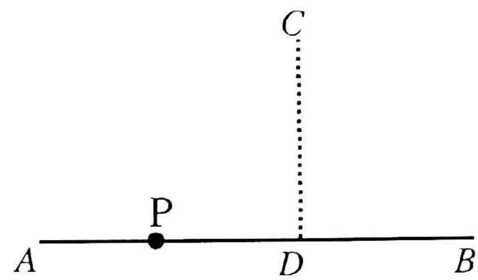
Unit 08 PROJECTION OF A SIDE OF A TRIANGLE

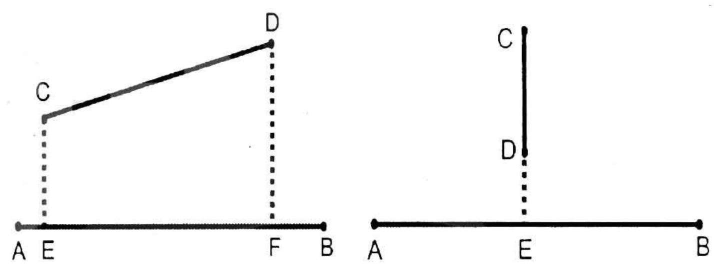
Projection of a Point:

The projection of a given point on a line segment is the foot of $\perp$ drawn from the point on that line segment. If $\overline{CD} \perp \overline{AB}$, then evidently D is the foot of perpendicular CD from the point C on the line segment AB. However projection of a point p lying on the $\overline{AB}$ is the point itself.



Projection of a Line Segment:

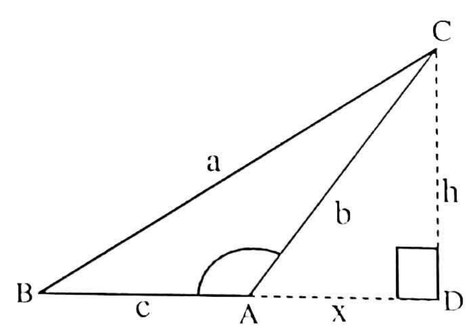
The projection of a line segment $\overline{CD}$ on a line segment $\overline{AB}$ is the portion $\overline{EF}$ of the latter intercepted between foots of the perpendiculars drawn from C and D. However projection of a vertical line segment $\overline{CD}$ on a line segment $\overline{AB}$ is a point on $\overline{AB}$ which is of zero dimension.



THEOREM 1

In an obtuse angled triangle, the square on the side opposite to the obtuse angle is equal to the sum of the squares on the sides containing the obtuse angle together with twice the rectangle contained by one of the sides, and the projection on it of the other.

Given: ABC is a triangle having an obtuse angle BAC at A. Draw $\overline{CD}$ perpendicular on $\overline{BA}$ produced , so that $\overline{AD}$ is the projection of $\overline{AC}$ on $\overline{BA}$ produced.



Take $m\overline{BC} = a$, $m\overline{CA} = b$, $m\overline{AB} = c$, $m\overline{AD} = x$ and $m\overline{CD} = h$

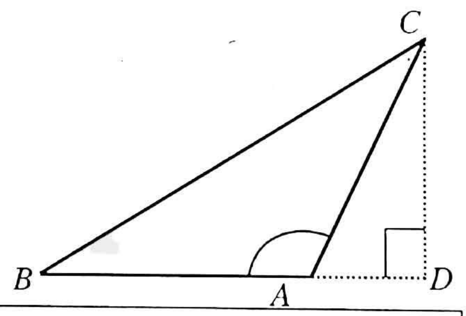
To prove: $\left(m\overline{BC}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{AB}\right)^2 + 2\left(m\overline{AB}\right)\left(m\overline{AD}\right)$

i.e., $a^2 = b^2 + c^2 + 2cx$

Proof:

Statements	Reasons
In $\angle$rt ΔCDA,	
$m\angle CDA = 90°$	Given
$(m\overline{AC})^2 + (m\overline{AD})^2 = (m\overline{CD})^2$	Pythagoras Theorem
or $b^2 = x^2 + h^2$ ....(i)	
In $\angle$rt ΔCDB,	
$m\angle CDB = 90°$	Given
$(m\overline{BC})^2 + (m\overline{BD})^2 = (m\overline{CD})^2$	Pythagoras Theorem
or $a^2 = (c + x)^2 + h^2$	$m\overline{BD} + m\overline{BA} = m\overline{AD}$
$a^2 = c^2 + 2cx + x^2 + h^2$ ....(ii)	
Hence $a^2 = c^2 + 2cx + b^2$	Using (i) and (ii)
i.e., $a^2 = b^2 + c^2 + 2cx$	
or $(m\overline{BC})^2 + (m\overline{AC})^2 + (m\overline{AB})^2 = 2(m\overline{AB})(m\overline{AD})$	

Example: **In a ΔABC with obtuse angle at A, if $\overline{CD}$ is an altitude on $\overline{BA}$ produced and $m\overline{AC} = m\overline{AB}$ then prove that $(m\overline{BC})^2 = 2(m\overline{AB})(m\overline{BD})$**



Given: In a ΔABC, $m\angle A$ is obtuse $m\overline{AC} = m\overline{AB}$ and $\overline{CD}$ being altitude on $\overline{BA}$ produced.

To prove: $(m\overline{BC})^2 = 2(m\overline{AB})(m\overline{BD})$

Proof: In a ΔABC, having obtuse angle BAC at A.

Statements	Reasons
$(m\overline{BC})^2 + (m\overline{BA})^2 + (m\overline{AC})^2 = 2(m\overline{BA})(m\overline{AD})$	By theorem 1
$= (m\overline{AB})^2 + (m\overline{AB})^2 + 2(m\overline{AB})(m\overline{AD})$	$m\overline{AC} = m\overline{AB}$ (Given)
$= 2(m\overline{AB})^2 + 2(m\overline{AB})(m\overline{AD})$	Taking $2(m\overline{AB})$ as common
$(m\overline{BC})^2 + 2m\overline{AB}(m\overline{AB} = m\overline{AD})$	On the line segment BD, Point A is between B and D.
$(m\overline{BC})^2 = 2(m\overline{AB})(m\overline{BD})$	

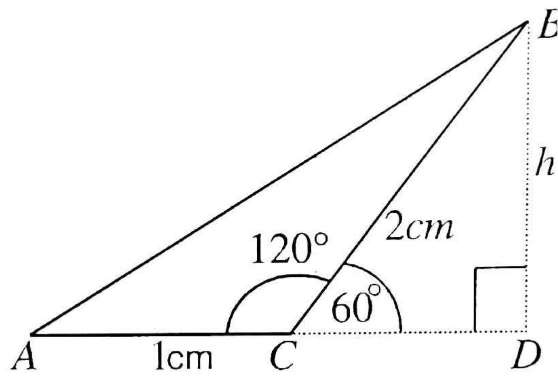
EXERCISE 8.1

Q. 1 Given $m\overline{AC}=1cm$, $m\overline{BC}=2cm$, $m\angle C=120°$

Compute the length AB and the area of ΔABC.

Hint : $(m\overline{AB})^2=(m\overline{AC})^2+(m\overline{BC})^2+2(m\overline{AC})(m\overline{CD})$

where $(m\overline{CD})=(m\overline{BC})\text{Cos}(180°-C)$ (Use theorem I)



Solution:

Given: In a ABC $m\overline{AC}=1cm$, $m\overline{BC}=2cm$, $m\angle C=120°$

To Find: (i) $m\overline{AB}$ **(ii) Area of** ABC

Calculations:

(i) In obtuse angled triangle ABC, by theorem I

$(m\overline{AB})^2 + (m\overline{AC})^2 + (m\overline{BC})^2 = 2(m\overline{AC}).(m\overline{CD})$ .... (i)

In right angled BCD

$$\text{Cos } 60° = \frac{m\overline{CD}}{m\overline{BC}}$$

$$\frac{1}{2} = \frac{m\overline{CD}}{/2}$$

$\overline{CD} = 1cm$ $\boxed{\overline{CD} = 1cm}$

Now putting the corresponding values in (i)

$(m\overline{AB})^2 = (1cm)^2 + (2cm)^2 + 2(1cm)(1cm)$

$= 1cm^2 + 4cm^2 + 2cm^2$

$= 7\ cm^2$

$\sqrt{(m\overline{AB})^2} = \sqrt{7cm^2}$ $\boxed{m\overline{AB} = 2.645\text{ cm}}$

(ii) Area of ΔABC = $\frac{1}{2}$ base × Altitude

Area of $\Delta ABC = \frac{1}{2}\ m\overline{AC} \times m\overline{BD}$

$= \frac{1}{2} \times 1cm \times h$ .... (ii)

In right angled triangle BCD

By Pythagoras theorem

$(2cm)^2 = (1cm)^2 + (h)^2$

$4cm^2 = 1\ cm^2 + h^2$

$h^2 = 3\ cm^2$ $h = \sqrt{3}$ cm

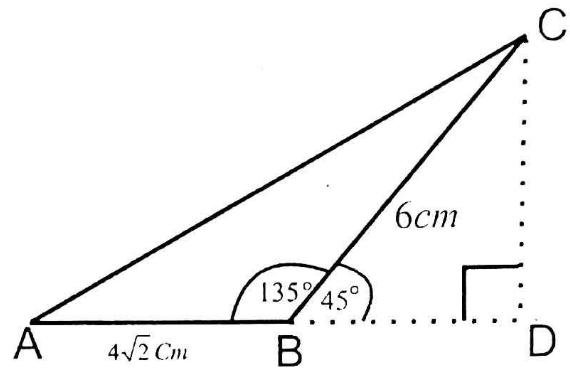
Thus equation (ii) becomes

$$\text{Area of } \Delta ABC = \frac{1}{2} \times 1cm \times \sqrt{3}\,cm$$

$$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{2} cm^2$$

Q. 2 Find $m\overline{AC}$ if in ABC, $m\overline{BC} = 6cm$, $m\overline{AB} = 4\sqrt{2}cm$ and $m\angle ABC = 135^{o}$.

Solution:



Given:

$$m\overline{BC} = 6cm$$

$$m\overline{AB} = 4\sqrt{2}cm$$

$$m\angle ABC = 135^{o}$$

To Find: $m\overline{AC} = ?$

Calculation:

In obtuse angled triangle ABC, by theorem 1

$$\left(m\overline{AC}\right)^2 = \left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2 + 2\left(m\overline{AB}\right)\left(m\overline{BD}\right) \text{..........(i)}$$

In right angled BCD

$$\text{Cos } 45^{o} = \frac{m\overline{BD}}{m\overline{BC}}$$

$$\frac{1}{\sqrt{2}} = \frac{m\overline{BD}}{6cm}$$

$$m\overline{BD} = \frac{6}{\sqrt{2}}\,cm$$

Now putting the corresponding values in equation (i) we get

$$\left(m\overline{AC}\right)^2 + \left(4\sqrt{2}\,cm\right)^2 \quad \left(6cm\right)^2 = 2\left(4\sqrt{2}\,cm\right)\frac{6}{\sqrt{2}}\,cm$$

$$= 16(2\text{ cm}^2) + 36\text{cm}^2 + 8\text{cm }(6\text{cm})$$

$$= 32\text{ cm}^2 + 36\text{ cm}^2 + 48\text{ cm}^2$$

$$= 116\text{ cm}^2$$

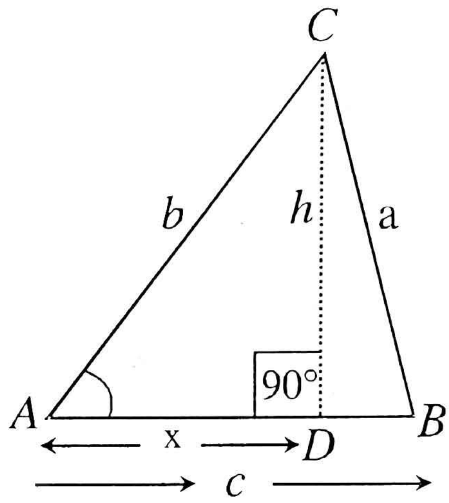
By taking square root of both sides, we get

$$\sqrt{\left(m\overline{AC}\right)^2} = \sqrt{116\text{cm}^2} = \sqrt{4\times 29\text{cm}^2}$$

$$m\overline{AC} = 2\sqrt{29}\text{ cm}$$

THEOREM 2

In any triangle, the square on the side opposite to acute angle is equal to sum of the squares on the sides containing that acute angle diminished by twice the rectangle contained by one of those sides and the projection on it of the other.



Given: ΔABC with an acute angle CAB at A.

Take $m\overline{BC} = a$ $m\overline{CA} = b$ and $m\overline{AB} = c$

Draw $\overline{CD} \perp \overline{AB}$ so that $\overline{AD}$ is projection of $\overline{AC}$ on $\overline{AB}$

Also, $m\overline{AD} = x$ and $m\overline{CD} = h$

To prove: $\left(m\overline{BC}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{AB}\right)^2 - 2\left(m\overline{AB}\right)\left(m\overline{AD}\right)$

i.e., $a^2 = b^2 + c^2 - 2cx$

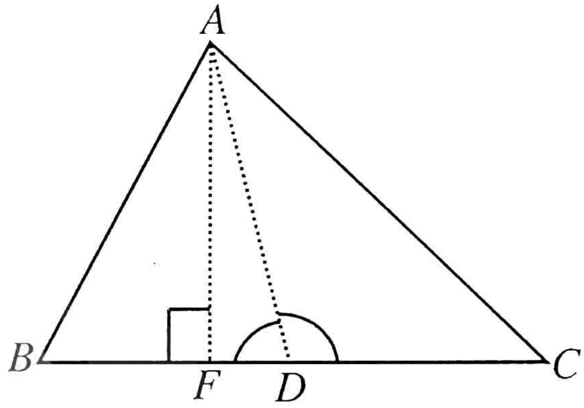
Proof:

Statements	Reasons
In $\angle$rt Δ CDA	
$m\angle CDA = 90°$	Given
$\left(m\overline{AC}\right)^2 + \left(m\overline{AD}\right)^2 = \left(m\overline{CD}\right)^2$	Pythagoras theorem
i.e., $b^2 = x^2 + h^2$ ……………….(i)	
In $\angle$rt Δ CDB,	
$m\angle CDB = 90°$	Given
$\left(m\overline{BC}\right)^2 + \left(m\overline{BD}\right)^2 = \left(m\overline{CD}\right)^2$	Pythagoras theorem
$a^2 = (c - x)^2 + h^2$	From the figure $m\overline{BD} = (c - x)$
or $a^2 = c^2 - 2cx + x^2 + h^2$ ……..(ii)	
$a^2 = c^2 - 2cx + b^2$	Using (i) and (ii)
Hence, $a^2 = b^2 + c^2 - 2cx$	
i.e., $\left(m\overline{BC}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{AB}\right)^2 - 2\left(m\overline{AB}\right)\left(m\overline{AD}\right)$	

THEOREM 3

(APOLLONIUS' THEOREM)

In any triangle, the sum of the squares on any two sides is equal to twice the square on half the third side together with twice the square on the median which bisects the third side.



Given: In a ΔABC, the median $\overline{AD}$ bisects $\overline{BC}$.

i.e., $m\overline{BD} = m\overline{CD}$

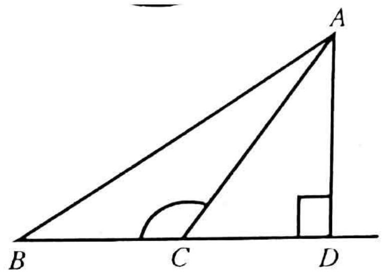
To prove: $\left(m\overline{AB}\right)^2 + \left(m\overline{AC}\right)^2 = 2\left(m\overline{BD}\right)^2 + 2\left(m\overline{AD}\right)^2$

Construction: Draw $\overline{AF} \perp \overline{BC}$

Proof:

Statements	Reasons
In ΔADB Since $\angle ADB$ is acute at D	
$\therefore \left(m\overline{AB}\right)^2 = \left(m\overline{BD}\right)^2 + \left(m\overline{AD}\right)^2 - 2\left(m\overline{BD}\right)\left(m\overline{FD}\right)$ ......(i)	Using theorem 2
Now in ΔADC since $\angle ADC$ is obtuse at D	
$\therefore \left(m\overline{AC}\right)^2 + \left(m\overline{CD}\right)^2 + \left(m\overline{AD}\right)^2 = 2\left(m\overline{CD}\right)\left(m\overline{FD}\right)$ ....(ii)	Using theorem 1
Then $\left(m\overline{AB}\right)^2 + \left(m\overline{AC}\right)^2 + \left(m\overline{BD}\right)^2 = \left(m\overline{CD}\right)^2 + 2\left(m\overline{AD}\right)^2$ $-2\left(m\overline{BD}\right)\left(m\overline{FD}\right) + 2\left(m\overline{CD}\right)\left(m\overline{FD}\right)$ ...(iii)	Adding (i) and (ii)
Also $m\overline{BD} = m\overline{CD}$ ....(iv)	Given
So $\left(m\overline{AB}\right)^2 + \left(m\overline{AC}\right)^2 + \left(m\overline{BD}\right)^2 = \left(m\overline{BD}\right)^2 + 2\left(m\overline{AD}\right)^2$ $-$ 2(mBD)(mFD) + 2(mBD)(mFD) $\left(m\overline{AB}\right)^2 + \left(m\overline{AC}\right)^2 = 2\left(m\overline{BD}\right)^2 + 2\left(m\overline{AD}\right)^2$	Using (iii) and (iv)

Example 1: **In ΔABC, $\angle C$ is obtuse, $\overline{AD} \perp \overline{BC}$ produced, whereas $\overline{BD}$ is projection of $\overline{AB}$ on $\overline{BC}$.**



Prove that $(m\overline{AC})^2 = (m\overline{AB})^2 + (m\overline{BC})^2 - 2(m\overline{BC})(m\overline{BD})$

Given:

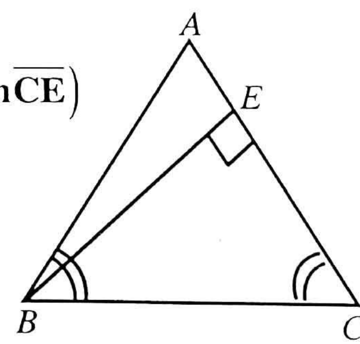
In a ΔABC, $\angle BCA$ is obtuse so that $\angle B$ is acute, $\overline{AD} \perp \overline{BC}$ produced whereas $\overline{BD}$ is projection of $\overline{AB}$ on $\overline{BC}$ produced.

To prove: $(m\overline{AC})^2 = (m\overline{AB})^2 + (m\overline{BC})^2 - 2(m\overline{BC})(m\overline{BD})$

Proof:

Statements	**Reasons**
In $\angle$rt ΔABD $(m\overline{AB})^2 + (m\overline{AD})^2 = (m\overline{BD})^2$ ....(i)	Pythagoras theorem
In $\angle$rt ΔACD $(m\overline{AC})^2 + (m\overline{AD})^2 = (m\overline{CD})^2$ ....(ii)	Pythagoras theorem $m\overline{BC} = m\overline{CD} + m\overline{BD}$
or $(m\overline{AC})^2 = (m\overline{AD})^2 + (m\overline{BD} - m\overline{BC})^2$	
$(m\overline{AC})^2 = (m\overline{AD})^2 + (m\overline{BD})^2 + (m\overline{BC})^2 - 2(m\overline{BC})(m\overline{BD})$ ....(iii)	Using (i) and (iii)
$(m\overline{AC})^2 = (m\overline{AB})^2 + (m\overline{BC})^2 - 2(m\overline{BC})(m\overline{BD})$	

Example 2: **In an isosceles ΔABC, if $m\overline{AB} = m\overline{AC}$ and $\overline{BE} \perp \overline{AC}$, then prove that $(m\overline{BC})^2 = 2(m\overline{AC})(m\overline{CE})$**



Given: In an isosceles ΔABC

$m\overline{AB} = m\overline{AC}$ and $\overline{BE} \perp \overline{AC}$

Whereas $\overline{CE}$ is the projection of $\overline{BC}$ on $\overline{AC}$

To prove: $(m\overline{BC})^2 = 2(m\overline{AC})(m\overline{CE})$

Proof:

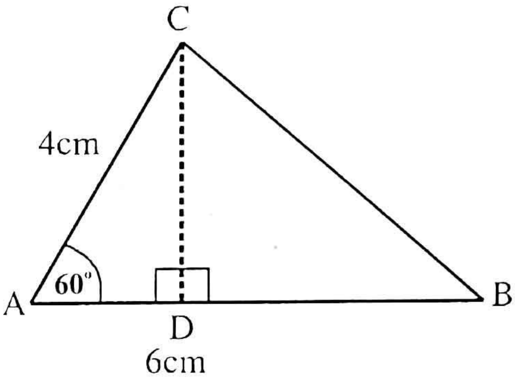
Statements	**Reasons**
In an isosceles ΔABC with $m\overline{AB} = m\overline{AC}$. If $\angle C$ is acute, then $(m\overline{AB})^2 = (m\overline{AC})^2 + (m\overline{BC})^2 - 2(m\overline{AC})(m\overline{CE})$	By theorem 2
$(m\overline{AC})^2 = (m\overline{AC})^2 + (m\overline{BC})^2 - 2(m\overline{AC})(m\overline{CE})$	Given $m\overline{AB} = m\overline{AC}$
$(m\overline{BC})^2 - 2(m\overline{AC})(m\overline{CE}) = 0$	Cancel $(m\overline{AC})^2$ on both sides
or $(m\overline{BC})^2 = 2(m\overline{AC})(m\overline{CE})$	

EXRCISE 8.2

Q. 1 In a ΔABC calculate $m\overline{BC}$

When $m\overline{AB} = 6cm$, $m\overline{AC} = 4cm$ and $m\angle A = 60°$

Solution:



Given: In a ΔABC, $m\overline{AB} = 6cm$, $m\overline{AC} = 4cm$ and $m\angle A = 60°$

To find: $m\overline{BC} = ?$

Calculations:

In acute angled triangle ABC, by theorem 2

$$\left(m\overline{BC}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{AB}\right)^2 - 2\left(m\overline{AB}\right)\left(m\overline{AD}\right) \ldots\ldots\ldots \text{(i)}$$

In right angle ΔACD

$$\text{Cos } 60° = \frac{m\overline{AD}}{m\overline{AC}}$$

$$\frac{1}{\cancel{2}} = \frac{m\overline{AD}}{\cancel{4}\,2}$$

$$\boxed{m\overline{AD} = 2cm}$$

Putting the corresponding values in equation (i), we get

$$\left(m\overline{BC}\right)^2 = (4cm)^2 + (6cm)^2 - 2(6cm)(2cm)$$

$$\left(m\overline{BC}\right)^2 = 16cm^2 + 36\ cm^2 - 24\ cm^2$$

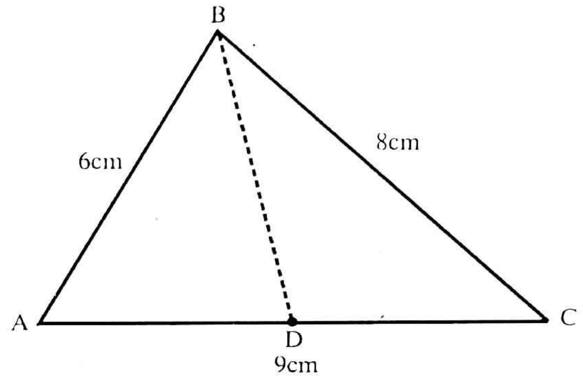
$$\left(m\overline{BC}\right)^2 = 28\ cm^2$$

$$\sqrt{\left(m\overline{BC}\right)^2} = \sqrt{28cm^2}$$

$$m\overline{BC} = 5.29\ cm$$

Q.2 In a $\Delta ABC, m\overline{AB} = 6cm, m\overline{BC} = 8cm, m\overline{AC} = 9cm$ and D is the mid-point of side $\overline{AC}$. Find length of the median $\overline{BD}$.

Solution:



Given:

In a ABC,

$m\overline{AB} = 6cm$

$m\overline{BC} = 8cm$

$m\overline{AC} = 9cm$

To Find: Length of median i.e. $m\overline{BD} = ?$

Calculations:

By Apollonius' theorem

In a ABC

$$\left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2 = 2\left(m\overline{AD}\right)^2 + 2\left(m\overline{BD}\right)^2 \quad \text{........(i)}$$

As $m\overline{AD} = \frac{1}{2} m\overline{AC}$

$$m\overline{AD} = \frac{1}{2}(9cm) = 4.5cm$$

Now, putting the corresponding value in equation (i)

$$(6cm)^2 + (8cm)^2 = 2(4.5cm)^2 + 2\left(m\overline{BD}\right)^2$$

$$36cm^2 + 64cm^2 = 2(20.25cm^2) + 2\left(m\overline{BD}\right)^2$$

$$100cm^2 + 40.5cm^2 = 2\left(m\overline{BD}\right)^2$$

$$100cm^2 - 40.5cm^2 = 2\left(m\overline{BD}\right)^2$$

$$59.5cm^2 = 2\left(m\overline{BD}\right)^2$$

$$\frac{59.5cm^2}{2} = \left(m\overline{BD}\right)^2$$

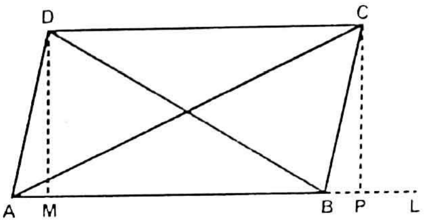
$$29.75cm^2 = \left(m\overline{BD}\right)^2$$

By taking square root

$$\sqrt{\left(m\overline{BD}\right)^2} = \sqrt{29.75cm^2}$$

$$\boxed{m\overline{BD} = 5.45cm}$$

Q.3 In a Parallelogram ABCD prove that $\left(m\overline{AC}\right)^2+\left(m\overline{BD}\right)^2=2\left(m\overline{AB}\right)^2+\left(m\overline{BC}\right)^2$



Given: ABCD is a Parallelogram.

To Prove: $\left(m\overline{AC}\right)^2+\left(m\overline{BD}\right)^2=2\left(m\overline{AB}\right)^2+\left(m\overline{BC}\right)^2$

Construction:

Extend $\overline{AB}$ beyond B. Draw $\overline{DM}\perp\overline{AB}$ and $\overline{CP}\perp\overline{AB}$ extended.

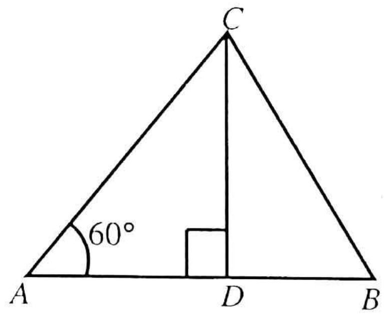
Proof:

Statements	Reasons
In $\Delta ABC, \angle ABC$ is obtuse	
$\left(m\overline{AC}\right)^2+\left(m\overline{AB}\right)^2+\left(m\overline{BC}\right)^2=2\left(m\overline{AB}\right)\left(m\overline{BP}\right)$ ....(i)	By theorem 1
In $\Delta ABD, \angle BAD$ is acute	
$\left(m\overline{BD}\right)^2=\left(m\overline{AB}\right)^2+\left(m\overline{AD}\right)^2-2\left(m\overline{AB}\right)\left(m\overline{AM}\right)$	By theorem 2
$=\left(m\overline{AB}\right)^2+\left(m\overline{BC}\right)^2-2\left(m\overline{AB}\right)\left(m\overline{BP}\right)$ ...(ii)	AMD ≅ BPC i.e $m\overline{AM}=m\overline{BP}$
$\left(m\overline{AC}\right)^2+\left(m\overline{BD}\right)^2=2\left(m\overline{AB}\right)^2+2\left(m\overline{BC}\right)^2$	By adding (i) and (ii)
$\left(m\overline{AC}\right)^2+\left(m\overline{BD}\right)^2=2\left(m\overline{AB}\right)^2+\left(m\overline{BC}\right)^2$	

MISCELLANEOUS EXERCISE – 8

Q. 1 In a ΔABC, $m\angle A = 60°$,

Prove that $\left(m\overline{BC}\right)^2=\left(m\overline{AB}\right)^2+\left(m\overline{AC}\right)^2-\left(m\overline{AB}\right)\left(m\overline{AC}\right)$



Solution:

Given: In a ABC, $m\angle A = 60°$

To Prove: $\left(m\overline{BC}\right)^2=\left(m\overline{AB}\right)^2+\left(m\overline{AC}\right)^2-\left(m\overline{AB}\right)\left(m\overline{AC}\right)$

Proof: In acute angled triangle ABC, by Theorem No. 2

$\left(m\overline{BC}\right)^2=\left(m\overline{AC}\right)^2+\left(m\overline{AB}\right)^2-2\left(m\overline{AB}\right)\left(m\overline{AD}\right)$ ....(i)

In right angled ΔACD

$$\text{Cos}60° = \frac{m\overline{AD}}{m\overline{AC}} \qquad \frac{1}{2} = \frac{m\overline{AD}}{m\overline{AC}} \qquad \left(\text{Cos}60° = \frac{1}{2}\right)$$

$$m\overline{AD} = \frac{1}{2}m\overline{AC}$$

Put it in equation (i)

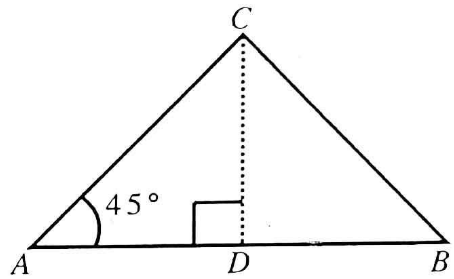
$$\left(m\overline{BC}\right)^2=\left(m\overline{AC}\right)^2+\left(m\overline{AB}\right)^2-2\left(m\overline{AB}\right)\frac{1}{2}m\overline{AC}$$

$$\left(m\overline{BC}\right)^2=\left(m\overline{AC}\right)^2+\left(m\overline{AB}\right)^2-\cancel{2}\left(m\overline{AB}\right)\frac{1}{\cancel{2}}m\overline{AC}$$

$$\left(m\overline{BC}\right)^2=\left(m\overline{AC}\right)^2+\left(m\overline{AB}\right)^2-\left(m\overline{AB}\right)\left(m\overline{AC}\right)$$ Hence proved

Q. 2 In a ΔABC, $m\angle A = 45°$, prove that $(m\overline{BC})^2 = (m\overline{AB})^2 + (m\overline{AC})^2 - \sqrt{2}(m\overline{AB})(m\overline{AC})$.

Solution:



Given: In a ABC, $m\angle A = 45°$

To prove: $(m\overline{BC})^2 = (m\overline{AB})^2 + (m\overline{AC})^2 - \sqrt{2}(m\overline{AB})(m\overline{AC})$

Proof: In triangle ABC, $\angle A$ is acute so by Theorem 2

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD}) \ldots\ldots (i)$$

In right angled ΔACD

$$\cos 45° = \frac{m\overline{AD}}{m\overline{AC}}$$

$$\frac{1}{\sqrt{2}} = \frac{m\overline{AD}}{m\overline{AC}}$$

$$m\overline{AD} = \frac{1}{\sqrt{2}} m\overline{AC}$$

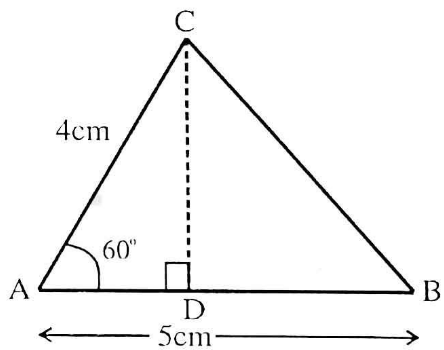
Put it in equation (i)

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB}) \frac{1}{\sqrt{2}} m\overline{AC}$$

$$(m\overline{BC})^2 = (m\overline{AB})^2 + (m\overline{AC})^2 - \sqrt{2}(m\overline{AB})(m\overline{AC}) \qquad \frac{2}{\sqrt{2}} = \sqrt{2}$$

Q. 3 In a ΔABC, calculate $m\overline{BC}$ when $m\overline{AB} = 5cm, m\overline{AC} = 4cm, m\angle A = 60°$

Solution:



Given: In a ΔABC $m\overline{AB} = 5cm, m\overline{AC} = 4cm, m\angle A = 60°$

To Find: $m\overline{BC} = ?$

Calculations: In triangle ABC, $\angle A$ is acute so by Theorem 2

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD})$$

$$(m\overline{BC})^2 = (4cm)^2 + (5cm)^2 - 2(5cm)(m\overline{AD}) \ldots\ldots (i)$$

In right angle ΔACD

$$\cos 60° = \frac{m\overline{AD}}{m\overline{AC}} \qquad \frac{1}{2} = \frac{m\overline{AD}}{4cm} \qquad \left(\cos 60° = \frac{1}{2}\right)$$

$$2cm = m\overline{AD} \qquad \boxed{m\overline{AD} = 2cm}$$

Putting the corresponding values in equation (i)

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD})$$

$$= (4cm)^2 + (5cm)^2 - 2(5cm)(2cm)$$

$$= 16cm^2 + 25cm^2 - 20cm^2$$

$$= 41cm^2 - 20cm^2$$

$$(m\overline{BC})^2 = 21cm^2$$

$$\sqrt{(m\overline{BC})^2} = \sqrt{21cm^2} \qquad \boxed{m\overline{BC} = 4.58 \text{ cm}}$$

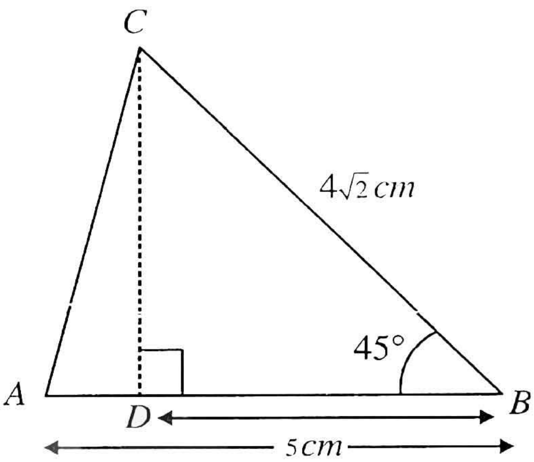
Q. 4 In a ΔABC, calculate $m\overline{AC}$ when $m\overline{AB} = 5cm, m\overline{BC} = 4\sqrt{2}cm, m\angle B = 45^\circ$

Solution:

Given: In a ΔABC $m\overline{AB} = 5cm, m\overline{BC} = 4\sqrt{2}cm, m\angle B = 45^\circ$

To Find: $m\overline{AC} = ?$

Calculations: In acute angled triangle ABC by theorem 2



$$\left(m\overline{AC}\right)^2 = \left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2 - 2\left(m\overline{AB}\right)\left(m\overline{BD}\right)$$

$$\left(m\overline{AC}\right)^2 = (5cm)^2 + \left(4\sqrt{2}cm\right)^2 - 2(5cm)\left(m\overline{BD}\right) \ldots\ldots(i)$$

$m\overline{BD} = ?$

In right angle ΔBCD

$$\text{Cos}45^\circ = \frac{m\overline{BD}}{m\overline{BC}}$$

$$\frac{1}{\sqrt{2}} = \frac{m\overline{BD}}{4\sqrt{2}}$$

$$1 = \frac{m\overline{BD}}{4}$$

$\boxed{m\overline{BD} = 4cm}$

Putting the value of $m\overline{BD}$ in equation (i)

$$\left(m\overline{AC}\right)^2 = (5cm)^2 + \left(4\sqrt{2}cm\right)^2 - 2(5cm)(4cm)$$

$$= 25cm^2 + 16(2cm^2) - 40cm^2$$

$$= 25cm^2 + 32cm^2 - 40cm^2$$

$$= 57cm^2 - 40cm^2$$

$$\left(m\overline{AC}\right)^2 = 17cm^2$$

$$\sqrt{\left(m\overline{AC}\right)^2} = \sqrt{17cm^2}$$

$\boxed{m\overline{AC} = 4.12\ cm}$

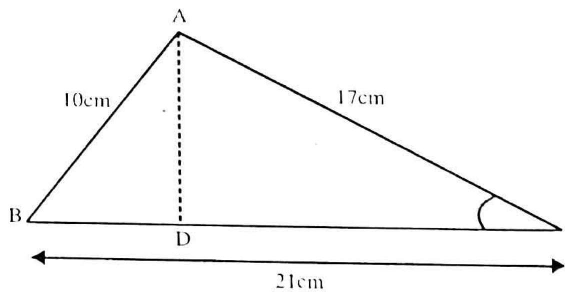
Q. 5 In a triangle ABC, $m\overline{BC} = 21cm$, $m\overline{AC} = 17cm$, $m\overline{AB} = 10cm$. Measure the length of projection of $\overline{AC}$ upon $\overline{BC}$.

Solution:

Given: In a triangle ABC, $m\overline{BC} = 21cm$, $m\overline{AC} = 17cm$, $m\overline{AB} = 10cm$

To Find: Projection of $\overline{AC}$ upon $\overline{BC}$ i.e., $m\overline{DC} = ?$

Calculations: In triangle ABC, $\angle C$ is acute so by theorem 2



$$\left(m\overline{AB}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{BC}\right)^2 - 2\left(m\overline{BC}\right)\left(m\overline{DC}\right)$$

$$(10cm)^2 = (17cm)^2 + (21cm)^2 - 2(21cm)\left(m\overline{DC}\right)$$

$$100cm^2 = 289cm^2 + 441cm^2 - 42cm\left(m\overline{DC}\right)$$

$$100cm^2 - 289cm^2 - 441cm^2 = -42cm\left(m\overline{DC}\right)$$

$$-630cm^2 = -42cm\left(m\overline{DC}\right)$$

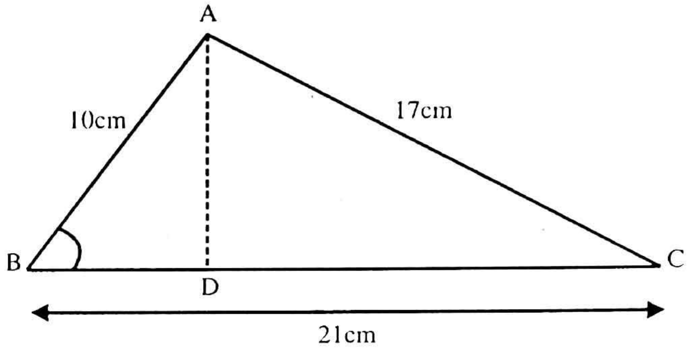
$$\frac{-630cm^2}{-42cm} = m\overline{DC}$$

$\boxed{m\overline{DC} = 15cm}$

Q. 6 In a triangle ABC, $m\overline{BC} = 21cm$, $m\overline{AC} = 17cm$, $m\overline{AB} = 10cm$. Calculate the projection of $\overline{AB}$ upon $\overline{BC}$.

Solution:

Given



$m\overline{BC} = 21cm$
$m\overline{AC} = 17cm$
$m\overline{AB} = 10cm$

To Find: Projection of $\overline{AB}$ upon $\overline{BC}$ i.e $m\overline{BD} = ?$

Calculations:

In triangle ABC , $\angle B$ is acute so by theorem 2

$$\left(m\overline{AC}\right)^2 = \left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2 - 2\left(m\overline{BC}\right)\left(m\overline{BD}\right)$$

$$(17cm)^2 = (10cm)^2 + (21cm)^2 - 2(21cm)\left(m\overline{BD}\right)$$

$$289cm^2 = 100cm^2 + 441cm^2 - 42cm\left(m\overline{BD}\right)$$

$$289cm^2 = 541cm^2 - 42cm\left(m\overline{BD}\right)$$

$$289cm^2 - 541cm^2 = -42cm\left(m\overline{BD}\right)$$

$$-252cm^2 = -42cm\left(m\overline{BD}\right)$$

$$\frac{-252cm^2}{-42cm} = m\overline{BD}$$

$$\boxed{m\overline{BD} = 6cm}$$

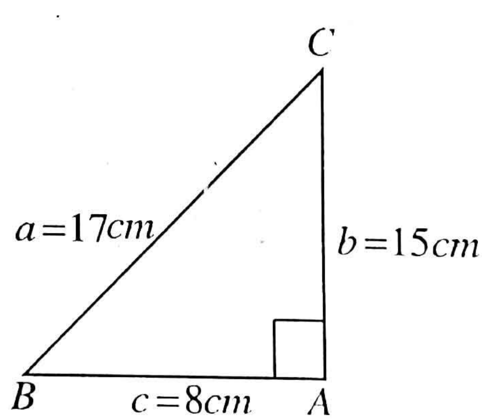
Q. 7 In a ΔABC , $a = 17cm$, $b = 15cm$ and $c = 8cm$. Find $m\angle A$.

Solution:

Given: In a ΔABC , $a = 17cm$, $b = 15cm$, $c = 8cm$

To Find: $m\angle A = ?$

Calculations:



Sum of squares of two sides $= b^2 + c^2$
$= (15cm)^2 + (8cm)^2$
$= 225cm^2 + 64\ cm^2$
$= 289\ cm^2$ ..... (i)

Square of length of third side $= a^2$
$= (17cm)^2$
$= 289cm^2$ ..... (ii)

From (i) and (ii)

$$a^2 = b^2 + c^2$$

The result show that the triangle ABC is right angled triangle with side a = 17cm as hypotenuse.

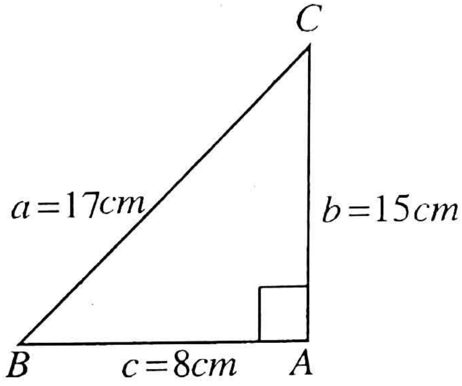
The angle opposite to the hypotenuse is right angle i.e $m\angle A = 90°$

8. In a ΔABC, $a = 17cm$, $b = 15cm$ and $c = 8cm$ find $m\angle B$.

Solution:

Given: In a ΔABC, $a = 17cm$, $b = 15cm$, $c = 8cm$

To Find: $m\angle B = ?$



Calculations:

Sum of squares of two sides $= b^2 + c^2$

$= (15cm)^2 + (8cm)^2$

$= 225cm^2 + 64\ cm^2$

$= 289\ cm^2$ ..... (i)

Square of length of third side $= a^2$

$= (17cm)^2$

$= 289cm^2$ ..... (ii)

From (i) and (ii)

$$a^2 = b^2 + c^2$$

The result show that the triangle ABC is right angled triangle with $m\angle A = 90°$

In triangle ABC, $\tan m\angle B = \frac{Per}{Base} = \frac{15cm}{8cm}$

$m\angle B = \tan^{-1}\frac{15}{8}$ $\quad m\angle B = (61.9)°$

Q.9 Whether the triangle with sides 5cm, 7cm, 8cm is acute, obtuse or right angled.

Solution:

In a triangle ABC, let $a = 5cm$, $b = 7cm$, $c = 8cm$

Sum of squares of two sides $= a^2 + b^2$

$= (5cm)^2 + (7cm)^2$

$= 25cm^2 + 49cm^2$

$= 74cm^2$ .... (i)

Square of length of 3^{rd} side $= c^2$

$= (8cm)^2$

$= 64cm^2$ .... (ii)

From (i) and (ii) $\quad 74cm^2 > 64cm^2$ i.e.

$a^2 + b^2 > c^2$

The result shows that the triangle with sides 5cm, 7cm, 8cm is acute angled triangle.

It is acute angled triangle.

Q.10 Whether the triangle with sides 8cm, 15cm, 17cm is acute, obtuse or right angled.

Solution:

In a triangle ABC let $a = 8cm$, $b = 15cm$, $c = 17cm$

Sum of squares of two sides $= a^2 + b^2$

$= (8cm)^2 + (15cm)^2$

$= 64cm^2 + 225\ cm^2$

$= 289\ cm^2$ ....(i)

Square of length of 3^{rd} side $= c^2$

$= (17cm)^2$

$= 289cm^2$ ....(ii)

From (i) and (ii)

i.e $\quad a^2 + b^2 = c^2$

Result shows that triangle with sides $a = 8cm$, $b = 15cm$ and $c = 17cm$ is right angled triangle.