel{\pi}}{\cancel{4}} \frac{\overset{45}{\cancel{180}}}{\cancel{\pi}} \text{ degrees}$$

$$= 3 \times 45 \text{ degrees}$$

$$= 135°$$

(ii) $\frac{5\pi}{6}$

Solution: $\frac{5\pi}{6}$

$$= \frac{5\pi}{6} \frac{180}{\pi} \text{ degrees}$$

$$= \frac{5\cancel{\pi}}{\cancel{6}} \frac{\overset{30}{\cancel{180}}}{\cancel{\pi}} \text{ degrees}$$

$$= 5 \times 30 \text{ degrees}$$

$$= 150°$$

(iii) $\frac{7\pi}{8}$

Solution: $\frac{7\pi}{8}$ radians

$$= \frac{7\pi}{8} \frac{180}{\pi} \text{ degrees}$$

$$= \frac{7\cancel{\pi}}{8} \frac{180}{\cancel{\pi}} \text{ degrees}$$

$$= \frac{7 \times 180}{8} \text{ degrees}$$

$$= \frac{1260}{8} \text{ degrees}$$

$$= 157.5°$$

(iv) $\frac{13\pi}{16}$

Solution: $\frac{13\pi}{16}$ radians

$= \frac{13\pi}{16}\ \frac{180}{\pi}$ degrees

$= \frac{13\cancel{\pi}}{16}\ \frac{180}{\cancel{\pi}}$ degrees

$= \frac{13\times180}{16}$ degrees

$= \frac{2340}{16}$ degrees

$= 146.25°$

(v) 3 radians

Solution: 3 radians

$= 3\ \frac{180}{\pi}$ degrees

$= \frac{540}{\pi}$ degrees

$= 171.887°$

(vi) **4.5**

Solution: 4.5 radians

$= 4.5\ \frac{180}{\pi}$ degrees

$= \frac{810}{\pi}$ degrees

$= 257.831°$

(vii) $-\frac{7\pi}{8}$

Solution: $-\frac{7\pi}{8}$ radians

$= -\frac{7\cancel{\pi}}{8}\ \frac{180}{\cancel{\pi}}$ degrees

$= \frac{-1260}{8}$ degrees

$= -157.5°$

(viii) $-\frac{13}{16}\pi$

Solution: $-\frac{13}{16}\pi$ radians

$= -\frac{13\cancel{\pi}}{16}\ \frac{180}{\cancel{\pi}}$ degrees

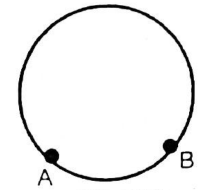
$= \frac{-2340}{16}$

$= -146.25°$

Sector of a Circle

(i) Arc of a Circle

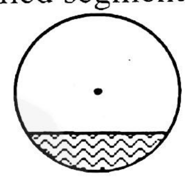
A part of the circumference of a circle is called an arc.



an arc

(ii) Segment of a circle

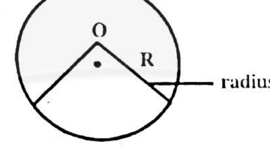
A part of the circular region bounded by an arc and a chord is called segment of a circle.



a segment

(iii) Sector of a circle

A part of the circular region bounded by the two radii and an arc is called sector of the circle.

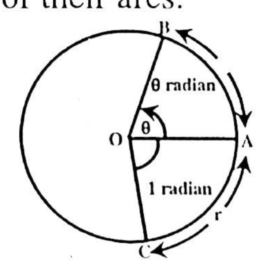


a sector

To establish the rule = r θ:

(where r is the radius of the circle, l the length of circular arc and θ the central angle measured in radians).

Let an arc AB denoted by l subtends an angle θ radian at the centre of the circle. It is a fact of plane geometry that measure of central angles of the arcs of a circle are proportional to the lengths of their arcs.



$$\frac{m\angle AOB}{m\angle AOC} = \frac{m\overline{AB}}{m\overline{AC}}$$

$$\frac{\theta \text{ radian}}{1 \text{ radian}} = \frac{\ }{r}$$

$$\frac{\ }{r} = \theta \quad \text{or} \quad l = r\,\theta$$

Example 1: **In a circle of radius 10m,**

(a) **Find the length of an arc intercepted by a central angle of 1.6 radian.**

(b) **Find the length of an arc intercepted by a central angle of 60°.**

Solution:

(a) Here $\theta = 1.6$ radian , $r = 10$ m and l ? =

Since

$$l = r\theta \qquad l = 10 \times 1.6 = 16 \text{ m}$$

(b) $\theta = 60° = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$ rad

$$\therefore \; l = r\theta = 10 \times \frac{\pi}{3} = \frac{10\pi}{3} \text{ m}$$

Example 2: **Find the distance travelled by a cyclist moving on a circle of radius 15m, if he makes 3.5 revolutions.**

Solution: θ = 1 revolution = 2π radians

3.5 revolution = $2\pi \times 3.5$ radian

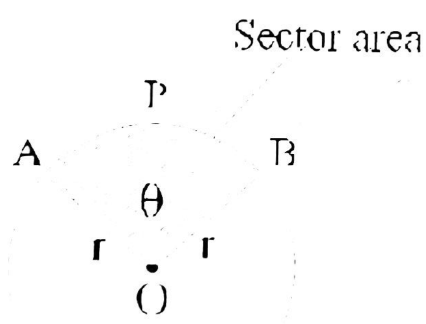
Distance traveled = $l = r\theta$

$= 15\text{m} \times 2\pi \times 3.5$

$= 105\ \pi\text{m}$

Area of circular sector

Consider a circle of radius r units and an arc of length l units, subtending an angle θ at O.



Area of the circle $= \pi r^2$

Angle of the circle $= 2\pi$

Angle of sector $= \theta$ radian

Then by elementary geometry we can use the proportion.

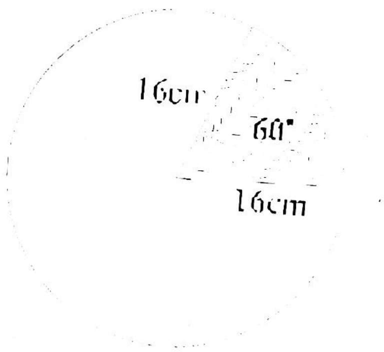
$$\frac{\text{rea of sector AOBP}}{\text{Area of Circle}} = \frac{\text{Angle of sector}}{\text{Angle of circle}}$$

or $$\frac{\text{Area of Sector AOBP}}{\pi r^2} = \frac{\theta}{2\pi}$$

$$\text{Area of Sector AOBP} = \frac{\theta}{2\pi} \times \pi r^2$$

$$\text{Area of Sector AOBP} = \frac{1}{2} r^2 \theta$$

Example 3: **Find area of the sector of a circle of radius 16 cm if the angle at the centre is 60°.**



Solution:

Here central angel = θ = 60°

Now $\theta = 60° \times \frac{\pi}{180}$

$$\theta = \frac{\pi}{3} \text{ rad}$$

$$r = 16\text{cm}$$

$$\text{Area of Sector} = \frac{1}{2} r^2 \theta$$

$$\text{Area of Sector} = \frac{1}{2} (16 \text{ cm})^2 \frac{\pi}{3}$$

$$= \frac{1}{2} (256\text{cm})^2 \times \frac{22}{7 \times 3}$$

$$\approx 1341.1 \text{ cm}^2$$

EXERCISE 7.2

Q.1. Find θ, when:

(i) $l = 2\text{cm}$, $r = 3.5$ cm

Solution : Using rule

$$l = r\theta,$$
$$2 = 3.5\,\theta$$
$$\frac{2}{3.5} = \theta$$
$$\theta = 0.57 \text{ radian}$$

(ii) $l = 4.5$ m, $r = 2.5$ m

Solution : Using rule

$$l = r\theta,$$
$$\frac{}{r} = \theta$$
$$\frac{4.5}{2.5} = \theta$$
$$\theta = 1.8 \text{ radian}$$

Q.2. Find l, when

(i) $\theta = 180°$, $r = 4.9$ cm

Solution: As θ should be in radians so

$$\theta = 180°$$
$$= \cancel{180}\frac{\pi}{\cancel{180}} \text{ radian}$$
$$= \pi \text{ radian}$$

Using rule
$$l = r\theta$$
$$= 4.9 \text{ cm} \times \pi$$
$$= 15.4 \text{ cm}$$

(ii) $\theta = $ **60°30′**, $r = $ **15 mm** 07(036)

Solution : As 'θ' should be in radians, so

$$\theta = 60°30'$$
$$= 60° + \frac{30}{60}^{\circ}$$
$$= 60° + 0.5°$$
$$= 60.5°$$
$$= 60.5\frac{\pi}{180} \text{ radian}$$
$$\theta = 1.056 \text{ radian}$$

Using rule
$$l = r\theta$$
$$= 15 \text{ mm} \times 1.056$$
$$= 15.84 \text{ mm}$$

Q.3. Find r, when

(i) $l = 4$ cm, $\theta = \frac{1}{4}$ radian

Solution: Using rule $l = r\theta$

$$4\text{cm} = r\frac{1}{4}$$
$$4\text{cm} \times 4 = r$$
$$r = 16 \text{ cm}$$

(ii) $l = 52$ cm , $\theta = 45°$

Solution : As θ should be in radians.

$$\theta = 45°$$
$$= 45\frac{\pi}{180} \text{ radian}$$
$$= \frac{\pi}{4} \text{ radian}$$

Now using rule $l = r\theta$

$$52 \text{ cm} = r\frac{\pi}{4}$$
$$\frac{52\text{cm} \times 4}{\pi} = r$$
$$r = 66.21 \text{ cm}$$

Q.4. In a circle of radius 12m, find the length of an arc which subtends a central angle θ = 1.5 radian.

Solution : Radius = $r = 12$m

Arc length = l ? =

Central angle = $\theta = 1.5$ radian

Using rule $l = r\theta$
$$l = 12\text{m} \times 1.5$$
$$l = 18\text{m}$$

Q.5. In a circle of radius 10m, find the distance travelled by a point moving on this circle if the point makes 3.5 revolution.

Solution: Radius = $r = 10$m

Number of revolutions = 3.5

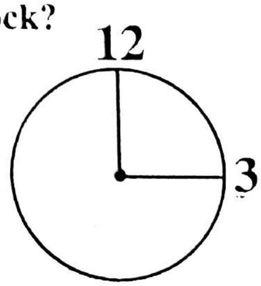
Angle of one revolution = 2π radian

Angle of 3.5 revolution = θ
$$= 3.5 \times 2\pi \text{ radian}$$
$$\theta = 7\pi \text{ radian}$$

Distance travelled = l ? =

Using rule $l = r\theta$
$$l = 10 \text{ m} \times 7\pi$$
$$l = 220 \text{ m}$$

Q.6. What is the circular measure of the angle between the hands of the watch at 3 O' clock?



Solution:

At 3 O' clock the minute hand will be at 12 and hour hand will be at 3 i.e the angle between the hands of watch will be one quarter of the central angle of full circle

$$\text{i.e} = \frac{1}{4} \text{ of } 360^\circ$$

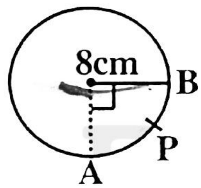
$$= \frac{1}{4} \times 360^\circ$$

$$= 90^\circ$$

$$= 90 \frac{\pi}{180} \text{ radian}$$

$$= \frac{\pi}{2} \text{ radian.}$$

Q.7. What is the length of arc APB?



Solution: From the figure we see that

Radius $= r = 8\text{cm}$

Central angle $= \theta$

$= 90^\circ$

$= \frac{\pi}{2}$ radian

Arc length $= l$? $=$

By rule $l = r\,\theta$

$$l = {}^{4}\cancel{8}\text{cm} \times \frac{\pi}{\cancel{2}}$$

$$l = 4\text{cm} \times \pi$$

$$l = 12.57 \text{ cm}$$

So, length of arc APB is 12.57 cm

Q.8. In a circle of radius 12 cm, how long an arc subtends a central angle of 84°?

Solution: Radius $= r = 12\text{cm}$

Arc length $= l$? $=$

Central angle $= \theta = 84^\circ$

$$84 \frac{\pi}{180} \text{ radian}$$

$$= 1.466 \text{ radian}$$

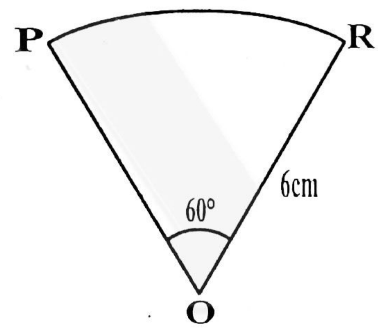
Now by rule $l = r\,\theta$

$$= 12\text{cm} \times 1.466$$

$$= 17.6 \text{ cm}$$

Q.9. Find the area of sector OPR.

(a)



Radius $= r = 6\text{cm}$

Central angle $= \theta = 60^\circ$

$$= 60 \frac{\pi}{180} \text{ radian}$$

$$= \frac{\pi}{3} \text{ radian}$$

Area of sector $=$?

As Area of sector $= \frac{1}{2} r^2 \theta$

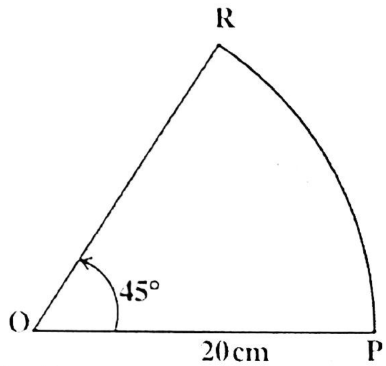
$$= \frac{1}{2} \times (6\text{cm})^2 \times \frac{\pi}{3}$$

$$= \frac{1}{\cancel{6}} \times {}^{6}\cancel{36}\text{cm}^2 \times \pi$$

$$= 6\pi \text{ cm}^2$$

$$= 18.85 \text{ cm}^2$$

(b)



Radius $= r = 20cm$

Central angle $= \theta = 45°$

$= 45 \frac{\pi}{180}$ radian

$= \frac{\pi}{4}$ radian

Area of sector = ?

Area of sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(20cm)^2 \times \frac{\pi}{4}$

$= \frac{400cm^2}{8} \times \pi$

$= 50 \pi \ cm^2$

$= 157.1 \ cm^2$

Q.10. Find area of sector inside a central angle of 20° in a circle of radius 7 m.

Solution: Area of sector = ?

Radius $= r = 7m$

Central angle $= \theta = 20°$

$= 20 \frac{\pi}{180}$ radian

$= \frac{\pi}{9}$ radian

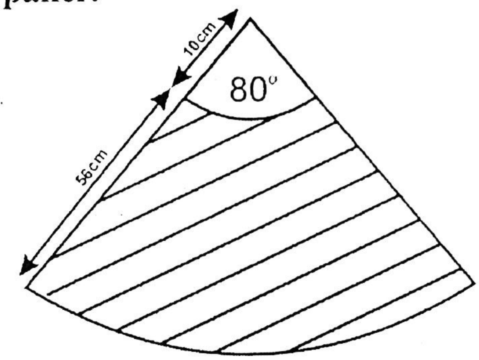
Area of sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times (7m)^2 \times \frac{\pi}{9}$

$= \frac{49\pi}{18} m^2$

$= 8.55 \ m^2$

Q.11. Sehar is making skirt. Each panel of this skirt is of the shape shown shaded in the diagram. How much material (cloth) is required for each panel?



Solution: Central angle $= \theta = 80°$

$= 80 \frac{\pi}{180}$ radian

$= \frac{4\pi}{9}$ radian

Radius of bigger sector $= R = (56 + 10)cm$

$R = 66 \ cm$

Radius of smaller sector $= r = 10 \ cm$

Shaded area = ?

Area of bigger sector $= \frac{1}{2} R^2 \theta$

$= \frac{1}{\cancel{2}} \times (66cm)^2 \times \frac{2\cancel{4}\pi}{9}$

$= \frac{484}{} \cancel{4356} cm^2 \times \frac{2\pi}{\cancel{9}}$

$= 968 \pi \ cm^2$

Area of smaller sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{\cancel{2}} (10cm)^2 \times \frac{2\cancel{4}\pi}{9}$

$= \frac{200}{9} \pi \ cm^2$

Shaded area $= 968 \pi - \frac{200}{9} \pi$

$= \frac{8712\pi - 200\pi}{9}$

$= \frac{8512}{9} \pi \ cm^2$

$= 2971.25 \ cm^2$

Q.12. Find the area of a sector with central angle of $\frac{\pi}{5}$ radian in a circle of radius 10 cm.

Solution: Area of sector = ?

Central angle = $\theta = \frac{\pi}{5}$ radian

Radius = r = 10cm

Area of sector = $\frac{1}{2}r^2\theta$

$= \frac{1}{2}(10\text{cm})^2 \times \frac{\pi}{5}$

$= \frac{1}{10} \times 100\text{cm}^2 \times \pi$

$= 10\pi \text{ cm}^2$

$= 31.43 \text{ cm}^2$

Q.13. The area of sector with central angle θ in a circle of radius 2m is 10 square meter. Find θ in radians.

Solution: Area of sector = 10 m^2

Radius = r = 2m

Central angle = θ ? =

As Area of sector = $\frac{1}{2}r^2\theta$

$10\text{m}^2 = \frac{1}{2}(2m)^2\theta$

$10 \text{ m}^2 = \frac{1}{2}(4m^2)\theta$

$10\text{m}^2 = 2\theta\text{m}^2$

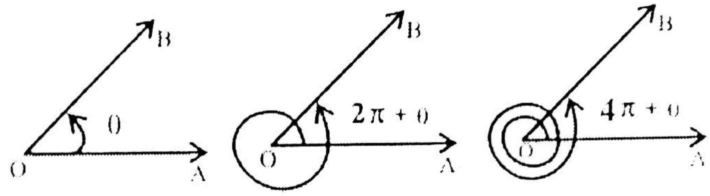
$\theta = \frac{10\text{m}^2}{2\text{m}^2}$

$\theta = 5$ radian

Trigonometric Ratios

General Angle (Coterminal angles)

An angle is indicated by a curved arrow that shows the direction of rotation from initial to the terminal side. Two or more than two angles may have the same initial and terminal sides. Consider an angle ∠AOB with $\overline{OA}$ as initial side and $\overline{OB}$ as terminal side with vertex O. Let m∠AOB = θ radian where $0 \le \theta \le 2\pi$.



If the terminal side $\overline{OB}$ comes to its original position after, one, two or more than two complete revolutions in the anti–clockwise direction, then m∠AOB in above four cases will be

(i)	θ rad	After zero revolution
(ii)	(2π+ θ) rad.	After one revolution
(iii)	(4π+ θ) rad.	After two revolutions.

Coterminal angle: Two or more than two angles with the same initial and terminal sides are called coterminal angles.

It means that terminal side comes to its original position after every revolution of 2π radian in anti clockwise or clockwise direction. In general if θ is in degrees, then 360°k+θ where k ∈ Z, is an angle conterminal with θ, if angle θ is in radian measure, then 2kπ+ θ where k∈Z is an angle coterminal with θ . Thus, the general angle θ = 2(k) π+ θ, where k ∈ Z.

Example 1: Which of following angles are coterminal with 120°?

$-240°$, $480°$, $\frac{14\pi}{3}$ and $-\frac{14\pi}{3}$

Solution:

- – 240° is coterminal with 120° as their terminal side is same
- 480° = 360° + 120°, the angle 480° terminates at 120° after one complete revolution.
- $\frac{14}{3}\pi = 4\pi + \frac{2\pi}{3} = 720° + 120°$ then angle $\frac{14\pi}{3}$ is coterminal with 120°.

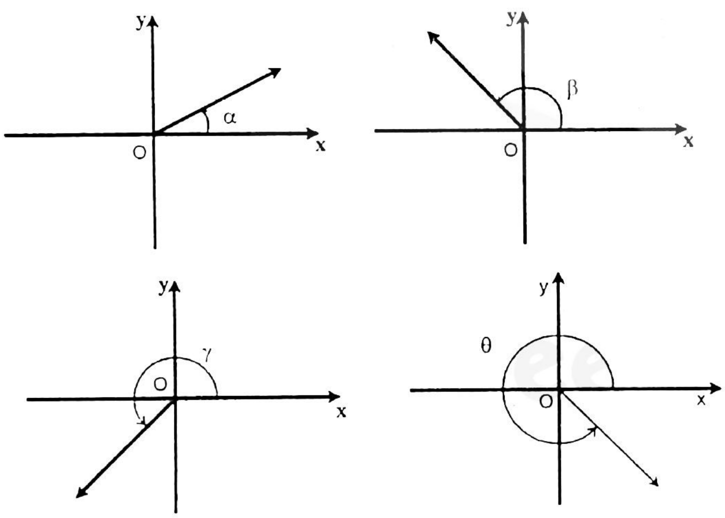
- $\dfrac{-14\pi}{3} = -4\pi + \dfrac{-2\pi}{3} = -720^\circ - 120^\circ$ So $\dfrac{-14\pi}{3}$ is not coterminal with 120°.

Angle in Standard Position:

A general angle is said to be in standard position if its vertex is at the origin and its initial side is directed along the positive direction of the x–axis of a rectangular coordinate system.

The position of the terminal side of a angle in standard position remains the same if measure of the angle is increased or decreased by a multiple of 2π .

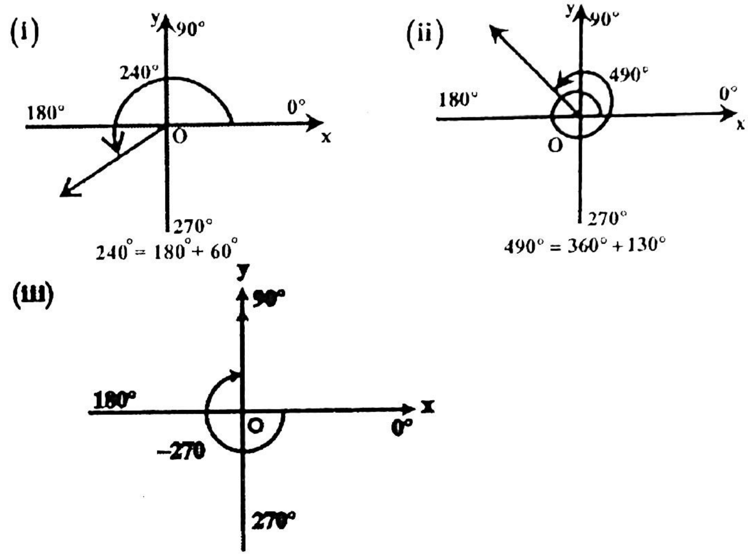
Some standard angles are shown in the following figures:



Example: Locate each angle in standard position.

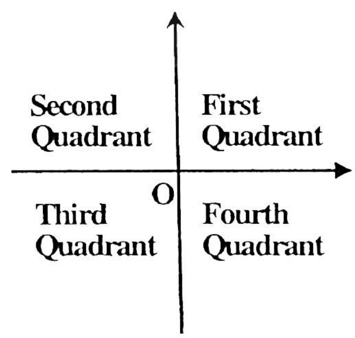
(i) 240° (ii) 490° (iii) – 270°

Solution: The angles are shown in figure.



The Quadrants and Quadrantal Angles:

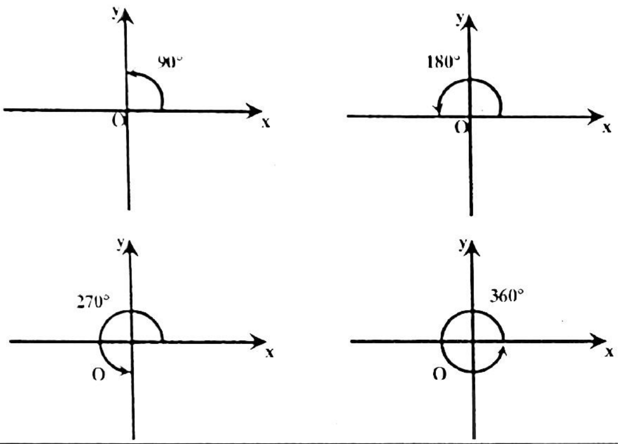
The x–axis and y–axis divides the plane in four regions, called quadrants, when they intersect each other at right angle. The point of intersection is called origin and is denoted by O.



- Angles between 0° and 90° are in the first quadrant.
- Angles between 90° and 180° are in the second quadrant.
- Angles between 180° and 270° are in the third quadrant.
- Angles between 270° and 360° are in the fourth quadrant.
- An angle in standard position is said to lie in a quadrant if its terminal side lies in that quadrant. Angles α, β, γ and θ lie in I, II, III and IV quadrant respectively.

Quadrantal Angles

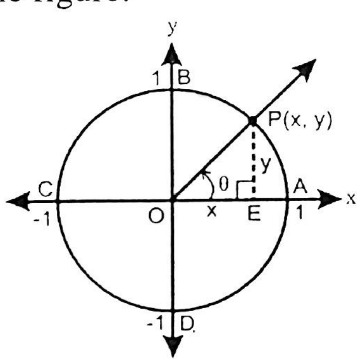
If the terminal side of an angle in standard position falls on x–axis or y–axis, then it is called a quadrantal angle i.e., 90°, 180°, 270° and 360° are quadrantal angles. The quadrantal angles are shown as below:



Trigonometric ratios and their reciprocals with the help of a unit circle:

There are six fundamental trigonometric ratios called sine, cosine, tangent, cotangent, secant and cosecant. To define these functions we use circular approach which involves the unit circle.

Let θ be a real number, which represents the radian measure of an angle in standard position. Let P(x, y) be any point on the unit circle lying on terminal side of θ as shown in the figure.



We define sine of θ, written as sinθ and cosine of θ written as cosθ, as:

$$\sin\theta = \frac{EP}{OP} = \frac{y}{1} \qquad \sin\theta = y$$

$$\cos\theta = \frac{OE}{OP} = \frac{x}{1} \qquad \cos\theta = x$$

i.e., cosθ and sinθ are the x–coordinate and y–coordinate of the point P on the unit circle. The equations x = cosθ and y = sinθ are called circular or trigonometric functions.

The remaining trigonometric functions tangent, cotangent, secant and cosecant will be denoted by tanθ, cotθ, secθ and cosecθ for any real angle θ .

- $\tan\theta = \frac{EP}{OE} = \frac{y}{x} \qquad \tan\theta = \frac{y}{x}\ (x \neq 0)$

As $y = \sin\theta$ and $x = \cos\theta \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$

- $\cot\theta = \frac{x}{y}\ (y \neq 0) \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$
- $\sec\theta = \frac{1}{x}(x \neq 0)$ and $\operatorname{cosec}\theta = \frac{1}{y}\ (y \neq 0)$
- $\sec\theta = \frac{1}{\cos\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

Reciprocal Identities

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$ or $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ or $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ or $\cot\theta = \frac{1}{\tan\theta}$

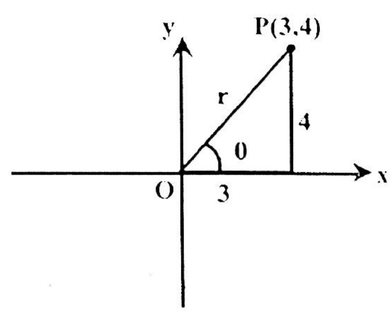
Example 2: Find the value of the trigonometric ratios at θ if point (3, 4) is on the terminal sides of θ .

Solution: We have x = 3 and y = 4

We shall also need value of r, which is found by using the fact that

$r = \sqrt{x^2+y^2}$; $r = \sqrt{(3)^2 \quad (4)^2} = \sqrt{25}+ \quad 5$

where r = OP



Thus $\sin\theta = \frac{y}{r} = \frac{4}{5}$; $\operatorname{cosec}\theta = \frac{5}{4}$

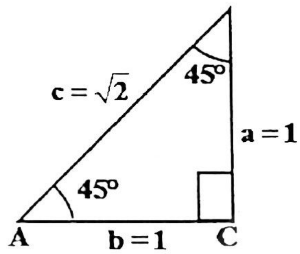
$\cos\theta = \frac{x}{r} = \frac{3}{5}$; $\sec\theta = \frac{5}{3}$

$\tan\theta = \frac{y}{x} = \frac{4}{3}$; $\cot\theta = \frac{3}{4}$

The values of trigonometric ratios for 45°, 30°, 60°

Consider a right triangle ABC with $m\angle C = 90°$. The sides opposite to the vertices A, B and C are denoted by a, b and c respectively.

Case I When $m\angle A = 45°$, where $45° = \frac{\pi}{4}$ radian. Since the sum of angles in a triangle is 180°, So $m\angle B = 45°$.



As values of trigonometric functions depends on the size of the angle only and not on the size of triangle. For convenience, we take $a = b = 1$. In this case the triangle is isosceles right triangle.

By Pythagorean theorem

$c^2 = a^2 + b^2 \qquad c^2 = (1)^2 + (1)^2 = 2$

$$c^2 = 2$$

$$c = \sqrt{2}$$

From this triangle we have

$$\sin 45° = \sin\frac{\pi}{4} = \frac{a}{c} = \frac{1}{\sqrt{2}}$$

$$\operatorname{cosec} 45° = \frac{1}{\sin 45°} = \sqrt{2}$$

$$\cos 45° = \cos\frac{\pi}{4} = \frac{b}{c} = \frac{1}{\sqrt{2}}$$

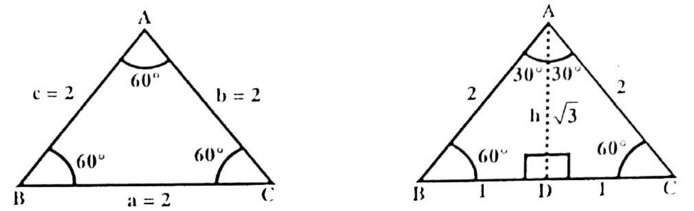
$$\sec 45° = \frac{1}{\cos 45°} = 2$$

$$\tan 45° = \tan\frac{\pi}{4} = \frac{a}{b} = \frac{1}{1} = 1$$

$$\cot 45° = \frac{1}{\tan 45°} = 1$$

Case II when $m\angle A = 30°$ or $m\angle A = 60°$

Consider an equilateral triangle with sides $a = b = c = 2$ for convenience. Since the angles in an equilateral triangle are equal and their sum is 180°, each angle has measure 60°. Bisecting an angle in the triangle, we obtain two right triangles with 30° and 60° angles. The height $|AD|$ of these triangles may be found by Pythagorean theorem, i.e.,



(i) (ii)

$$(m\overline{AD})^2 = (m\overline{BD})^2 + (m\overline{AB})^2$$

$$(m\overline{AD})^2 = (m\overline{AB})^2 - (m\overline{BD})^2$$

$$h^2 = (2)^2 - (1)^2 = 3$$

$$h = \sqrt{3}$$

$\therefore$ Using triangle ADB with $m\angle A = 30°$, we have

$$\sin 30° = \sin\frac{\pi}{6} = \frac{m\overline{BD}}{m\overline{AB}} = \frac{1}{2}$$

$$\operatorname{cosec} 30° = \frac{1}{\sin 30°} = 2$$

$$\cos 30° = \cos\frac{\pi}{6} = \frac{m\overline{AD}}{m\overline{AB}} = \frac{\sqrt{3}}{2}$$

$$\sec 30° = \frac{1}{\cos 30°} = \frac{2}{\sqrt{3}}$$

$$\tan 30° = \tan\frac{\pi}{6} = \frac{m\overline{BD}}{m\overline{AD}} = \frac{1}{\sqrt{3}}$$

$$\cot 30° = \frac{1}{\tan 30°} = \sqrt{3}$$

Now using triangle ABD with $m\angle B = 60°$

$$\operatorname{Sin} 60° = \frac{m\overline{AD}}{m\overline{AB}} = \frac{\sqrt{3}}{2}$$

$$\operatorname{cosec} 60° = \frac{1}{\sin 60°} = \frac{2}{\sqrt{3}}$$

$$\cos 60° = \frac{m\overline{BD}}{m\overline{AB}} = \frac{1}{2}$$

$$\sec 60° = \frac{1}{\cos 60°} = 2$$

$$\tan 60° = \frac{m\overline{AD}}{m\overline{BD}} = \frac{\sqrt{3}}{1}$$

$$\cot 60° = \frac{1}{\tan 60°} = \frac{1}{\sqrt{3}}$$

Signs of trigonometric ratios in different quadrants

In case of trigonometric ratios like $\sin\theta$, $\cos\theta$ and $\tan\theta$ if θ is not a quadrantal angle, then θ will lie in a particular quadrant. Since

$r = \sqrt{x^2 + y^2}$ is always +ve, the signs of ratios can be found if the quadrant of θ is known.

(i) If θ lies in first quadrant then a point P(x, y) on its terminal side has x and y co–ordinate positive.

Therefore, all trigonometric functions are positive in quadrant I.

(ii) If θ lies in second quadrant then a point P(x, y) on its terminal side has negative x–coordinate and positively y–coordinate i.e.,

- $\sin\theta = \dfrac{y}{r}$ is +ve or > 0,
- $\cos\theta = \dfrac{x}{r}$ is –ve or < 0
- $\tan\theta = \dfrac{y}{x}$ is –ve or < 0

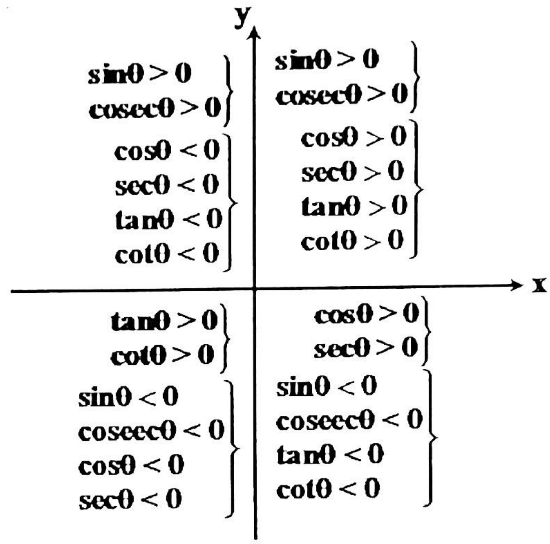
(iii) When θ lies in third quadrant, then a point P(x, y) on its terminal side has negative x–coordinate and negative y–coordinate.

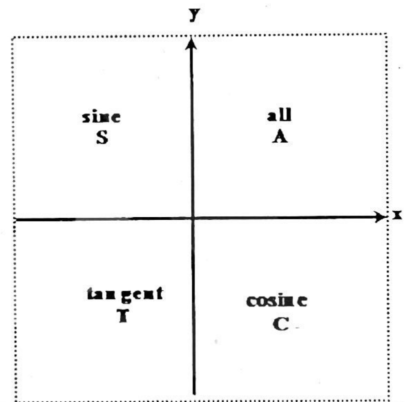
- $\sin\theta = \dfrac{y}{r}$ is –ve or < 0,
- $\cos\theta = \dfrac{x}{r}$ is –ve or < 0 and
- $\tan\theta = \dfrac{y}{x}$ is +ve or > 0

(iv) When θ lies in fourth quadrant, then the point P(x, y) on the terminal side of θ has positive x–coordinate and negative y–coordinate.

- $\sin\theta = \dfrac{y}{r}$ is –ve or < 0,
- $\cos\theta = \dfrac{x}{r}$ is +ve or > 0 and
- $\tan\theta = \dfrac{y}{x}$ is –ve or < 0

The sign of all trigonometric functions are summarized as below.





Values of remaining trigonometric ratios if one trigonometric ratio is given

Example 1:

If $\sin\theta = \dfrac{-3}{4}$ and $\cos\theta = \dfrac{\sqrt{7}}{4}$, then find the values of tanθ, cotθ, secθ and cosecθ .

Solution: Applying the identities that express the remaining trigonometric functions in terms of sine and cosine, we have

$$\sin\theta = \frac{-3}{4}$$

$$\therefore \quad \text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{-3}{4}} = \frac{-4}{3}$$

$$\text{cosec}\,\theta = \frac{-4}{3}$$

$$\cos\theta = \frac{\sqrt{7}}{4}$$

$\therefore \quad \sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{\sqrt{7}}{4}}$

$\sec\theta = \dfrac{4}{\sqrt{7}} = \dfrac{4\sqrt{7}}{7}$

Now $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{-3}{4}}{\frac{\sqrt{7}}{4}} = \dfrac{-3}{\sqrt{7}}$

$\tan\theta = \dfrac{-3}{\sqrt{7}}$

And $\quad \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{-\sqrt{7}}{3}$

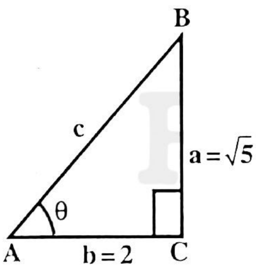
Example 2:

If $\tan\theta = \dfrac{\sqrt{5}}{2}$, then find the values of other trigonometric ratios at θ.

Solution: In any right triangle ABC

$\tan\theta = \dfrac{\sqrt{5}}{2} = \dfrac{a}{b}$

$a = \sqrt{5}, \quad b = 2$



Now by Pythagorean theorem

$a^2 + b^2 = c^2$

$\left(\sqrt{5}\right)^2 \quad (2)^2 + \quad c^2$

$c^2 = 5 + 4 = 9$

$c = \pm 3 \quad \text{or} \quad c = 3$

$\cot\theta = \dfrac{1}{\tan\theta}$

$\cot\theta = \dfrac{1}{\frac{\sqrt{5}}{2}} \qquad \cot\theta = \dfrac{2}{\sqrt{5}}$

$\sin\theta = \dfrac{a}{c} = \dfrac{\sqrt{5}}{3}$,

$\text{cosec}\theta = \dfrac{1}{\sin\theta}$

$\text{cosec}\theta = \dfrac{1}{\frac{\sqrt{5}}{3}}$

$\therefore \quad \text{cosec}\theta = \dfrac{3}{\sqrt{5}}$

Also $\quad \cos\theta = \dfrac{b}{c} = \dfrac{2}{3}$,

$\sec\theta = \dfrac{1}{\cos\theta}$

$\sec\theta = \dfrac{1}{\frac{2}{3}}$

$\therefore \quad \sec\theta = \dfrac{3}{2}$

Calculate the values of trigonometric ratios for 0°, 90°, 180°, 270°, 360°

An angle θ is called a quadrantal angle if its terminal side lies on the x–axis on the y–axis.

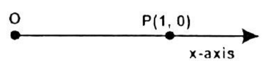
Case I when $\theta = 0°$

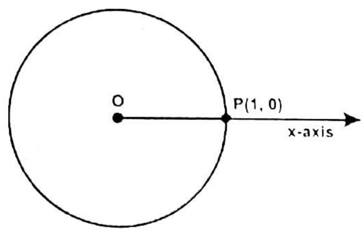
The point (1, 0) lies on the terminal side of angle $\theta°$. We may consider the point on the unit circle on the terminal side of the angle.

P (1, 0)

$x = 1$ and $y = 0$

so $\quad r \quad \sqrt{x^2 \pm y^2} = \sqrt{1+0} = 1$





$\sin 0° = \dfrac{y}{r} = \dfrac{0}{1} = 0,$

$\text{cosec}\,0° = \dfrac{1}{\sin 0°} = \dfrac{1}{0} = \infty$ (Undefined)

$\cos 0° = \dfrac{x}{r} \quad \dfrac{1}{1} = 1,$

$$\sec 0^\circ = \frac{1}{\cos 0^\circ} = 1$$

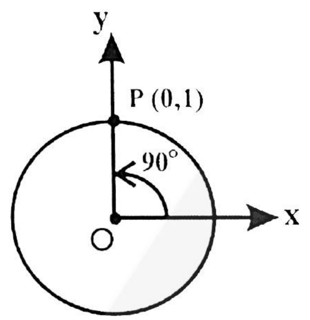
$$\tan 0^\circ = \frac{y}{x} = \frac{0}{1} = 0,$$

$$\cot 0^\circ = \frac{1}{\tan 0^\circ} = \frac{1}{0} = \infty \quad \text{(Undefined)}$$

Case II when θ = 90°

The point P(0, 1) lies on the terminal side of angle 90°. $r \quad \sqrt{x^2 \; y^2}$

Here $x = 0$ and $y = 1$ $\quad r = \sqrt{0^2 \quad (1)^2} \; 1$



$$\sin 90^\circ = \frac{y}{r} = \frac{1}{1} = 1$$

i.e.

$$\sin 90^\circ = 1 \text{ and } \operatorname{cosec} 90^\circ = \frac{r}{y} = 1$$

Using reciprocal identities, we have

$$\cos 90^\circ = \frac{x}{r} = \frac{0}{1} = 0$$

$$\sec 90^\circ = \frac{r}{x} = \frac{1}{0} = \infty \quad \text{(Undefined)}$$

$$\tan 90^\circ = \frac{y}{x} = \frac{1}{0} = \infty \quad \text{(Undefined)},$$

$$\cot 90^\circ = \frac{x}{y} = \frac{0}{1} = 0$$

Case III when θ = 180°

When θ = 180° and the point P (−1, 0) lies on x′−axis or on terminal side of angle 180°

Here $\quad x = -1$ and $y = 0$

$$r = \sqrt{x^2 + y^2} = 1$$

$$\sin 180^\circ = \frac{y}{r} = \frac{0}{1} = 0$$

$$\operatorname{cosec} 180^\circ = \frac{r}{y} = \frac{1}{0} = \infty \text{ (undefined)}$$

$$\cos 180^\circ = \frac{x}{r} = \frac{-1}{1} = -1$$

$$\sec 180^\circ = \frac{r}{x} = \frac{1}{-1} = -1$$

$$\tan 180^\circ = \frac{y}{x} = \frac{0}{-1} = 0$$

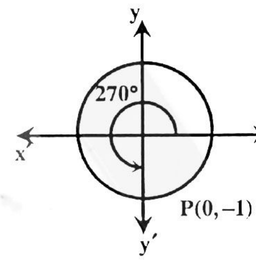
$$\cot 180^\circ = \frac{x}{y} = \frac{-1}{0} = \infty \text{ (Undefined)}$$

Case IV when θ = 270°

When θ = 270° and the point P(0, −1) lis on y′−axis or on the terminal side of angle 270°.

The point P(0, −1) shows that x=0 and y= −1

So $\quad r = \sqrt{(0)^2 + (-1)^2} = 1$



$$\operatorname{Sin} 270^\circ = \frac{y}{r} = \frac{-1}{1} = -1$$

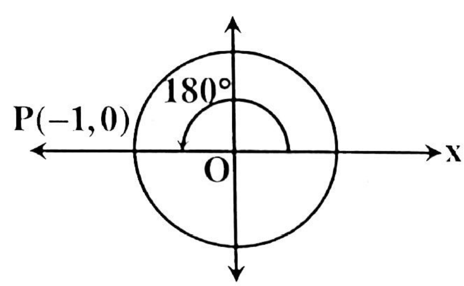
$$\operatorname{cosec} 270^\circ = \frac{r}{y} = \frac{1}{-1} = -1$$

$$\operatorname{Cos} 270^\circ = \frac{x}{r} = \frac{0}{1} = 0$$

$$\sec 270^\circ = \frac{r}{x} = \frac{1}{0} = \infty$$

$$\tan 270^\circ = \frac{y}{x} = \frac{-1}{0} = -\infty$$

$$\cot 270 = \frac{x}{y} = \frac{0}{-1} = 0$$



Case V When θ° = 360°

Now the point P (1, 0) lies once again on x−axis

We know that $\theta + 2k\pi = \theta$ where $k \in Z$.

Now $\theta = 360^\circ = 0^\circ + (360^\circ)1 = 0^\circ$ where $k = 1$

So $\quad \sin 360^\circ = \sin 0^\circ = 0$

$$\text{cosec}360° = \frac{1}{\sin 360°} = \frac{1}{\sin 0°} = \frac{1}{0} = \infty \text{ (undefined)}$$

$$\cos 360° = \cos 0° = 1$$

$$\sec 360° = \frac{1}{\cos 0°} = \frac{1}{1} = 1,$$

$$\tan 360° = \tan 0° = 0$$

$$\cot 360° = \frac{1}{\tan 0°} = \frac{1}{0} = \infty \text{ (undefined)}$$

Example: **Find each of the following without using table or calculator:**

(i) Cos540° (ii) sin315° (iii) sec(–300)°

Solution:

We know that $2k\pi + \theta = \theta$, where $k \in Z$.

(i) $540° = (360° + 180°) = 2\,(1)\pi + 180°$

$$\text{Cos}540° = \cos(2\pi + \pi) = \cos\pi = -1$$

(ii) $\sin 315° = \sin(360° - 45°) = \sin\ 2\pi - \frac{\pi}{4}$

$$= \sin \frac{-\pi}{4} = -\sin\frac{\pi}{4} = \frac{-1}{\sqrt{2}}$$

(iii) $\sec(-300°) = \sec(-360° + 60°)$

$$= \sec[2(-1)\pi + 60]$$

$$= \sec 60° = \frac{1}{\cos 60°} = \frac{1}{\frac{1}{2}} = 2$$

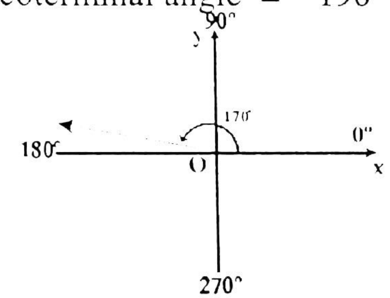
EXERCISE 7.3

Q.1. Locate each of the following angles in standard position using a protractor or fair free hand guess. Also find a positive and a negative angle coterminal with each given angle:

(i) 170°

Positive coterminal angle $= 360° + 170°$
$= 530°$

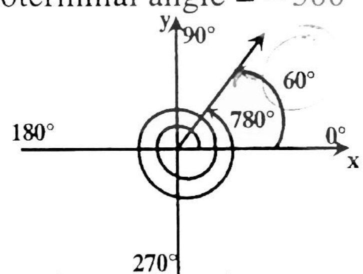
Negative coterminal angle $= -190°$



(ii) **780°**

Positive coterminal angle $780° + 2(360°) + 60° = 60°$

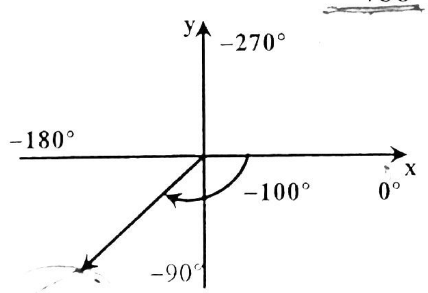
Negative coterminal angle $= -300°$



(iii) **– 100°**

Positive coterminal angle $= 260°$

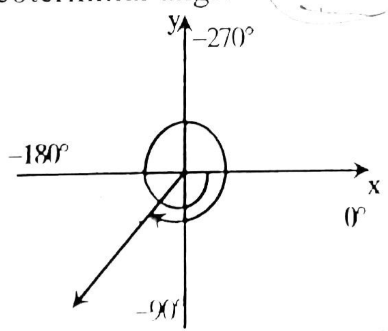
Negative coterminal angle $= -360° - 100°$
$= -460°$



(iv) – 500°

Positive coterminal angle $= 220°$

Negative coterminal angle $= -140°$



Q.2. Identify closest quadrantal angles between which the following angles lie.

(i) 156°

Ans: 90° and 180°

(ii) 318°

Ans: 270° and 360°

(iii) 572°

Ans: 540° and 630°

(iv) –330°

Ans: 0° and 90°

Q.3. Write the closest quadrantal angles between which the angles lie. Write your answer in radian measure.

(i) $\frac{\pi}{3}$

Ans: 0 and $\frac{\pi}{2}$

(ii) $\frac{3\pi}{4}$

Ans: $\frac{\pi}{2}$ and π

(iii) $\frac{-\pi}{4}$

Ans: 0 and $-\frac{\pi}{2}$

(iv) $-\frac{3\pi}{4}$

Ans: $-\frac{\pi}{2}$ and $-\pi$

Q.4. In which quadrant θ lies, when

(i) $\sin\theta > 0, \tan\theta < 0$

Ans: II quadrant

(ii) $\cos\theta < 0, \sin\theta < 0$

Ans: III quadrant

(iii) $\sec\theta > 0, \sin\theta < 0$

Ans: IV quadrant

(iv) $\cos\theta < 0, \tan\theta < 0$

Ans: II quadrant

(v) $\text{cosec}\,\theta > 0, \cos\theta > 0$

Ans: I quadrant

(vi) $\sin\theta < 0, \sec\theta < 0$

Ans: III quadrant

Q.5. Fill in the blanks:

(i) $\cos(-150°) =$ ________ $\cos 150°$

(ii) $\sin(-310°) =$ ________ $\sin 310°$

(iii) $\tan(-210°) =$ ________ $\tan 210°$

(iv) $\cot(-45°) =$ ________ $\cot 45°$

(v) $\sec(-60°) =$ ________ $\sec 60°$

(vi) $\text{cosec}(-137°) =$ ________ $\text{cosec}137°$

Answers:

(i) +ve (ii) −ve (iii) −ve

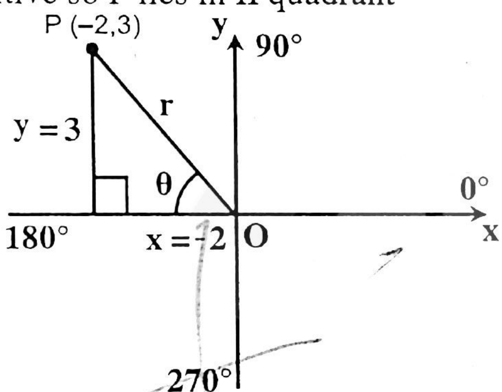
(iv) −ve (v) +ve (vi) −ve

Q.6. The given point P lies on the terminal side of θ. Find quadrant of θ and all six trigonometric ratios.

(i) (−2, 3)

Solution: $P(x, y) = P(-2, 3)$

As x −coordinate is negative and y− coordinate is positive so P lies in II quadrant



The point P can be shown in II quadrant.

By Pythagorean theorem

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-2)^2 + (3)^2}$$

$$r = \sqrt{4+9}$$

$$r = \sqrt{13}$$

Now, $\sin\theta = \frac{y}{r} = \frac{3}{\sqrt{13}}$

$$\text{cosec}\,\theta = \frac{r}{y} = \frac{\sqrt{13}}{3}$$

$$\cos\theta = \frac{x}{r} = -\frac{2}{\sqrt{13}}$$

$$\sec\theta = \frac{r}{x} = \frac{-\sqrt{13}}{2}$$

$$\tan\theta = \frac{y}{x} = -\frac{3}{2}$$

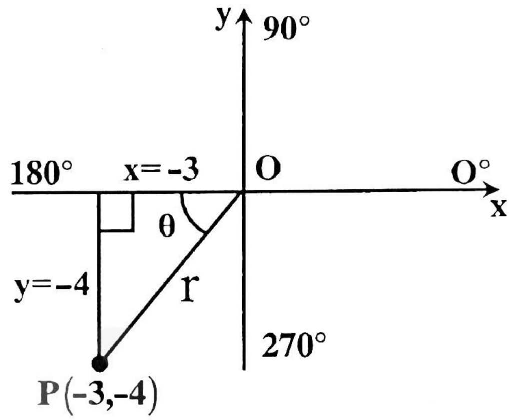
$$\cot\theta = \frac{x}{y} = -\frac{2}{3}$$

(ii) (−3, −4)

Solution: $P(x, y) = P(-3, -4)$

As x and y both coordinates are negative, so 'P' lies in III quadrant.

The point P can be shown in III quadrant.



By Pythagorean, theorem

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-3)^2 + (-4)^2}$$

$$r = \sqrt{9+16}$$

$$r = \sqrt{25}$$

$$r = 5$$

Now $\sin\theta = \frac{y}{r} = \frac{-4}{5}$

$$\text{cosec}\,\theta = \frac{r}{y} = \frac{-5}{4}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{5}$$

$$\sec\theta = \frac{r}{x} = \frac{-5}{3}$$

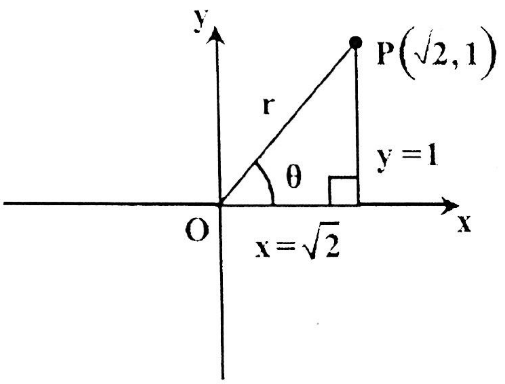
$$\tan\theta = \frac{y}{x} = \frac{-4}{-3} = \frac{4}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4}$$

(iii) $\left(\sqrt{2},1\right)$

Solution: $P(x, y) = P\left(\sqrt{2},1\right)$

As x and y both coordinates are positive, so P lies in I quadrant.



The point 'P' can be shown in quadrant I.

By Pythagorean theorem

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2+y^2}$$

$$r = \sqrt{\left(\sqrt{2}\right)^2+(1)^2}$$

$$r = \sqrt{2+1}$$

$$r = \sqrt{3}$$

Now $\sin\theta = \dfrac{y}{r} = \dfrac{1}{\sqrt{3}}$

$$\operatorname{cosec}\theta = \frac{r}{y} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\cos\theta = \frac{x}{r} = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{3}}{\sqrt{2}} = \sqrt{\frac{3}{2}}$$

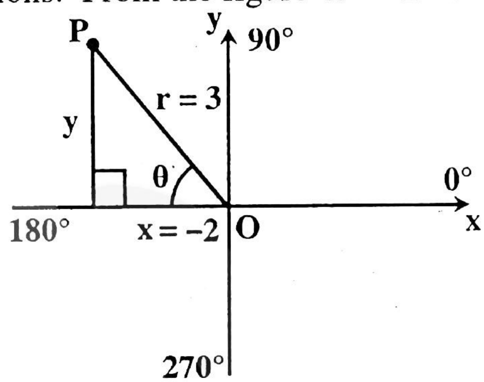
$$\tan\theta = \frac{y}{x} = \frac{1}{\sqrt{2}}$$

$$\cot\theta = \frac{x}{y} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

Q.7. If $\cos\theta = \dfrac{-2}{3}$ and terminal arm of the angle θ is in quadrant II, find the values of remaining trigonometric functions.

Solution:

As $\cos\theta = \dfrac{-2}{3}$ and θ is in quadrant II, so we complete the figure according to conditions. From the figure $x = -2$ and $r = 3$



By Pythagorean theorem

$$r^2 = x^2 + y^2$$

$$y^2 = r^2 - x^2$$

$$y = \sqrt{r^2 - x^2}$$

$$y = \sqrt{(3)^2 - (-2)^2}$$

$$y = \sqrt{9-4}$$

$$y = \sqrt{5}$$

Now

$$\sin\theta = \frac{y}{r} = \frac{\sqrt{5}}{3}$$

$$\operatorname{cosec}\theta = \frac{r}{y} = \frac{3}{\sqrt{5}}$$

$$\cos\theta = \frac{x}{r} = \frac{-2}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{-3}{2}$$

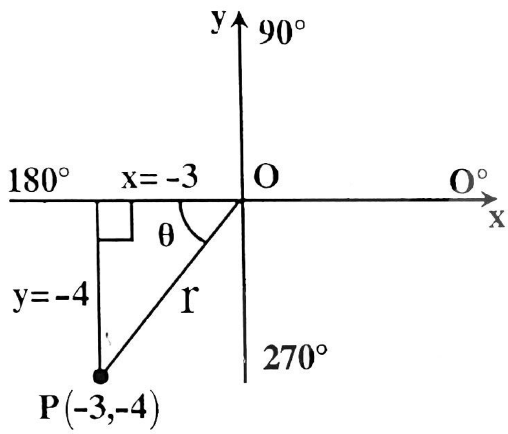
$$\tan\theta = \frac{y}{x} = \frac{-\sqrt{5}}{2}$$

$$\cot\theta = \frac{x}{y} = \frac{-2}{\sqrt{5}}$$

Q.8. If $\tan\theta = \frac{4}{3}$ and $\sin\theta < 0$, find the values of other trigonometric functions at θ.

Solution:

As $\tan\theta = \frac{4}{3}$ and sinθ is –ve, which is possible in quadrant III only. We complete the figure.



From the figure x = – 3 and y = – 4

By pathagorean theorem

$$r^2 = x^2 + y^2$$
$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{(-3)^2 + (-4)^2}$$
$$r = \sqrt{9+6}$$
$$r = \sqrt{25}$$
$$r = 5$$

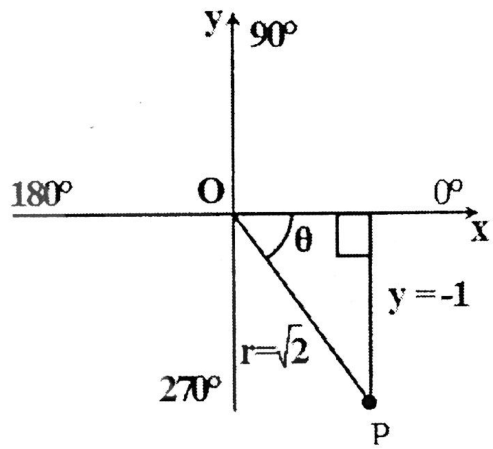
Now

$$\sin\theta = \frac{y}{r} = \frac{-4}{5}$$
$$\text{cosec}\theta = \frac{r}{y} = \frac{-5}{4}$$
$$\cos\theta = \frac{x}{r} = \frac{-3}{5}$$
$$\sec\theta = \frac{r}{x} = \frac{-5}{3}$$
$$\tan\theta = \frac{y}{x} = \frac{4}{3}$$
$$\cot\theta = \frac{x}{y} = \frac{3}{4}$$

Q.9. If $\sin\theta = -\frac{1}{\sqrt{2}}$ and terminal side of the angle is not in quadrant III, find the values of tanθ, secθ and cosecθ.

Solution:

As $\sin\theta = \frac{-1}{\sqrt{2}}$ and terminal side of angle is not in III quadrant, so it lies in quadrant IV.



From the figure y = –1 and $r = \sqrt{2}$

By Pathagorean theorem

$$r^2 = x^2 + y^2$$
$$x^2 = r^2 - y^2$$
$$x = \sqrt{r^2 - y^2}$$
$$x = \sqrt{(\sqrt{2})^2 - (-1)^2}$$
$$x = \sqrt{2-1}$$
$$x = \sqrt{1}$$
$$x = 1$$

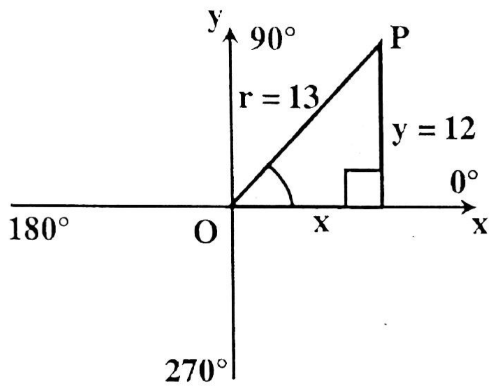
Now

$$\tan\theta = \frac{y}{x} = \frac{-1}{1} = -1$$
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{2}}{1} = \sqrt{2}$$
$$\text{cosec}\,\theta = \frac{r}{y} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$$

Q.10. If $\text{cosec}\theta = \frac{13}{12}$ and $\sec\theta > 0$, find the remaining trigonometric functions.

Solution:

As $\text{cosec}\theta = \frac{13}{12}$ and also $\sec\theta$ is +ve, which is only possible in quadrant I.



From the figure $y = 12$ and $r = 13$

By Pythagorean theorem

$$r^2 = x^2 + y^2$$
$$x^2 = r^2 - y^2$$
$$x = \sqrt{r^2 - y^2}$$
$$x = \sqrt{(13)^2 - (12)^2}$$
$$x = \sqrt{169 - 144}$$
$$x = \sqrt{25}$$
$$x = 5$$

Now

$$\sin\theta = \frac{y}{r} = \frac{12}{13}$$

$$\text{cosec}\theta = \frac{r}{y} = \frac{13}{12}$$

$$\cos\theta = \frac{x}{r} = \frac{5}{13}$$

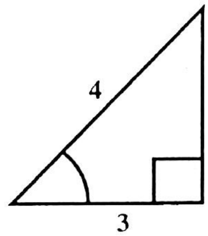
$$\sec\theta = \frac{r}{x} = \frac{13}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{12}{5}$$

$$\cot\theta = \frac{x}{y} = \frac{5}{12}$$

Q.11. Find the values of trigonometric functions at the indicated angle θ in the right triangle.

(i)



From the figure Hypotenuse = 4 and Base = 3

By Pythagorean theorem we can find perpendicular

$$(\text{Per.})^2 + (\text{Base})^2 = (\text{Hyp.})^2$$
$$(\text{Per.})^2 + (3)^2 = (4)^2$$
$$(\text{Per.})^2 = 16 - 9$$
$$(\text{Per.})^2 = 7$$
$$\text{Perpendicular} = \sqrt{7}$$

Now $\sin\theta = \frac{\text{Per.}}{\text{Hyp.}} = \frac{\sqrt{7}}{4}$

$$\text{cosec}\,\theta = \frac{\text{Hyp.}}{\text{Per.}} = \frac{4}{\sqrt{7}}$$

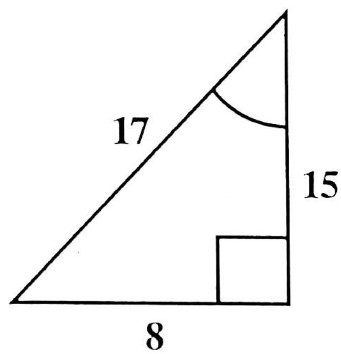
$$\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{3}{4}$$

$$\sec\theta = \frac{\text{Hyp.}}{\text{Base}} = \frac{4}{3}$$

$$\tan\theta = \frac{\text{Per.}}{\text{Base}} = \frac{\sqrt{7}}{3}$$

$$\cot\theta = \frac{\text{Base}}{\text{Per.}} = \frac{3}{\sqrt{7}}$$

(ii)



From the figure

Hypotenuse = 17

Perpendicular = 8

Base = 15

Now

$$\sin\theta = \frac{\text{Per.}}{\text{Hyp.}} = \frac{8}{17}$$

$$\text{cosec}\ \theta = \frac{\text{Hyp.}}{\text{Per.}} = \frac{17}{8}$$

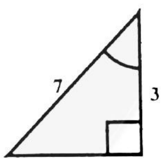
$$\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{15}{17}$$

$$\sec\theta = \frac{\text{Hyp.}}{\text{Base}} = \frac{17}{15}$$

$$\tan = \frac{\text{Per.}}{\text{Base}} = \frac{8}{15}$$

$$\cot\theta = \frac{\text{Base}}{\text{Per.}} = \frac{15}{8}$$

(iii)



From the figure Hypotenuse = 7, Base = 3

We can find perpendicular by Pythagorean theorem.

$$(\text{Base})^2 + (\text{Per.})^2 = (\text{Hyp.})^2$$

$$(\text{Per.})^2 + (3)^2 = (7)^2$$

$$(\text{Per.})^2 = 49 - 9$$

$$(\text{Per.})^2 = 40$$

$$\text{Per.} = \sqrt{40}$$

$$\text{Per.} = \sqrt{4\times10}$$

$$\text{Per.} = 2\sqrt{10}$$

Now, $\sin\theta = \dfrac{\text{Per.}}{\text{Hyp.}} = \dfrac{2\sqrt{10}}{7}$

$$\text{cosec}\theta = \frac{\text{Hyp.}}{\text{Per.}} = \frac{7}{2\sqrt{10}}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{3}{7}$$

$$\sec\theta = \frac{\text{Hyp.}}{\text{Base}} = \frac{7}{3}$$

$$\tan\theta = \frac{\text{Per.}}{\text{Base}} = \frac{2\sqrt{10}}{3}$$

$$\cot\theta = \frac{\text{Base}}{\text{Per.}} = \frac{3}{2\sqrt{10}}$$

Q.12. Find the values of the trigonometric functions. Do not use trigonometric table or calculator.

Solution:

We know that $2k\pi + \theta = \theta$, where $k \in Z$

(i) tan 30°

$$30° = 30\ \frac{\pi}{180}\ \text{radian} = \frac{\pi}{6}\ \text{radian}$$

$$\tan 30° = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

(ii) tan 330°

$$\tan 330° = \tan(360° - 30°)$$

$$= \tan\ 2\pi - \frac{\pi}{6}$$

$$= \tan\ -\frac{\pi}{6}$$

$$= -\tan\frac{\pi}{6}$$

$$= -\frac{1}{\sqrt{3}}$$

(iii) sec 330°

$$\sec 330° = \sec(360° - 30°)$$

$$= \sec\ 2\pi - \frac{\pi}{6}$$

$$= \sec\ -\frac{\pi}{6}$$

$$= \sec\frac{\pi}{6}$$

$$= \frac{2}{\sqrt{3}}$$

(iv) $\cot\frac{}{4}$

$$\cot\frac{\pi}{4} = 1$$

(v) $\cos\frac{2}{3}$

$$\cos\frac{2\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

(vi) $\text{cosec}\ \frac{2\pi}{3}$

$\text{cosec}\frac{2\pi}{3} = \text{cosec}\ \frac{\pi}{3} = \frac{2}{\sqrt{3}}$

(vii) $\cos(-450°)$

$\cos(-450°) = \cos(-360° - 90°)$

$= \cos\ -2\pi - \frac{\pi}{2}$

$= \cos\ 2(-1)\pi - \frac{\pi}{2}$

$= \cos\ -\frac{\pi}{2}$

$= \cos\ \frac{\pi}{2} = 0$

(viii) $\tan(-9\pi)$

$\tan(-9\pi) = \tan(-8\pi - \pi)$

$= \tan[2(-4)\pi - \pi]$

$= \tan(-\pi)$

$= -\tan\pi$

$= -(0) = 0$

(ix) $\cos\ \frac{-5\pi}{6}$

$\cos\ -\frac{5\pi}{6} = -\cos\ \frac{\pi}{6}$

$= -\frac{\sqrt{3}}{2}$

(x) $\sin 7\frac{}{6}$

$\sin\frac{7\pi}{6} = \sin\ \ +\frac{}{6}$

$= -\sin\frac{\pi}{6} = -\frac{1}{2}$

(xi) $\cot\frac{7}{6}$

$\cot\frac{7\pi}{6} = \cot\ \pi + \frac{\pi}{6}$

$= \cot\frac{\pi}{6} = \sqrt{3}$

(xii) $\cos 225°$

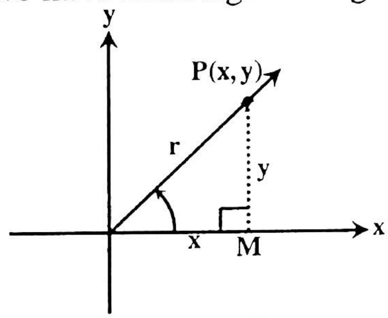
$\cos 225° = \cos(180° + 45°)$

$= \cos\ \pi + \frac{\pi}{4}$

$= -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

Trigonometric Identities

Consider an angle $\angle MOP = \theta$ radian in standard position. Let point P (x, y) be on the terminal side of the angle. By Pythagorean theorem, we have from right triangle OMP.



$(m\overline{OM})^2 = (m\overline{MP})^2 + (m\overline{OP})^2$

$x^2 + y^2 = r^2$ ....(i)

Dividing both sides by r^2, we get

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$

$\left(\sin\theta = \frac{y}{r},\ \cos\theta = \frac{x}{r},\ \tan\theta = \frac{y}{x}\right)$

$\frac{x}{r}^2 \quad \frac{y}{r}^2 + 1$

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

$\boxed{\cos^2\theta + \sin^2\theta = 1}$ ....(i)

Dividing (i) by x^2, we have

$\frac{x^2}{x^2}\ \frac{y^2}{x^2} + \frac{r^2}{x^2}$

$1 + \frac{y}{x}^2 = \frac{r}{x}^2$

As $\tan\theta = \frac{y}{x}$ and $\text{see}\theta = \frac{r}{x}$

$1 + (\tan\theta)^2 = (\sec\theta)^2$

$1 + \tan^2\theta = (\sec\theta)^2$

$1 + \tan^2\theta = \sec^2\theta$ .... (ii)

or $\sec^2\theta - \tan^2\theta = 1$

Again dividing both sides of (i) by y^2, we get

$\frac{x^2}{y^2} = \frac{y^2}{y^2} + \frac{r^2}{y^2}$

$\frac{x}{y}^2 = 1 + \frac{r}{y}^2$

$\cot\theta = \frac{x}{y}$ and $\text{cosec}\theta = \frac{r}{y}$

$(\cot\theta)^2 + 1 = (\text{cosec}\theta)^2$

$1 + \cot^2\theta = \text{cosec}^2\theta$ .... (iii)

or $\text{cosec}^2\theta - \cot^2\theta = 1$

The identities (1), (2) and (3) are also known as Pythagorean Identities.

Example 1: **Verify that cotθsecθ = cosecθ**

Solution:

L.H.S $= \cot\theta\sec\theta$

$= \dfrac{\cos\theta}{\sin\theta}\ \dfrac{1}{\cos\theta}$

$= \dfrac{1}{\sin\theta}$

$= \text{cosec}\theta$

L.H.S = R.H. S

Example 2: **Verify that**

$\tan^4\theta + \tan^2\theta = \tan^2\theta\sec^2\theta$

Solution:

L.H.S $= \tan^4\theta + \tan^2\theta$

$= \tan^2\theta(\tan^2\theta + 1)$ $\quad \tan^2\theta + 1 = \sec^2\theta$

$= \tan^2\theta\sec^2\theta$

L.H.S = L.H.S

Example 3:

Show that $\dfrac{\cot^2}{\text{cosec}\ -1} = \text{cosec}\,\alpha+1$

Solution:

L.H.S $= \dfrac{\cot^2\alpha}{\text{cosec}\,\alpha - 1}$ $\qquad \text{cosec}^2\theta - \cot^2\theta = 1$

$\qquad \cot^2\theta = \text{cosec}^2\theta - 1$

$= \dfrac{(\text{cosec}^2\alpha - 1)}{\text{cosec}\,\alpha - 1}$

$= \dfrac{(\text{cosec}\,\alpha - 1)(\text{cosec}\,\alpha + 1)}{(\text{cosec}\,\alpha - 1)}$

$= \text{cosec}\,\alpha + 1$

L.H.S = R.H.S

Example 4:

Express the trigonometric functions in terms of tanθ.

Solution:

- By using reciprocal identity, we can express cotθ in terms of tanθ .

  i.e $\cot\theta = \dfrac{1}{\tan\theta}$

- By solving the identity $1+\tan^2\theta = \sec^2\theta$

We have expressed secθ in terms of tanθ

$\sec\theta = \pm\sqrt{\tan^2\theta + 1}$

$\cos\theta = \dfrac{1}{\sec\theta}$

$\cos\theta = \dfrac{1}{\pm\sqrt{\tan^2\theta + 1}}$

Because

$\sin\theta = \tan\theta\cos\theta$, we have

$\sin\theta = \tan\theta\ \dfrac{1}{\pm\sqrt{\tan^2\theta + 1}}$

$\sin\theta = \dfrac{\tan\theta}{\pm\sqrt{\tan^2\theta + 1}}$

$\text{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{\pm\sqrt{\tan^2\theta + 1}}{\tan\theta}$

EXERCISE 7.4

In problem 1— 6, simply each expression to single trigonometric function:

Q.1. $\dfrac{\sin^2 x}{\cos^2 x}$

Solution: $\dfrac{\sin^2 x}{\cos^2 x} = \tan x^2$

Q.2. tan$x$ sin$x$ sec$x$

Solution: $\tan x \sin x \sec x$

$= \dfrac{\sin x}{\cos x} . \sin x . \dfrac{1}{\cos x}$

$= \dfrac{\sin^2 x}{\cos^2 x}$

$= \tan^2 x$

Q.3. $\dfrac{\tan x}{\sec x}$

Solution: $\dfrac{\tan x}{\sec x} = \tan x \div \sec x$

$= \dfrac{\sin x}{\cos x} \div \dfrac{1}{\cos x}$

$= \dfrac{\sin x}{\cancel{\cos x}} \times \cancel{\cos x}$

$= \sin x$

Q.4. 1 – cos²x

Solution: $1 - \cos^2 x$

$= \sin^2 x + \cancel{\cos^2} x - \cancel{\cos^2} x$

$= \sin^2 x$

Q.5. $\sec^2 x - 1$

Solution: $\sec^2 x - 1$

$= 1 + \tan^2 x - 1$

$= \tan^2 x$

Q.6. $\sin^2 x.\cot^2 x$

Solution:

$\sin^2 x.\cot^2 x$

$= \sin^2 x \dfrac{\cos^2 x}{\sin^2 x}$

$= \cos^2 x$

In problem 7 — 24, verify the identities

Q.7. **(1–sinθ) (1+sinθ) = cos²θ**

Solution:

L.H.S $= (1 - \sin\theta)(1 + \sin\theta)$

$= (1)^2 - (\sin\theta)^2$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

L.H.S = R.H.S

Q.8. $\dfrac{\sin\theta + \cos\theta}{\cos\theta} = 1 + \tan\theta$

Solution: Let

L.H.S $= \dfrac{\sin\theta + \cos\theta}{\cos\theta}$

$= \dfrac{\cos\theta + \sin\theta}{\cos\theta}$

$= \dfrac{\cos\theta}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= 1 + \tan\theta \qquad \because \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

L.H.S = R.H.S

Q.9. **(tanθ + cotθ) tanθ = sec²θ**

Solution: Let

L.H.S $= (\tan\theta + \cot\theta)\tan\theta$

$= \tan^2\theta + \cot\theta.\tan\theta$

$= \tan^2\theta + \dfrac{1}{\tan\theta}.\tan\theta$

$= 1 + \tan^2\theta$

$= \sec^2\theta \qquad (1 + \tan^2\theta = \sec^2\theta)$

L.H.S = R.H.S

Q.10. **(Cotθ +cosecθ)(tanθ– sinθ) =secθ –cosθ**

Solution: Let

L.H.S $= (\cot\theta + \operatorname{cosec}\theta)(\tan\theta - \sin\theta)$

$= \dfrac{1}{\tan\theta} + \dfrac{1}{\sin\theta} (\tan\theta - \sin\theta)$

$= \dfrac{\sin\theta + \tan\theta}{\tan\theta.\sin\theta} (\tan\theta - \sin\theta)$

$= \dfrac{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}{\tan\theta.\sin\theta}$

$= \dfrac{(\tan\theta)^2 - (\sin\theta)^2}{\tan\theta.\sin\theta}$

$= \dfrac{\tan^2\theta - \sin^2\theta}{\tan\theta.\sin\theta}$

$= \dfrac{\tan^2\theta}{\tan\theta.\sin\theta} - \dfrac{\sin^2\theta}{\tan\theta.\sin\theta}$

$= \dfrac{\tan\theta}{\sin\theta} - \dfrac{\sin\theta}{\tan\theta}$

$= (\tan\theta \div \sin\theta) - (\sin\theta \div \tan\theta)$

$= \dfrac{\sin\theta}{\cos\theta} \div \sin\theta - \sin\theta \div \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} \times \dfrac{1}{\sin\theta} - \sin\theta \times \dfrac{\cos\theta}{\sin\theta}$

$= \sec\theta - \cos\theta$

L.H.S = R.H.S

Q.11. $\dfrac{\sin\theta + \cos\theta}{\tan^2\theta - 1} = \dfrac{\cos^2\theta}{\sin\theta - \cos\theta}$

Solution: Let

L.H.S $= \dfrac{\sin\theta + \cos\theta}{\tan^2\theta - 1}$

$= (\sin\theta + \cos\theta) \div (\tan^2\theta - 1)$

$= (\sin\theta + \cos\theta) \div \dfrac{\sin^2\theta}{\cos^2\theta} - 1$

$= (\sin\theta + \cos\theta) \div \dfrac{\sin^2 - \cos^2}{\cos^2}$

$= (\sin\theta + \cos\theta) \times \dfrac{\cos^2}{(\sin^2\theta - \cos^2\theta)}$

$= \dfrac{(\sin\theta + \cos\theta) \times \cos^2\theta}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}$

$= \dfrac{\cos^2\theta}{\sin\theta - \cos\theta}$

L.H.S = R.H.S

Q.12. $\dfrac{\cos^2\theta}{\sin\theta} + \sin\theta = \text{cosec}\ \theta$

Solution: Let

$$\text{L.H.S} = \frac{\cos^2\theta}{\sin\theta} + \sin\theta$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta}$$

$$= \text{cosec}\ \theta$$

L.H.S = R.H.S

Q.13. $\sec\theta - \cos\theta = \tan\theta\ .\sin\theta$

Solution: Let

$$\text{L.H.S} = \sec\theta - \cos\theta$$

$$= \frac{1}{\cos\theta} - \cos\theta$$

$$= \frac{1 - \cos^2\theta}{\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}\sin\theta$$

$$= \tan\theta\ .\ \sin\theta$$

L.H.S = R.H.S

Q.14. $\dfrac{\sin^2\theta}{\cos\theta} + \cos\theta = \sec\theta$

Solution: Let

$$\text{L.H.S} = \frac{\sin^2\theta}{\cos\theta} + \cos\theta$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\ \theta$$

L.H.S = R.H.S

Q.15. $\tan\theta + \cot\theta = \sec\theta.\ \text{cosec}\theta$

Solution: Let

$$\text{L.H.S} = \tan\theta + \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta.\sin\theta}$$

$$= \frac{1}{\cos\theta.\sin\theta} \quad (\ \sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{1}{\cos\theta}.\frac{1}{\sin\theta}$$

$$= \sec\theta.\text{cosec}\theta$$

L.H.S = R.H.S

Q.16. $(\tan\theta+\cot\theta)(\cos\theta+\sin\theta) = \sec\theta+\text{cosec}\theta$

Solution: Let

$$\text{L.H.S} = (\tan\theta + \cot\theta)(\cos\theta + \sin\theta)$$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)(\cos\theta + \sin\theta)$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta.\sin\theta}(\cos\theta + \sin\theta)$$

$$= \frac{1}{\cos\theta.\sin\theta}(\cos\theta + \sin\theta)$$

$$= \frac{\cos\theta + \sin\theta}{\cos\theta.\sin\theta}$$

$$= \frac{\cancel{\cos\theta}}{\cancel{\cos\theta}.\sin\theta} + \frac{\cancel{\sin\theta}}{\cos\theta.\cancel{\sin\theta}}$$

$$= \frac{1}{\sin\theta} + \frac{1}{\cos\theta}$$

$$= \text{cosec}\theta + \sec\theta$$

$$= \sec\theta + \text{cosec}\theta$$

L.H.S = R.H.S

Q.17. $\sin\theta(\tan\theta + \cot\theta) = \sec\theta$

Solution: Let

$$\text{L.H.S} = \sin\theta(\tan\theta + \cot\theta)$$

$$= \sin\theta\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \cancel{\sin\theta}\ \frac{\sin^2\theta + \cos^2\theta}{\cos\theta.\cancel{\sin\theta}}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\ \theta$$

L.H.S = R.H.S

Q.18. $\dfrac{1+\cos}{\sin}+\dfrac{\sin}{1+\cos}=2\text{cosec}$

Solution: Let

$$\text{L.H.S} = \frac{1+\cos\theta}{\sin\theta}+\frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{(1+\cos\theta)^2+(\sin\theta)^2}{(\sin\theta)(1+\cos\theta)}$$

$$= \frac{(1)^2+2(1)(\cos\theta)+\cos^2\theta+\sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1 \quad 2\cos\theta+1}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}}$$

$$= \frac{2}{\sin\theta}$$

$= 2\text{cosec}\theta$

L.H.S = R.H.S

Q.19. $\dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta} = 2\text{cosec}^2\theta$

Solution:

$$\text{L.H.S} = \frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}$$

$$= \frac{1+\cancel{\cos\theta}+1-\cancel{\cos\theta}}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{2}{(1)^2-(\cos^2\theta)}$$

$$= \frac{2}{1-\cos^2\theta}$$

$$= \frac{2}{\sin^2\theta}$$

$= 2\ \text{cosec}^2\theta$

L.H.S = R.H.S

Q.20. $\dfrac{1+\sin}{1-\sin}-\dfrac{1-\sin}{1+\sin}=4\tan\ \sec$

Solution: Let

$$\text{L.H.S} = \frac{1+\sin\theta}{1-\sin\theta}=\frac{1-\sin\theta}{1+\sin\theta}$$

$$= \frac{(1+\sin\theta)^2-(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{(1+\sin^2\theta+2\sin\theta)-(1+\sin^2\theta-2\sin\theta)}{(1)^2-(\sin\theta)^2}$$

$$= \frac{1+\sin^2\theta+2\sin\theta-1-\sin^2\theta+2\sin\theta}{1-\sin^2\theta}$$

$$= \frac{4\sin\theta}{\cos^2\theta}$$

$$= \frac{4\sin\theta}{\cos\theta.\cos\theta}$$

$$= 4\frac{\sin\theta}{\cos\theta}.\frac{1}{\cos\theta}$$

$= 4\tan\theta.\ \sec\theta$

L.H.S = R.H.S

Q.21. $\sin^3\theta = \sin\theta - \sin\theta.\cos^2\theta$

Solution: Let

L.H.S $= \sin^3\theta$

$= \sin\theta\ .\ \sin^2\theta$

$= \sin\theta(1-\cos^2\theta)$

$= \sin\theta - \sin\theta.\cos^2\theta$

L.H.S = R.H.S

Q.22. $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

Solution: Let

L.H.S $= \cos^4\theta - \sin^4\theta$

$= (\cos^2\theta)^2 - (\sin^2\theta)^2$

$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$= (1)(\cos^2\theta - \sin^2\theta)$

$= \cos^2\theta - \sin^2\theta$

L.H.S = R.H.S

Q.23. $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}} = \dfrac{\sin\theta}{1-\cos\theta}$

Solution: Let

$$\text{L.H.S} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{(1)^2-(\cos\theta)^2}} = \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} = \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta}\times\frac{1-\cos\theta}{1-\cos\theta}$$

$$= \frac{(1)^2-(\cos\theta)^2}{\sin\theta(1-\cos\theta)}$$

$$= \frac{1-\cos^2\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{\sin^2\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{\sin\theta}{1-\cos\theta}$$

L.H.S = R.H.S

Q.24. $\sqrt{\dfrac{\sec\theta+1}{\sec\theta-1}} = \dfrac{\sec\theta+1}{\tan\theta}$

Solution: Let

$$\text{L.H.S} = \sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$$

$$= \sqrt{\frac{\sec\theta+1}{\sec\theta-1}\times\frac{\sec\theta+1}{\sec\theta+1}}$$

$$= \sqrt{\frac{(\sec\theta+1)^2}{(\sec\theta)^2-(1)^2}}$$

$$= \sqrt{\frac{(\sec\theta+1)^2}{\sec^2\theta-1}}$$

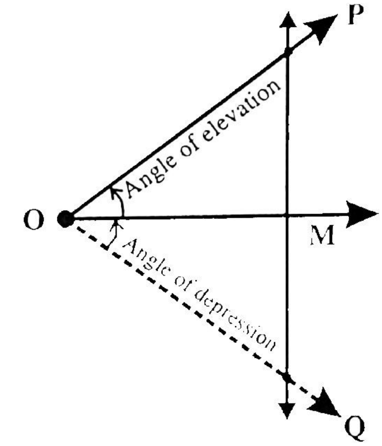
$$= \sqrt{\frac{(\sec\theta+1)^2}{\tan^2\theta}}$$

$$= \frac{\sec\theta+1}{\tan\theta}$$

L.H.S = R.H.S

Angle of Elevation and Angle of Depression

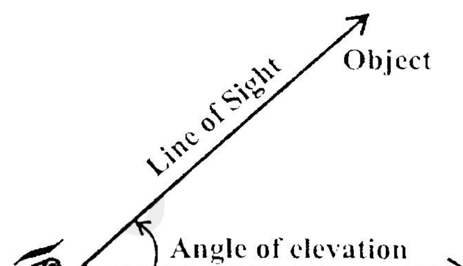
Suppose O, P and Q are three points, P being at a higher level of O and Q being at lower level than O. Let a horizontal line drawn through O meet in M, the vertical line drawn through P and Q.



The angle MOP is called the angle of elevation of point P as seen from O. For looking at Q below the horizontal line we have to lower our eyes and ∠ MOQ is called the angle of depression.

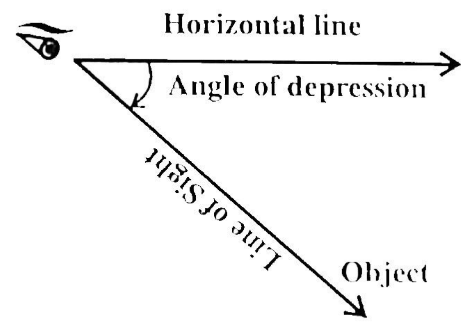
Angle of Elevation:

The angle between the horizontal line through eye and a line from eye to the object, above the horizontal line is called angle of elevation.



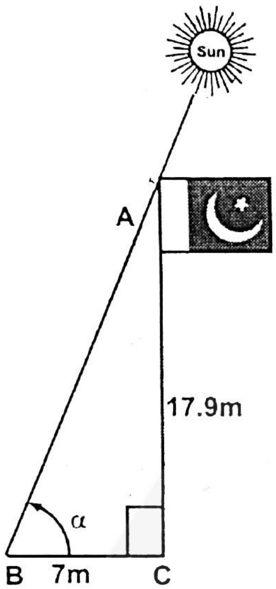
Angle of depression:

The angle between the horizontal line through eye and a line from eye to the object, below the horizontal line is called angle of depression.



Example 1: A flagpole 17.9 meter high casts a 7 meter shadow. Find the angle of elevation of the sun.

Solution:



From the figure, we observe that α is the angle of elevation.

Using the fact that

$$\tan\alpha = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan\alpha = \frac{17.9m}{7m}$$

$$\tan\alpha \quad 2.55714$$

Solving for α gives us

$$\alpha \quad \tan^{-1}(2.55714)$$

$$\alpha \quad (68.6666)^\circ$$

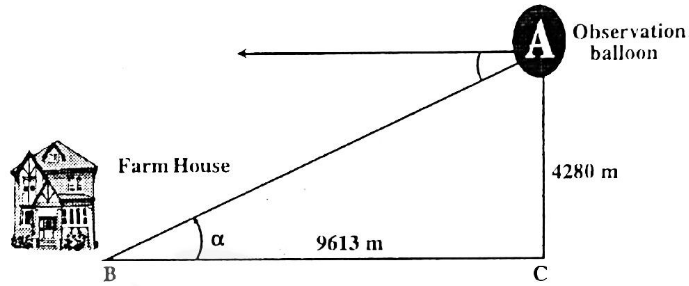
$$68^\circ 40'$$

$$\alpha \quad 68^\circ 40'$$

So angle of elevation is $68^\circ 40'$.

Example 2: An observation balloon is 4280 meter above the ground and 9613 meter away from a farmhouse. Find the angle of depression of the farmhouse as observed from the observation balloon.

Solution:



For problems of this type the angle of elevation of A from B is considered equal to the angle of depression of B from A, as shown in the diagram.

$$\tan\alpha = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan\alpha = \frac{4280m}{9613m}$$

$$\tan\alpha = 0.44523$$

$$\alpha = \tan^{-1}(0.144523)$$

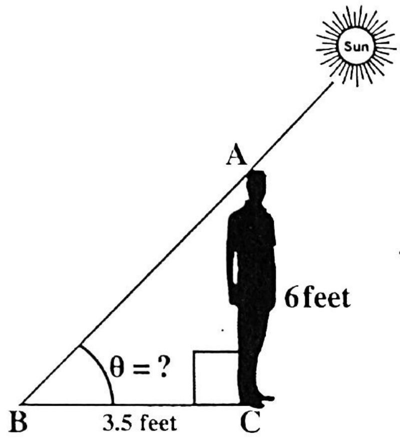
$$\alpha = 24^\circ$$

So, angle of depression is 24°.

EXERCISE 7.5

Q.1. Find the angle of elevation of the sun if a 6 feet man casts a 3.5 feet shadow.

Solution:



From the figure we observe that

Height of man $= m\overline{AC} = 6$ feet

Length of shadow $= m\overline{BC} = 3.5$ feet

Angle of elevation $= \theta\ ? =$

Using the fact that

$$\tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan\theta = \frac{6}{3.5}$$

$$\theta = \tan^{-1}\frac{6}{3.5}$$

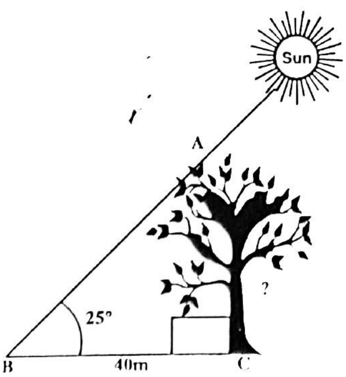
$$\theta = 59.7436$$

$$\theta = 59.74^\circ$$

So, the angle of elevation is $59^\circ 44' 37''$.

Q.2. A tree casts a 40 meters shadow when the angle of elevation of the sun is 25°. Find the height of the tree.

Solution:



From the figure

Height of tree $= m\overline{AC}\ ? =$

Length of shadow $= m\overline{BC} = 40$m

Angle of elevation $= \theta = 25^\circ$

Using the fact that

$$\tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan 25^\circ = \frac{m\overline{AC}}{40}$$

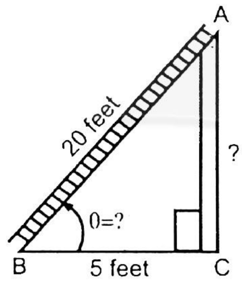
$$m\overline{AC} = 40 \times \tan 25^\circ$$

$$m\overline{AC} = 18.65 \text{ m}$$

So, height of tree is 18.65 m

Q.3. A 20 feet long ladder is leaning against a wall. The bottom of the ladder is 5 feet from the base of the wall. Find the acute angle (angle of elevation) the ladder makes with the ground.

Solution:



From the figure

Length of ladder $= m\overline{AB} = 20$ feet

Distance of ladder from the wall $= m\overline{BC} = 5$ feet

Angle of elevation $= \theta\ ? =$

Using the fact that

$$\cos\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\cos\theta = \frac{5\text{ ft.}}{20\text{ ft.}}$$

$$\cos\theta = 0.25$$

$$\theta = \cos^{-1}(0.25)$$

$$\theta = 75.5225$$

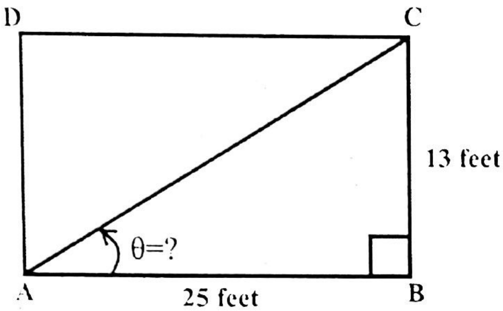
$$\theta = 75.5^\circ$$

or $\theta = 75^\circ\ 30'$

So, angle of elevation is $75^\circ 31' 21''$

Q.4. The base of rectangle is 25 feet and the height of rectangle is 13 feet. Find the angle that the diagonal of the rectangle makes with the base.

Solution:



From the figure

Base of rectangle = $m\overline{AB}$ = 25 feet

Height of rectangle = $m\overline{BC}$ = 13 feet

Diagonal $\overline{AC}$ is taken

Angle between diagonal and base = θ ? =

Using the fact that

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\tan\theta = \frac{13}{25}$$

$$\theta = \tan^{-1}\frac{13}{25}$$

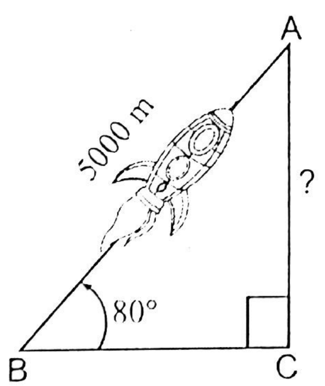
$$\theta = 27.4744$$

$$\theta = 27.47°$$

So, angle between diagonal and base is 27°28′28″.

Q.5. A rocket is launched and climbs at a constant angle of 80°. Find the altitude of the rocket after it travels 5000 meter.

Solution:



From the figure

Distance travelled by rocket = $m\overline{AB}$ = 5000m

Altitude of rocket = $m\overline{AC}$? =

Angle of elevation = θ = 80°

Using $$\sin\theta = \frac{m\overline{AC}}{m\overline{AB}}$$

$$\text{Sin}80° = \frac{m\overline{AC}}{5000}$$

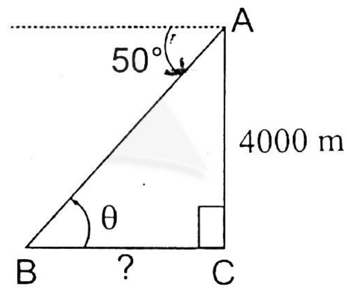
$$m\overline{AC} = 5000 \times \sin 80°$$

$$m\overline{AC} = 4924.04\text{m}$$

So, the altitude of rocket is 4924.04m

Q.6. An aeroplane pilot flying at an altitude of 4000m wishes to make an approach to an airport at an angle of 50° with the horizontal. How far from the airport will the plane be when the pilot begins to descend?

Solution:



From the figure

Altitude of aeroplane = $m\overline{AC}$ = 4000m

Distance of plane from airport = $m\overline{BC}$? =

Angle of depression = 50°

As the alternate angles of parallel lines are equal, so angle

$$\theta = 50°$$

Using the fact that, $$\tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan 50° = \frac{4000\text{m}}{m\overline{BC}}$$

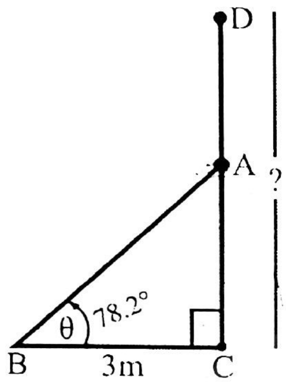
$$m\overline{BC} = \frac{4000\text{m}}{\tan 50°}$$

$$m\overline{BC} = 3356.4\text{ m}$$

So, the distance of aeroplane from airport is 3356.4 m.

Q.7. A guy wire (supporting wire) runs from the middle of a utility pole to the ground. The wire makes an angle of 78.2° with the ground and touch the ground 3 meters from the base of the pole. Find the height of the pole.

Solution:



From the figure

Height of pole $= m\overline{CD}$? =

Distance of wire from the base of the pole

$= m\overline{BC} = 3m$

Angle of elevation $= \theta = 78.2°$

As the wire is attached with the pole at its middle point A, so, first we find $m\overline{AC}$

Using the fact that

$$\tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan 78.2 = \frac{m\overline{AC}}{3}$$

$$m\overline{AC} = 3m \times \tan 78.2°$$

$$m\overline{AC} = 14.36 \text{ m}$$

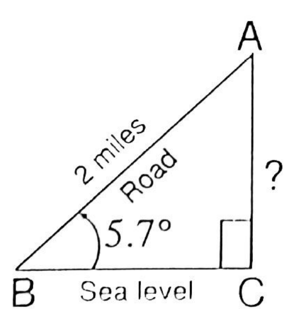
So Height of pole is $= m\overline{DC} = 2(m\overline{AC})$

$= 2\times 14.36$ m

$= 28.72$ m

Q.8. A road is inclined at an angle 5.7°. Suppose that we drive 2 miles up this road starting from sea level. How high above sea level are we? 07(121)

Solution:



From the figure

Distance covered on road $= m\overline{AB} = 2$ miles

Angle of inclination $= \theta = 5.7°$

Height from sea level $= m\overline{AC}$? =

Using the fact that,

$$\sin\theta = \frac{m\overline{AC}}{m\overline{AB}}$$

$$\sin 5.7° = \frac{m\overline{AC}}{2}$$

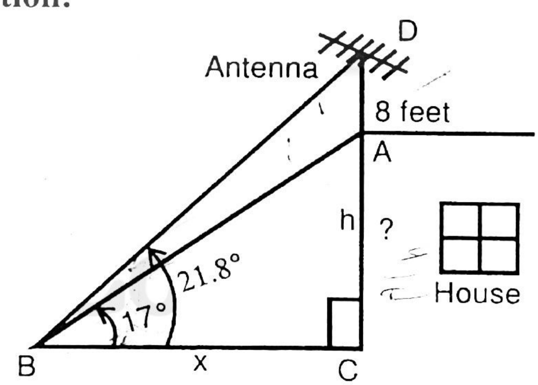
$$m\overline{AC} = 2 \times \sin 5.7°$$

$$m\overline{AC} = 0.199 \text{ mile}$$

Hence, we are at the height of 0.199 mile from the sea level.

Q.9. A television antenna of 8 feet height is located on the top of a house. From a point on the ground the angle of elevation to the top of the house is 17° and the angle of elevation to the top of antenna is 21.8°. Find the height of the house.

Solution:



From the figure

Distance of point from house $= m\overline{BC} = x$

Height of house $= m\overline{AC} = h = ?$

Height of antenna $= m\overline{AD} = 8$ feet

Angle of elevation of top of house = 17°

Angle of elevation of top of antenna= 21.8°

In right angled ΔABC

$$\tan 17° = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan 17° = \frac{h}{x}$$

$$x = \frac{1}{\tan 17°} \times h$$

$x = 3.271 \times h$ ....(i)

Now in right angle ΔDBC

$$\tan 21.8 = \frac{m\overline{CD}}{m\overline{BC}}$$

$$\tan 21.8 = \frac{m\overline{AD} + m\overline{AC}}{m\overline{BC}}$$

$$\tan 21.8 = \frac{8+h}{x}$$

$$0.40 = \frac{8+h}{3.271h} \quad \text{[From (i)]}$$

$$0.40 \times 3.271h = 8 + h$$

$$1.3084\ h - h = 8$$

$$(1.3084 - 1)\ h = 8$$

$$0.3084\ h = 8$$

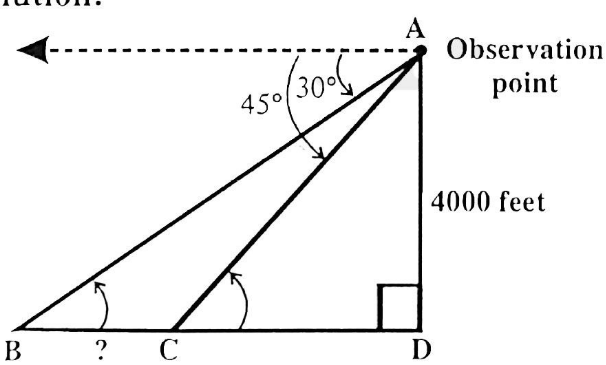
$$h = \frac{8}{0.3084}$$

$$h = 25.94 \text{ feet}$$

So, the height of the house is 25.94 feet

Q.10. From an observation point, the angles of depression of two boats in line with this point are found to 30° and 45°. Find the distance between the two boats if the point of observation is 4000 feet high.

Solution:



From the figure

Height of observation point= $m\overline{AD}$ =4000 feet

Distance between boats = $m\overline{BC}$? =

Angles of depression of points B and C are 30° and 45° respectively from point A.

As the alternate angles of parallel lines are equal, so

$m\angle B = 30°$ and $m\angle C = 45°$

Now in right angled ΔACD

$$\tan 45° = \frac{m\overline{AD}}{m\overline{CD}}$$

$$1 = \frac{4000}{m\overline{CD}}$$

$$m\overline{CD} = 4000 \text{ feet}$$

Now in right angled ΔBCD

$$\tan 30° = \frac{m\overline{AD}}{m\overline{BD}}$$

$$\frac{1}{\sqrt{3}} = \frac{4000}{m\overline{BC} + m\overline{CD}}$$

$$\frac{1}{\sqrt{3}} = \frac{4000}{m\overline{BC} + 4000}$$

$$m\overline{BC} + 4000 = 4000\sqrt{3}$$

$$m\overline{BC} = 4000\sqrt{3} - 4000$$

$$m\overline{BC} = 6928.20 - 4000$$

$$m\overline{BC} = 2928.20 \text{ feet}$$

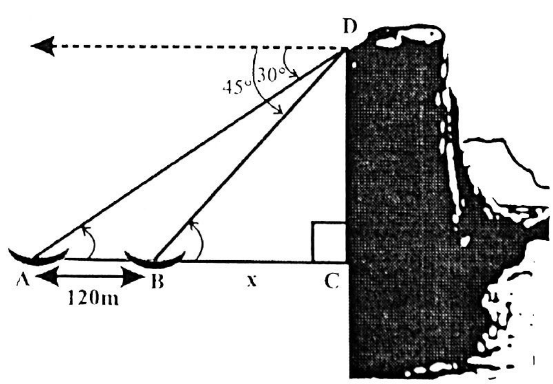
So, the distance between boats is 2928.2 feet.

Q.11. Two ships, which are in line with the base of a vertical cliff are 120 meters apart. The angles of depression from the top of the cliff to the ships are 30° and 45°, as shown in the diagram.

(a) Calculate the distance BC

(b) Calculate the height CD of the cliff.

Solution:



From the figure

Height of cliff = $\overline{CD}$ = h = ?

Distance = $\overline{BC}$ = x = ?

Distance between boats = $\overline{AB}$ = 120 m

Angles of depression from point D to points A and B are 30° and 45° respectively.

As the alternate angles of parallel lines are equal, so $m\angle A = 30°$ and $m\angle B = 45°$

In right angled ΔBCD

$$\tan 45^\circ = \frac{m\overline{CD}}{m\overline{BC}}$$

$$1 = \frac{h}{x}$$

$$x = h \qquad \ldots\ldots. (i)$$

Now in right angled ΔACD

$$\tan 30^\circ = \frac{m\overline{CD}}{m\overline{AC}}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{m\overline{AB} + m\overline{BC}}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{120 + x}$$

$$120 + x = \sqrt{3}\,h$$

$$120 + h = \sqrt{3}\,h \qquad (\because x = h)$$

$$120 = \sqrt{3}h - h$$

$$120 = \left(\sqrt{3} - 1\right)h$$

$$120 = (1.7321 - 1)\,h$$

$$120 = 0.7321\,h$$

$$\frac{120}{0.7321} = h$$

$$h = 163.91 \text{ m}$$

As x= h, so

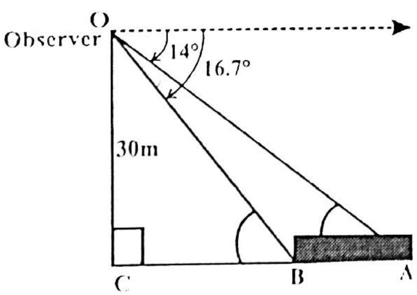
$x = 163.91$ m or 164m

Thus Distance $m\overline{BC} = 164m$

Height of cliff $= m\overline{CD} = 164m$

Q.12. Suppose that we are standing on a bridge 30 meter above a river watching a log (piece of wood) floating towards us. If the angle with the horizontal to the front of the log is 16.7° and angle with the horizontal to the back of the log is 14°, how long is the log?

Solution:



From the figure

Height of observer's position $= m\overline{OC} = 30m$

Length of log of wood $= m\overline{AB} = x = ?$

Angles of depression from point O of the points A and B are 14° and 16.7° respectively.

In right angled ΔOBC

$$\tan 16.7^\circ = \frac{m\overline{OC}}{m\overline{BC}}$$

$$0.30 = \frac{30}{m\overline{BC}}$$

$$m\overline{BC} = \frac{30}{0.30}$$

$$m\overline{BC} = 100m$$

Now in right angled ΔOAC

$$\tan 14^\circ = \frac{m\overline{OC}}{m\overline{AC}}$$

$$0.249 = \frac{30}{m\overline{AB} + m\overline{BC}}$$

$$0.249 = \frac{30}{(x + 100)}$$

$$0.249(x + 100) = 30$$

$$x + 100 = \frac{30}{0.249}$$

$$x + 100 = 120.482$$

$$x = 120.482 - 100$$

$$x = 20.482 \text{ m}$$

So the length of log is 20.482 m.

MISCELLANEOUS EXERCISE –7

Q. 1 Multiple choice questions:
Four possible answers are given for the following questions. Tick ( ) the correct answer.

1. The union of two non–collinear rays, which have common end point is called
   (a) an angle (b) a degree
   (c) a minute (d) a radian
2. The system of measurement in which the angle is measured in radians is called
   (a) CGS system
   (b) sexagesimal system
   (c) MKS system
   (d) circular system
3. $\sec\theta \cot\theta =$
   (a) $\sin\theta$ (b) $\frac{1}{\cos\theta}$
   (c) $\frac{1}{\sin\theta}$ (d) $\frac{\sin\theta}{\cos\theta}$
4. $\text{cosec}^2\theta - \cot^2\theta =$
   (a) -1 (b) 1
   (c) 0 (d) $\tan\theta$
5. If 'r' is the radius of a circle, then its circumference is:
   (a) $\frac{\pi}{2}r$ (b) πr
   (c) $2\pi r$ (d) $4\pi r$
6. $\sec^2\theta =$
   (a) $1 - \sin^2\theta$ (b) $1 + \tan^2\theta$
   (c) $1 + \cos^2\theta$ (d) $1 - \tan^2\theta$
7. $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$
   (a) $2\sec^2\theta$ (b) $2\cos^2\theta$
   (c) $\sec^2\theta$ (d) $\cos\theta$
8. How many right angles are there in 360 degrees?
   (a) two (b) four
   (c) six (d) eight
9. $20° = \ldots\ldots\ldots$
   (a) $360'$ (b) $630'$
   (c) $1200'$ (d) $3600'$
10. $\frac{3\pi}{4}$ radians =
    (a) $115°$ (b) $135°$
    (c) $150°$ (d) $30°$
11. In degree measurement, 1° is equal to:
    (a) $1'$ (b) $60'$
    (c) $90'$ (d) $360'$
12. In degree measurement, $1'$ is equal to:
    (a) $1''$ (b) $60''$
    (c) $90''$ (d) $360''$
13. $\frac{1}{2}\,\text{cosec}45°$
    (a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$
    (c) $\sqrt{2}$ (d) $\frac{\sqrt{3}}{2}$
14. If $\tan\theta = \sqrt{3}$, then θ is equal to
    (a) $90°$ (b) $45°$
    (c) $60°$ (d) $30°$
15. The radian measure of an angle that form a complete circle is:
    (a) $\frac{\pi}{2}$ (b) π
    (c) 2π (d) 4π
16. $\frac{\pi}{2}$ radians =
    (a) $30°$ (b) $45°$
    (c) $60°$ (d) $90°$
17. $\frac{\pi}{3}$ radians =
    (a) $30°$ (b) $45°$
    (c) $60°$ (d) $90°$
18. $1° =$
    (a) 180π radian (b) π radian
    (c) $\frac{\pi}{180}$ radian (d) $\frac{180}{\pi}$ radian

19. Area of a circular sector =

(a) $r\theta$ (b) $r^2\theta$

(c) $\frac{1}{2}r\theta$ (d) $\frac{1}{2}r^2\theta$

20. 2π radians =

(a) 0° (b) 90°

(c) 180° (d) 360°

21. π radians =

(a) 0° (b) 90°

(c) 180° (d) 360°

22. $\frac{1}{\text{Cos}\theta} =$

(a) $\text{Sin}\theta$ (b) $\text{Sec}\theta$

(c) $\text{Cosec}\,\theta$ (d) $\text{Cot}\theta$

23. $\frac{\pi}{6}$ radians =

(a) 30° (b) 45°

(c) 60° (d) 90°

24. $\text{Sin}45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

25. $1^\circ =$

(a) 0.0175 radians

(b) 0.175 radian

(c) 1.75 radians

(d) 175 radians

26. A part of circumference of a circle is called:

(a) radius (b) chord

(c) sector (d) arc

27. $\tan 45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

28. 1 radian =

(a) $(180\pi)^\circ$ (b) $(180)^\circ$

(c) $\frac{\pi}{180}^\circ$ (d) $\frac{180}{\pi}^\circ$

29. $\frac{1}{\text{Sin}\theta} =$

(a) $\text{Cos}\theta$ (b) $\text{Sec}\theta$

(c) $\text{Cosec}\,\theta$ (d) $\text{Cot}\,\theta$

30. $\frac{\pi}{4}$ radians =

(a) 30° (b) 45°

(c) 60° (d) 90°

31. $\frac{1}{\tan\theta} =$

(a) $\tan\theta$ (b) $\text{Sec}\theta$

(c) $\text{Cosec}\,\theta$ (d) $\text{Cot}\theta$

32. $\frac{3\pi}{2}$ radians=

(a) 90° (b) 180°

(c) 270° (d) 360°

33. $\text{Cos}45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

34. Formula for arc length is:

(a) $= r\theta$ (b) $r = \theta$

(c) $\theta = r$ (d) $= \frac{r}{\theta}$

35. $\text{Cosec}\,45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

36. $\text{Sin}60^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

37. $\text{Cot}45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

19. Area of a circular sector =

(a) $r\theta$ (b) $r^2\theta$

(c) $\frac{1}{2}r\theta$ (d) $\frac{1}{2}r^2\theta$

20. 2π radians =

(a) 0° (b) 90°

(c) 180° (d) 360°

21. π radians =

(a) 0° (b) 90°

(c) 180° (d) 360°

22. $\frac{1}{\text{Cos}\theta} =$

(a) $\text{Sin}\theta$ (b) $\text{Sec}\theta$

(c) $\text{Cosec}\,\theta$ (d) $\text{Cot}\theta$

23. $\frac{\pi}{6}$ radians =

(a) 30° (b) 45°

(c) 60° (d) 90°

24. $\text{Sin}45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

25. $1^\circ =$

(a) 0.0175 radians

(b) 0.175 radian

(c) 1.75 radians

(d) 175 radians

26. A part of circumference of a circle is called:

(a) radius (b) chord

(c) sector (d) arc

27. $\tan 45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

28. 1 radian =

(a) $(180\pi)^\circ$ (b) $(180)^\circ$

(c) $\left(\frac{\pi}{180}\right)^\circ$ (d) $\left(\frac{180}{\pi}\right)^\circ$

29. $\frac{1}{\text{Sin}\theta} =$

(a) $\text{Cos}\theta$ (b) $\text{Sec}\theta$

(c) $\text{Cosec}\,\theta$ (d) $\text{Cot}\,\theta$

30. $\frac{\pi}{4}$ radians =

(a) 30° (b) 45°

(c) 60° (d) 90°

31. $\frac{1}{\tan\theta} =$

(a) $\tan\theta$ (b) $\text{Sec}\theta$

(c) $\text{Cosec}\,\theta$ (d) $\text{Cot}\theta$

32. $\frac{3\pi}{2}$ radians=

(a) 90° (b) 180°

(c) 270° (d) 360°

33. $\text{Cos}45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

34. Formula for arc length is:

(a) $= r\theta$ (b) $r = \theta$

(c) $\theta = r$ (d) $= \frac{r}{\theta}$

35. $\text{Cosec}\,45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

36. $\text{Sin}60^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

37. $\text{Cot}45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

38. $\text{Cos}60^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

39. $\text{Cos}30^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

40. $\tan 30^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

41. $\text{Cot}30^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

42. In which quadrant only $\text{Cos}\theta$ and $\text{Sec}\theta$ are positive?

(a) I (b) II

(c) III (d) IV

43. $\text{Cosec}\,30^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

44. $\text{Sec}45^\circ =$

(a) 1 (b) $\sqrt{2}$

$\frac{1}{\sqrt{2}}$

45. $\text{Sin}30^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

46. In which quadrant θ lie when $\text{Cos}\theta < 0, \text{Sin}\theta < 0$?

(a) I (b) II

(c) III (d) IV

47. In which quadrant θ lie when $\text{Sec}\theta > 0, \text{Sin}\theta < 0$?

(a) I (b) II

(c) III (d) IV

48. $\text{Sec}60^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

49. $\text{Cosec}\,60^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

50. In which quadrant only $\text{Sin}\theta$ and $\text{Cosec}\,\theta$ are positive?

(a) I (b) II

(c) III (d) IV

51. $\text{Sec}30^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

52. In which quadrant only $\tan\theta$ and $\text{Cot}\theta$ are positive?

(a) I (b) II

(c) III (d) IV

53. In which quadrant θ lie when $\text{Sin}\theta > 0, \tan\theta < 0$?

(a) I (b) II

(c) III (d) IV

54. $\tan 60^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

55. $\text{Cot}60^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

56. In which quadrant θ lie when $\text{Cos}\theta < 0, \tan\theta < 0$?

(a) I (b) II

(c) III (d) IV

57. Cos . Sec=

(a) 1 (b) tan

(c) 0 (d) Cot

58. In which quadrant θ lie when $\text{Sin}\theta < 0, \text{Sec}\theta < 0$?

(a) I (b) II

(c) III (d) IV

59. $\text{Sin}^2\theta + \text{Cos}^2\theta =$

(a) $\tan^2\theta$ (b) $\text{Cot}^2\theta$

(c) 1 (d) 0

60. $1 + \tan^2\theta =$

(a) $\text{Sin}^2\theta$ (b) $\text{Cos}^2\theta$

(c) $\text{Cosec}^2\theta$ (d) $\text{Sec}^2\theta$

61. Angles between 180° and 270° are in which quadrant?

(a) I (b) II

(c) III (d) IV

62. Angles between 0° and 90° are in which quadrant?

(a) I (b) II

(c) III (d) IV

63. Fundamental trigonometric ratios are

(a) 3 (b) 4

(c) 5 (d) 6

64. Which one is a quadrantal angle?

(a) 30° (b) 45°

(c) 60° (d) 90°

65. Sin .cosec=

(a) 1 (b) 0

(c) Sin (d) Cos

66. In which quadrant θ lie when $\text{Cosec}\theta > 0, \text{Cos}\theta > 0$?

(a) I (b) II

(c) III (d) IV

67. $\tan\theta\cot\theta =$

(a) $\sin\theta$ (b) $\text{Sec}\theta$

(c) 1 (d) 0

68. $1 + \text{Cot}^2\theta =$

(a) $\text{Sin}^2\theta$ (b) $\text{Cos}^2\theta$

(c) $\text{Cosec}^2\theta$ (d) $\text{Sec}^2\theta$

69. In which quadrate all trigonometric ratios are positive?

(a) I (b) II

(c) III (d) IV

70. Sin (-310°)=.......

(a) Sin310° (b) –Sin310°

(c) Cos310° (d) tan310°

71. Sec (-60°).......

(a) –Sec60° (b) Sec60°

(c) Cos60° (d) Cot60°

ANSWER KEY

1	a	**2**	d	**3**	c	**4**	b	**5**	c	**6**	b	**7**	a	**8**	b
9	c	**10**	b	**11**	b	**12**	b	**13**	b	**14**	c	**15**	c	**16**	d
17	c	**18**	c	**19**	d	**20**	d	**21**	c	**22**	b	**23**	a	**24**	c
25	a	**26**	d	**27**	a	**28**	d	**29**	c	**30**	b	**31**	d	**32**	c
33	c	**34**	a	**35**	b	**36**	b	**37**	a	**38**	a	**39**	b	**40**	d
41	c	**42**	d	**43**	c	**44**	b	**45**	a	**46**	c	**47**	d	**48**	c
49	d	**50**	b	**51**	d	**52**	c	**53**	b	**54**	c	**55**	d	**56**	b
57	a	**58**	c	**59**	c	**60**	d	**61**	c	**62**	a	**63**	d	**64**	d
65	a	**66**	a	**67**	c	**68**	c	**69**	a	**70**	b	**71**	b		

Q.1. Write short answers of the following question:

(i) Define an angle.

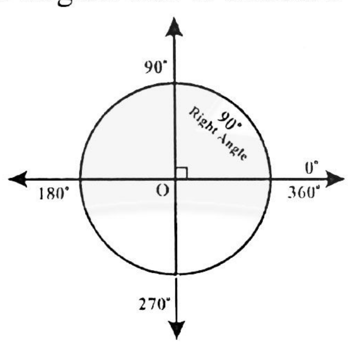
Ans: Angle:

An angle is defined as the union of two non–collinear rays with some common end point. The rays are called arms of the angle and the common end point is known as vertex of the angle.

(ii) What is sexagesimal system of measurement of angles?

Ans: Measurement of an angle in sexagesimal system (degree, minute and second)

Degree: We divide the circumference of a circle into 360 equal arcs. The angle subtended at the centre of the circle by one arc is called one degree and is denoted by 1°.



The symbols 1°, 1′ and 1″ are used to denote a degree, a minute and a second respectively.

Thus 60 seconds (60″) make one minute (1′)
60 minutes (60′) make one degree (1°)
90 degrees (90°) make one right angle.
360 degrees (360°) make 4 right angles.
An angle of 360° denotes a complete

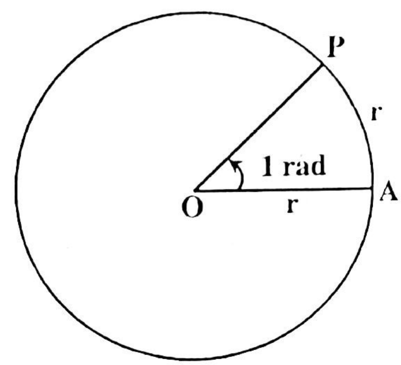
(iii) How many minutes are in two right angles?

Ans: As we know that one right angle = 90°
Two right angles = 180°
Minutes in 1° = 60′
Minutes in 180° = 180×60
Minutes in two right angles = 10800′

(iv) Define radian measure of an angle.

Ans: 07(200)

Radian: The angle subtended at the centre of the circle by an arc, whose length is equal to the radius of the circle is called one Radian.



Consider a circle of radius r whose centre is O. From any point A on the circle cut off an arc AP whose length is equal to the radius of the circle. Join O with A and O with P. The ∠AOP is one radian. This means that when

Length of arc AP = length of radius $\overline{OA}$ then m∠AOP = 1 radian

(v) Convert $\frac{\pi}{4}$ radian to degree measure.

Ans: $\frac{\pi}{4}$ radian $= \frac{\pi}{4}\,\frac{180}{\pi}$ degrees

$= \frac{\pi}{4}\,\frac{4\times 45}{\pi}$ degrees

$= 45°$

(vi) Convert 15° to radians

Ans: $15° = 15\,\frac{\pi}{180}$ radian

$= 15\,\frac{\pi}{15\times 12}$ radian

$= \frac{\pi}{12}$ radian

(vii) What is the radian measure of the central angle of an arc 50m long on the circle of radius 25m.

Solution: Central angle = θ ? =

Arc length = l = 50m

Radius = r = 25m

Using formula

$L = r\theta$

$\theta = \frac{}{r}$

$= \frac{50}{25}$

$\theta = 2$ radian

(viii) **Find r when l = 56cm and θ = 45°**

Solution:

r = ?

L= 56 cm

θ = 45°

$= 45 \frac{\pi}{180}$ radian

$= \frac{\pi}{4}$ radian

Using formula

L = r θ

$56 = r \frac{\pi}{4}$

$r = \frac{56\times4}{\pi}$

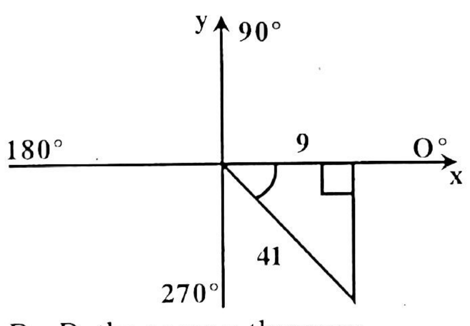
r = 71.3 cm

(ix) **Find tanθ when cosθ = $\frac{9}{41}$ and terminal side of the angle θ is in fourth quadrant.**

Solution: tanθ ? =

As $\cos\theta = \frac{9}{41}$ i.e Base= 9 and hypotenuse = 41

and terminal side of θ is in quadrant IV.



By Pythagorean theorem.

$(Base)^2 + (Perpendicular)^2 = (Hypotenuse)^2$

$(9)^2 + (Per.)^2 = (41)^2$

$(Per.)^2 = 1681 - 81$

$(Per.)^2 = 1600$

$Per. = \sqrt{1600}$

$Per. = \pm 40$

So $\tan\theta = \frac{Per.}{Base}$

$= \frac{-40}{9}$

–ve sign shows tanθ is –ve in quadrant IV.

(x) Prove that $(1-\sin^2\theta)(1+\tan^2\theta) = 1$

Solution: Let

L.H.S $= (1-\sin^2\theta)(1+\tan^2\theta)$

$= \cos^2\theta.\sec^2\theta$

$= \cos^2\theta.\frac{1}{\cos^2\theta}$

$= 1$

L.H.S = R.H.S

Q.2. Fill in the blanks:

(i) π radians =__________ degree.

(ii) The terminal side of angle 235° lies in _______quadrant.

(iii) Terminal side of the angle – 30° lies in _______quadrant.

(iv) Area of a circular sector is ________.

(v) If r = 2cm and θ = 3 radian, then area of the circular sector is______.

(vi) The general form of the angle 480° is ._________________

(vii) If $\sin\theta = \frac{1}{2}$, then θ .___________=

(viii) If θ = 300°, then see (–300)° = _____

(ix) $1 + \cot^2\theta$ _________=

(x) $\sec\theta - \tan\theta$ ____________ =

ANSWER KEY

(i)	180°	**(ii)**	III	**(iii)**	IV	**(iv)**	$\frac{1}{2}r^2\theta$	**(v)**	6cm²
(vi)	2kπ + 120°, where k = 1	**(vii)**	30° or $\frac{\pi}{6}$ *radian*	**(viii)**	2	**(ix)**	$\csc^2\theta$	**(x)**	$\frac{1-\sin\theta}{\cos\theta}$