Unit 11

PARALLELOGRAMS AND TRIANGLES

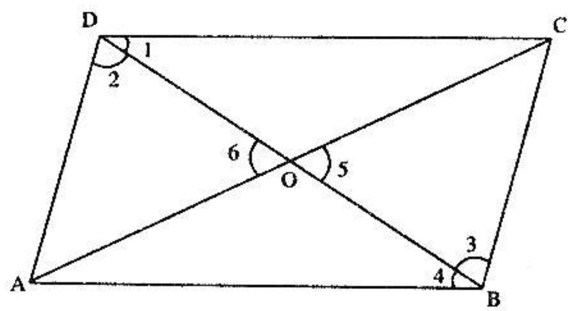
Theorem

In a parallelogram

(i) Opposite sides are congruent.

(ii) Opposite angles are congruent.

(iii) The diagonals bisect each other.



Given

In a quadrilateral ABCD, $\overline{AB} \parallel \overline{DC}, \overline{BC} \parallel \overline{AD}$ and the diagonals $\overline{AC}$, $\overline{BD}$ meet each other at point O.

To Prove

(i) $\overline{AB} \cong \overline{DC}$. $\overline{AD} \cong \overline{BC}$

(ii) $\angle ADC \cong \angle ABC, \angle BAD \cong \angle BCD$

(iii) $\overline{OA} \cong \overline{OC}$. $\overline{OB} \cong \overline{OD}$

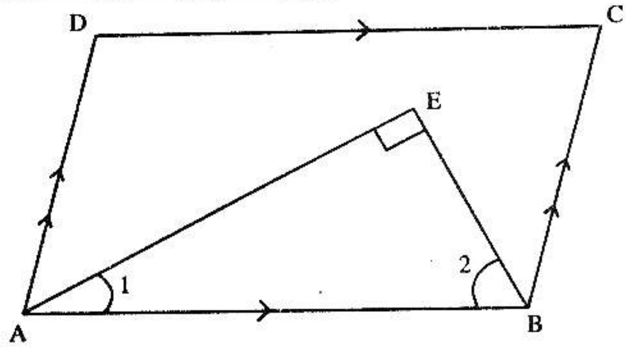
Construction

In the figure as shown, we label the angles as $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$.

Proof

	Statements		Reasons
(i)	In $\Delta ABD \leftrightarrow \Delta CDB$		
	$\angle 4 \cong \angle 1$		Alternate angles
	$\overline{BD} \cong \overline{BD}$		Common
	$\angle 2 \cong \angle 3$		Alternate angles
$\therefore$	$\Delta ABD \cong \Delta CDB$		A.S.A. $\cong$ A.S.A.
So,	$\overline{AB} \cong \overline{DC}$. $\overline{AD} \cong \overline{BC}$		(corresponding sides of congruent triangles)
and	$\angle A \cong \angle C$		(corresponding angles of congruent triangles)
(ii)	Since		
	$\angle 1 \cong \angle 4$	......(a)	Proved
and	$\angle 2 \cong \angle 3$	......(b)	Proved
$\therefore$	$m\angle 1 + m\angle 2 = m\angle 4 + m\angle 3$		From (a) and (b)
or	$m\angle ADC = m\angle ABC$		
or	$\angle ADC \cong \angle ABC$		

and $\angle BAD = \angle BCD$	Proved in (i)
(iii) In $\Delta BOC \leftrightarrow \Delta DOA$	
$\overline{BC} \cong \overline{AD}$	Proved in (i)
$\angle 5 \cong \angle 6$	Vertical angles
$\angle 3 \cong \angle 2$	Proved
$\therefore$ $\Delta BOC \cong \Delta DOA$	A.A.S $\cong$ A.A.S
Hence $\overline{OC} \cong \overline{OA}$, $\overline{OB} \cong \overline{OD}$	Corresponding sides of congruent triangles)



Corollary

Each diagonal of a parallelogram bisects it into two congruent triangles.

Example

The bisectors of two angles on the same side of a parallelogram cut each other at right angles.

Given

A parallelogram ABCD, in which

$\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$

The bisectors of $\angle A$ and $\angle B$ cut each other at E.

To prove

$m\angle E = 90^\circ$

Construction

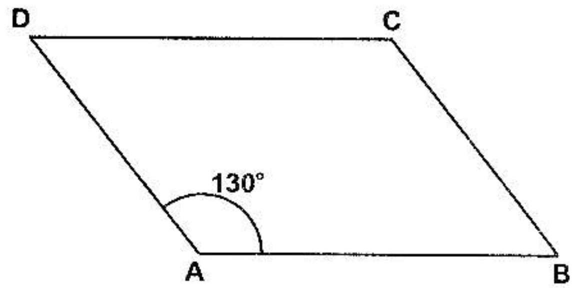
Name the angles $\angle 1$ and $\angle 2$ as shown in the figure.

Proof

Statements	Reasons
$m\angle 1 + m\angle 2$ $= \frac{1}{2}(m\angle BAD + m\angle ABC)$	$\left\{ \begin{array}{l} m\angle 1 = \frac{1}{2} m\angle BAD, \\ m\angle 2 = \frac{1}{2} mABC \end{array} \right.$
$= \frac{1}{2}(180^\circ)$ $= 90^\circ$	$\left\{ \begin{array}{l} \text{Int.angles on the same side of } \overline{AB} \\ \text{Which cuts} \parallel \text{segments } \overline{AD} \text{ and } \overline{BC} \\ \text{are supplementary.} \end{array} \right.$
Hence in ΔABE, $m\angle E = 90^\circ$	$m\angle 1 + m\angle 2 = 90^\circ$ (proved)

EXERCISE 11.1

(1) One angle of a parallelogram is 130°. Find the measures of its remaining angles.



Given

ABCD is a parallelogram that

$m\angle A = 130^\circ$

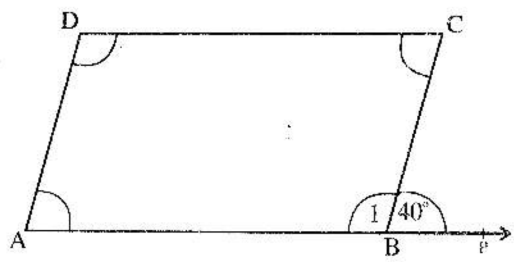
To Prove

(Required) To find the measures of $\angle B$, $\angle C$, $\angle D$

Proof

Statements	Reasons
$m\angle C = m\angle A$	Opposite angles of parallelogram.
$m\angle C = 130^\circ$	Given, $m\angle A = 130^\circ$
$m\angle B + m\angle A = 180^\circ$	$\overline{AD} \parallel \overline{BC}$ and $\overline{AB}$ is transversal. ∴ sum of interior angles.
$m\angle B + 130^\circ = 180^\circ$	Given $m\angle A = 130^\circ$
$m\angle B = 180^\circ - 130^\circ$	
$m\angle B = 50^\circ$	
$m\angle D = m\angle B$	Opp. angles
$m\angle D = 50^\circ$	As $m\angle B = 50^\circ$
∴ $m\angle B = 50^\circ$, $m\angle C = 130^\circ$, $m\angle D = 50^\circ$	

(2) One exterior angle formed on producing one side of a parallelogram is 40°. Find the measures of its interior angles.



Given

ABCD is a parallelogram, side AB has been produced to p to form exterior angle $m\angle CBP = 40^\circ$ and name the interior angles as $\angle 1$, $\angle C$, $\angle D$, $\angle A$.

Required

To find the degree measures of $\angle 1$, $\angle C$, $\angle D$, $\angle A$

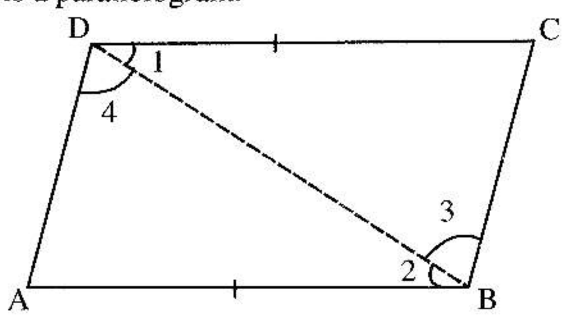
Proof

Statements	Reasons
$m\angle 1 + m\angle CBP = 180^\circ$	Supp.angles.
$m\angle 1 + 40^\circ = 180^\circ$	$m\angle CBP = 40^\circ$ given

$\therefore$	$m\angle 1 = 180^\circ - 40^\circ$		
	$m\angle 1 = 140^\circ$ (i)		
	$m\angle D = m\angle 1$	Opp.angles of llm	
	$m\angle D = 140^\circ$.....(ii)	From (i)	
	$m\angle A + m\angle 1 = 180^\circ$	$\overline{AD} \parallel \overline{BC}$ and $\overline{AB}$ is transversal. (Interior angles)	
	$m\angle A + 140^\circ = 180^\circ$	From (i)	
	$m\angle A = 180^\circ - 140^\circ$		
	$m\angle A = 40^\circ$..............(iii)		
	$m\angle C = m\angle A$	Opp. angles	
	$m\angle C = 40^\circ$	From (iii)	
Thus	$m\angle 1 = 140^\circ, m\angle C = 40^\circ$		

Theorem

If two opposite sides of a quadrilateral are congruent and parallel, it is a parallelogram.



Given

In a quadrilateral ABCD,

$\overline{AB} \cong \overline{DC}$ and $\overline{AB} \parallel \overline{DC}$

To prove

ABCD is a parallelogram.

Construction

Join the point B to D and in the figure, name the angles as indicated:

$\angle 1, \angle 2, \angle 3$ and $\angle 4$

Proof

	Statements	Reasons
In	$\Delta ABD \leftrightarrow \Delta CDB$	
	$\overline{AB} \cong \overline{DC}$	Given
	$\angle 2 \cong \angle 1$	Alternate angles
	$\overline{BD} \cong \overline{BD}$	Common
$\therefore$	$\Delta ABD \cong \Delta CDB$	S.A.S. postulate
Now	$\angle 4 \cong \angle 3$ .....(i)	(corresponding angles of congruent triangles)
$\therefore$	$\overline{AD} \parallel \overline{BC}$ ... .(ii)	From (i)

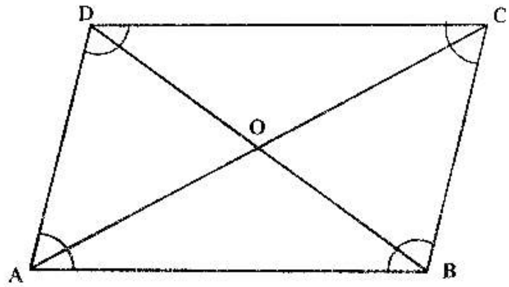
and $\overline{AD} \cong \overline{BC}$ ....(iii)	Corresponding sides of congruent Δs
Also $\overline{AB} \parallel \overline{DC}$ ....(iv)	Given
Hence ABCD is a parallelogram	From (ii) – (iv)

EXERCISE 11.2

(1) Prove that a quadrilateral is a parallelogram if its

(a) Opposite angles are congruent.

(b) Diagonals bisect each other.



Given Given ABCD is a quadrilateral.

$m\angle A = m\angle C$,

$m\angle B = m\angle D$

To prove ABCD is a parallelogram.

Proof

Statements	Reasons
$m\angle A = m\angle C$ (i)	Given
$m\angle B = m\angle D$ (ii)	Given
Now	
$m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$	Angles of a quad.
$m\angle A + m\angle B + m\angle A + m\angle B = 360^\circ$	From (i), (ii)
$m\angle A + m\angle A + m\angle B + m\angle B = 360^\circ$	Rearranging
$2m\angle A + 2m\angle B = 360^\circ$	
$(m\angle A + m\angle B) = 360^\circ/2 = 180^\circ$	Dividing by 2
$\therefore$ $\overline{AD} \parallel \overline{BC}$	As $m\angle A + m\angle B = 180^\circ$
Similarly it can be	(sum of interior angles)
Proved that $\overline{AB} \parallel \overline{CD}$	
Hence ABCD is a parallelogram.	

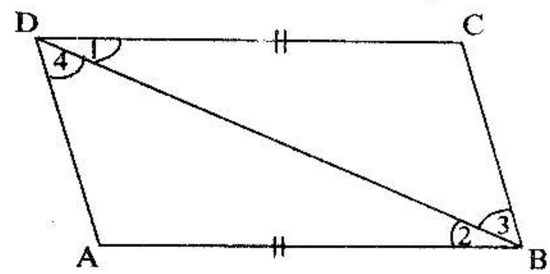
(2) prove that a quadrilateral is a parallelogram if its opposite sides are congruent.

Given

In quadrilateral

ABCD, $\overline{AB} \cong \overline{DC}$,

$\overline{AD} \cong \overline{BC}$



Required

ABCD is a $\parallel$ gm

$\overline{AB} \parallel \overline{CD}$, $\overline{AD} \parallel \overline{BC}$

Construction

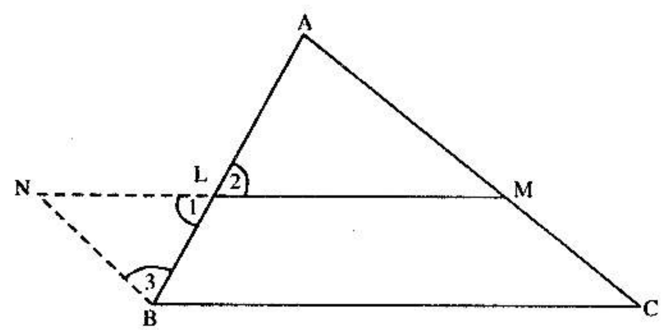
Join point B to D and name the angles $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$

Proof

Statements			Reasons
	$\Delta ABD \leftrightarrow \Delta CDB$		
	$\overline{AD} \cong \overline{CB}$		Given
	$\overline{AB} \cong \overline{CD}$		Given
	$\overline{BD} \cong \overline{BD}$		Common
$\therefore$	$\Delta ABD \cong \Delta CDB$		S.S.S $\cong$ S.S.S
So	$\angle 2 \cong \angle 1$	(i)	Corresponding angles of Congruent triangles
	$\angle 4 \cong \angle 3$	(ii)	Alternate angles
Hence	$\overline{AB} \parallel \overline{CD}$	(iii)	$\angle 2$ and $\angle 1$ are congruent
Similarly	$\overline{BC} \parallel \overline{AD}$	(iv)	Alternate angles $\angle 3$, $\angle 4$ congruent
$\therefore$	ABCD is a parallelogram.		From iii, iv

Theorem

The line segment, joining the mid-points of two sides of a triangle, is parallel to the third side and is equal to one half if its length.



Given In ΔABC, the mid-points of $\overline{AB}$ and $\overline{AC}$ are L and M respectively.

To Prove

$\overline{LM} \parallel \overline{BC}$ and $m\overline{LM} = \frac{1}{2} m\overline{BC}$

Construction

Join M to L and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$ .Join N to B. and in the figures name the angles $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$ as shown.

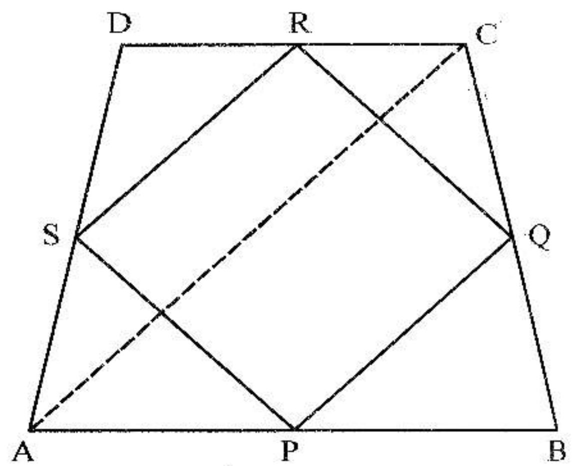
Proof

Statements		Reasons
In	$\Delta BLN \leftrightarrow \Delta ALM$	
	$\overline{BL} \cong \overline{AL}$,	Given
	$\angle 1 \cong \angle 2$	Vertical angles
	$\overline{NL} \cong \overline{ML}$	Construction

$\therefore$ $\Delta BLN \cong \Delta ALM$		S.A.S. postulate
$\angle A \cong \angle 3$	......(i)	(corresponding angles of congruent triangles)
and $\overline{NB} \cong \overline{AM}$	.....(ii)	(corresponding sides of congruent triangles)
But $\overline{NB} \parallel \overline{AM}$		From (i), alternate $\angle$s
Thus $\overline{NB} \parallel \overline{MC}$	....(iii)	(M is a point of $\overline{AC}$)
$\overline{MC} \cong \overline{AM}$	....(iv)	Given
$\overline{NB} \cong \overline{MC}$	... (v)	{from (ii) and (iv)}
$\therefore$ BCMN is a parallelogram		From (iii) and (v)
$\therefore$ $\overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$		(Opposite sides of a parallelogram BCMN)
$\overline{BC} \cong \overline{NM}$	.....(vi)	(Opposite sides of parallelogram)
$m\overline{LM} = \frac{1}{2}\, m\overline{NM}$	.....(vii)	Construction
Hence $m\overline{LM} = \frac{1}{2}\, m\overline{BC}$		{from (vi) and (vii)}

Example

The line segments, joining the mid-points of the sides of a quadrilateral, taken in order, form a parallelogram.



Given

A quadrilateral ABCD, in which P is the mid-point of $\overline{AB}$, Q is the mid-point of $\overline{BC}$, R is the mid-point of $\overline{CD}$, S is the mid-point of $\overline{DA}$.

P is joined to Q, Q is joined to R. R is joined to S and S is joined to P.

To prove

PQRS is a parallelogram.

Construction

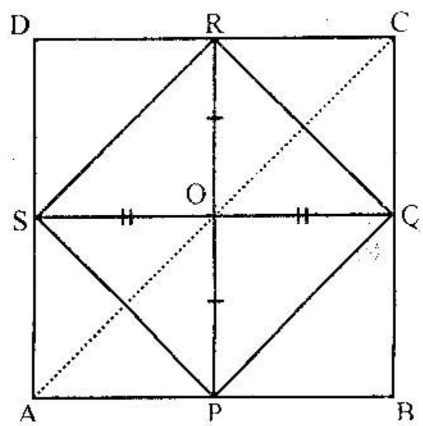
Join A to C.

Proof

Statements	Reasons
In ΔDAC,	
$\left.\begin{array}{l}\overline{SR} \parallel \overline{AC} \\ m\overline{SR} = \frac{1}{2} m\overline{AC}\end{array}\right\}$	S is the mid-point of $\overline{DA}$ R is the mid-point of $\overline{CD}$
In ΔBAC,	
$\left.\begin{array}{l}\overline{PQ} \parallel \overline{AC} \\ m\overline{PQ} = \frac{1}{2} m\overline{AC}\end{array}\right\}$	P is the mid-point of $\overline{AB}$ Q is the mid-point of $\overline{BC}$
$\overline{SR} \parallel \overline{PQ}$	Each $\parallel$ $\overline{AC}$
$m\overline{SR} = m\overline{PQ}$	Each $= \frac{1}{2} m\overline{AC}$
Thus PQRS is a parallelogram	$\overline{SR} \parallel \overline{PQ}, m\overline{SR} = m\overline{PQ}$ (proved)

EXERCISE 11.3

(1) Prove that the line-segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.



Given

ABCD is a quadrilateral.

P, Q, R, S are the mid-points of $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$ respectively.

P is joined to R, Q is joined to S. $\overline{SQ}, \overline{PR}$ intersect at point "O"

To Prove

$\overline{OP} \cong \overline{OR}, \overline{OS} \cong \overline{OQ}$

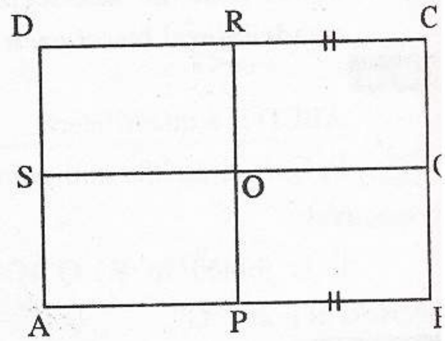
Construction Join P, Q, R, S in order, join A to C.

Proof

Statements	Reasons
$\overline{SR} \parallel \overline{AC}$ (i)	In ΔADC. S, R are mid-points Of $\overline{AD}, \overline{DC}$
$m\overline{SR} = \frac{1}{2} m\overline{AC}$ (ii)	

And $\overline{PQ} \parallel \overline{AC}$ (iii)	In ΔABC; P, Q are mid-points of $\overline{AB}, \overline{BC}$
$m\overline{PQ} = \frac{1}{2} m\overline{AC}$ (iv)	
∴ $\overline{PQ} \parallel \overline{SR}$ (v)	from (i), and (iii)
$m\overline{PQ} = m\overline{SR}$ (vi)	From (ii) and (iv)
Similarly $\overline{PS} \parallel \overline{QR}$	
$m\overline{PS} = m\overline{QR}$	
Hence PQRS is a parallelogram	
Now $\overline{PR}$, $\overline{SQ}$ are the diagonals Of PQRS that intersect at point O.	
∴ $\overline{OP} \cong \overline{OR}$	
∴ $\overline{OS} \cong \overline{OQ}$	Diagonals of a parallelogram Bisect each other.

(2) Prove that the line-segments joining the mid-points of the opposite sides of a rectangle are the right-bisectors of each other.



Given

ABCD is a rectangle.

and P, Q, R, S are the mid-points of sides $\overline{AB}, \overline{BC}$, $\overline{CD}$ and $\overline{DA}$, respectively.

P is joined to R, S to Q These intersect at "O"

To Prove

$\overline{OQ} \cong \overline{OS}, \overline{OR} \cong \overline{OP}$ and $\overline{RP} \perp \overline{SQ}$

Proof

Statements	Reasons
$\overline{AB} \parallel \overline{CD}$	opposite sides of rectangle
$\overline{AP} = \overline{DR}$ (i)	
$m\overline{AB} = m\overline{CD}$	
$\frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$	
$m\overline{AP} = m\overline{DR}$ (ii)	
∴ APRD is rectangle	

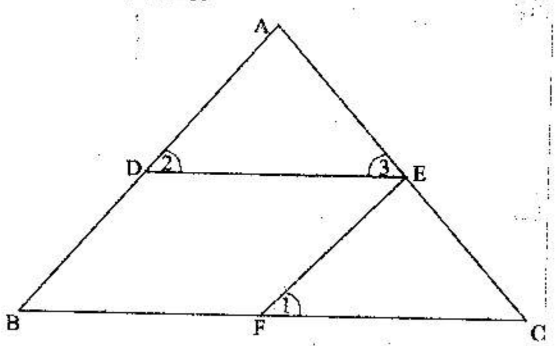
$\therefore \quad \overline{OR} \cong \overline{OP}$	As $m\angle A = m\angle D = 90^\circ$
Similarly $\quad \overline{OQ} \cong \overline{OS}$	
Now In rectangle APRD	
$m\overline{DA} = m\overline{RP}$	
$\frac{1}{2} m\overline{DA} = m\overline{RP}$	
$m\overline{DS} = m\overline{RO}$	
$\therefore \quad \overline{DS} \parallel \overline{RO}$,	
Hence SORD is rectangle.	
$\therefore \quad m\angle SOR = 90^\circ, \quad \overline{RP} \perp \overline{SQ}$.	

Note: Diagonals of a rectangle are congruent.]

9) Prove that the line-segment passing through the mid-point of one side and parallel to another side of a triangle also bisects the third side.

iven In ΔABC, D is mid-point of $\overline{AB}$, $\overline{DE} \parallel BC$ which meets $\overline{AC}$ at E.

equired E is mid-point of $\overline{AE}$ and $\overline{EA} \cong EC$



onstruction

Take $\overline{EF} \parallel \overline{AB}$ which meets $\overline{BC}$ at F.

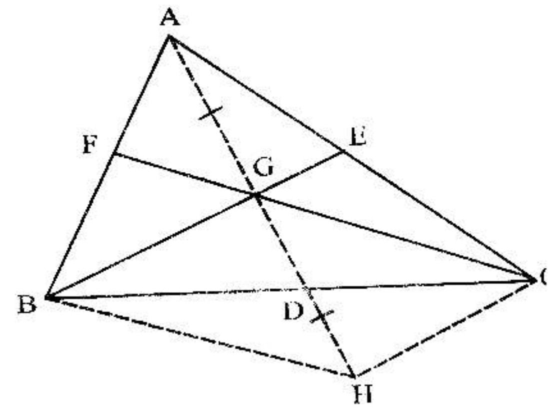
roof

Statements		Reasons
Now BDEF is parallelogram		$\overline{DE} \parallel \overline{BF}$ given, $\overline{EF} \parallel \overline{DB}$ const.
$\therefore \quad \overline{EF} \cong \overline{DB}$	(i)	Opposite sides of parallelogram
$\overline{EF} \cong \overline{AD}$	(ii)	Given
$\angle 1 \cong \angle B$		Corresponding angles.
$\angle 2 \cong \angle B$	(iii)	Corresponding angles.
$\therefore \quad \angle 1 \cong \angle 2$	(iv)	Form (iii)
Now In $\Delta ADE \leftrightarrow \Delta EFC$		
$\angle 1 \cong \angle 2$		Form (iv)
$\angle 3 \cong \angle C$		Corresponding angles.
$\overline{AD} \cong \overline{EF}$		Form (ii)
Hence $\Delta ADE \cong \Delta EFC$		A.A.S $\cong$ A.A.S

$\therefore \quad \overline{AE} \cong \overline{CE}$	Corresponding sides of congruent triangles.

Theorem

The medians of a triangle are concurrent and their point of concurrency is the point of trisection of each median.



Given

ΔABC

To Prove

The medians of the ΔABC are concurrent and the point of concurrency is the point of trisection of each median.

Construction

Draw two medians $\overline{BE}$ and $\overline{CF}$ of the ΔABC which intersect each other at point G. Join A to G and produce it to point H such that $\overline{AG} \cong \overline{GH}$. Join H to the points B and C.

$\overline{AH}$ Intersects $\overline{BC}$ at the point D.

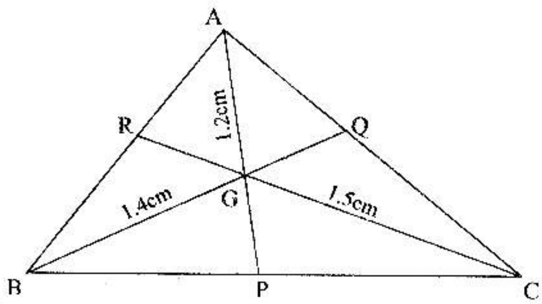
Proof

Statements	Reasons
In ΔACH, $\overline{GE} \parallel \overline{HC}$,	G and E are mid-points of sides $\overline{AH}$ and $\overline{AC}$ respectively
or $\overline{BE} \parallel \overline{HC}$ .....(i)	G is a point of $\overline{BE}$
Similarly $\overline{CF} \parallel \overline{HB}$ .....(ii)	
$\therefore$ BHCG is a parallelogram	from (i) and (ii)
and $m\overline{GD} = \frac{1}{2} m\overline{GH}$ .....(iii) $\overline{BD} \cong \overline{CD}$	(Diagonals $\overline{BC}$ and $\overline{GH}$ of a parallelogram BHCG intersect each other at point D).
$\overline{AD}$ is a median of ΔABC	
Medians $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ pass through the point G	(G is the intersecting point of $\overline{BE}$ and $\overline{CF}$ and $\overline{AD}$ pass through it.)
Now $\overline{GH} \cong \overline{AG}$ .....(iv)	Construction

$\therefore \quad m\overline{GD} = \frac{1}{2} m\overline{AG}$ and G is the point of trisection of $\overline{AD}$ –(v) similarly it can be proved that G is also the point of trisection of $\overline{CF}$ and $\overline{BE}$.	from (iii) and (iv)

EXERCISE 11.4

(1) The distances of the point of concurrency of the medians of a triangle from its vertices are respectively 1.2cm; 1.4 cm and 1.5 cm. Find the lengths of its medians.



Solution: Let ABC be a triangle with center of gravity at G where $m\overline{AG} = 1.2\text{cm}$, $\overline{BG} = 1.4\text{cm}$, $m\overline{CG} = 1.5\text{cm}$

Required To find the length of AP, BQ, CR

Proof:

$$m\overline{AP} = \frac{3}{2} \times (mAG)$$

$$= \frac{3}{2} \times 1.2 = 1.8\,\text{cm}$$

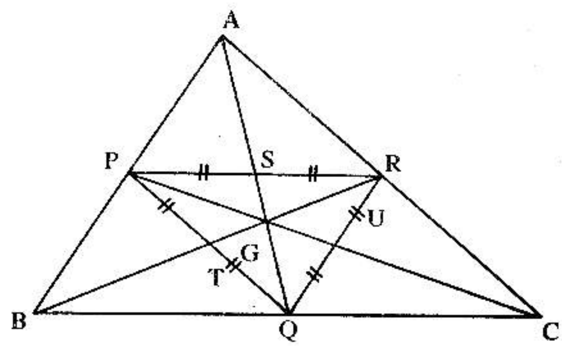
$$m\overline{BQ} = \frac{3}{2} \times (m\overline{BG})$$

$$= \frac{3}{2} \times 1.4 = 2.1\,\text{cm}$$

$$m\overline{CR} = \frac{3}{2} \times (mCG)$$

$$= \frac{3}{2} \times 1.5 = 2.25\,\text{cm}$$

(2) Prove that the point of concurrency of the medians of a triangle and the triangle which is made by joining the mid-points of its sides is the same.



Given

In ΔABC, $\overline{AQ}, \overline{BR}, \overline{CP}$ are its medians that are concurrent at point G. ΔPQR is formed by joining mid-points of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$

To Prove

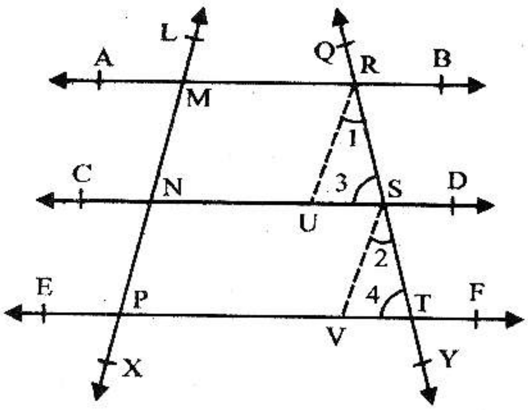
Point G is point of concurrency of triangle PQR.

Proof

Statements	Reasons
$\overline{PR} \parallel \overline{BC}$	P, R are mid-points of $\overline{AB}$ and $\overline{AC}$
$\Rightarrow$ $\overline{PR} \parallel \overline{BQ}$ (i)	
$\overline{RQ} \parallel \overline{AB}$	P, Q are mid-points of $\overline{AB}$ and $\overline{BC}$
$\Rightarrow$ $\overline{RQ} \parallel \overline{PB}$ (ii)	
$\therefore$ PBQR is a parallelogram.	
$\overline{BR}$, $\overline{PQ}$ are its diagonals, that bisect each other at T.	
T is mid-point $\overline{PQ}$, similarly	
S is mid-point of $\overline{PR}$ and U is mid-point of $\overline{PQ}$.	

Theorem

If three or more parallel lines make congruent segments on a transversal, they also intercept congruent segments on any other line that cuts them.



Given

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

The transversal $\overleftrightarrow{LX}$ intersects $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ at the points M, N and P respectively, such that $\overline{MN} \cong \overline{NP}$. The transversal $\overleftrightarrow{QY}$ intersects them at points R, S and T respectively.

To Prove

$\overline{RS} \cong \overline{ST}$

Construction

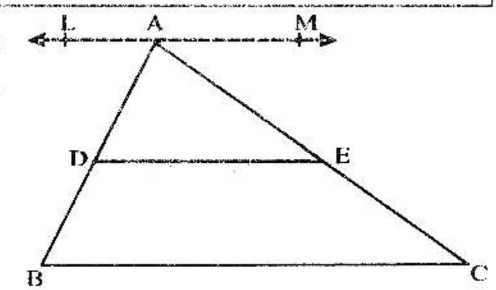
From R, draw $\overline{RU} \parallel \overline{LX}$, which meets $\overline{CD}$ at U. From S, draw $\overline{SV} \parallel \overline{LX}$ which meets $\overline{EF}$ at V. as shown in the figure let the angles be labeled as

$\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$

Proof

Statements	Reasons
MNUR is a parallelogram	$\overline{RU} \parallel \overline{LX}$ (construction) $\overleftrightarrow{AB} \parallel \overline{CD}$ (given)
$\therefore$ $\overline{MN} \cong \overline{RU}$ .....(i)	(opposite sides of a parallelogram)

Similarly,	
$\overline{NP} \cong \overline{SV}$ .....(ii)	
But $\overline{MN} \cong \overline{NP}$ .....(iii)	Given
$\therefore$ $\overline{RU} \cong \overline{SV}$	{from (i), (ii) and (iii)}
Also $\overline{RU} \parallel \overline{SV}$	Each is $\parallel \overleftrightarrow{LX}$ (construction)
$\therefore$ $\angle 1 \cong \angle 2$	Corresponding angles
and $\angle 3 \cong \angle 4$	Corresponding angles
In $\Delta RUS \leftrightarrow \Delta SVT$,	
$\overline{RU} \cong \overline{SV}$	Proved
$\angle 1 \cong \angle 2$	Proved
$\angle 3 \cong \angle 4$	Proved
$\therefore$ $\Delta RUS \cong \Delta SVT$	S.A.A. $\cong$ S.A.A.
Hence $\overline{RS} \cong \overline{ST}$	(corresponding sides of a congruent triangles)



Corollaries (i) A line, through the mid-point of one side, parallel to another side of a triangle, bisects the third side.

Given In ΔABC, D is the mid-point of $\overline{AB}$. $\overline{DE} \parallel \overline{BC}$ which cuts $\overline{AC}$ at E.

To prove

$\overline{AE} \cong \overline{EC}$

Construction

Through A, draw $\overleftrightarrow{LM} \parallel \overline{BC}$.

Proof

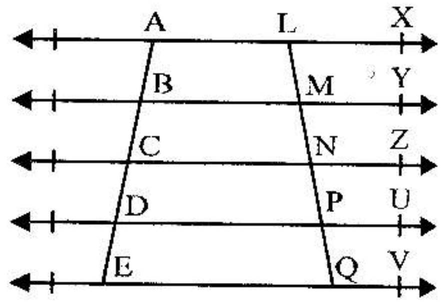
Statements	Reasons
Intercepts cut by $\overleftrightarrow{LM}$, $\overleftrightarrow{DE}$, $\overleftrightarrow{BC}$ on $\overleftrightarrow{AC}$ are congruent. i.e., $\overline{AC} \cong \overline{EC}$	{Intercepts cut by parallels $\overleftrightarrow{LM}$, $\overline{DE}$, $\overline{BC}$ on AB are congruent (given)

(ii) The parallel line from the mid-point of one non-parallel side of a trapezium to the parallel sides bisects the other non-parallel side.

(iii) If one side of a triangle is divided into congruent segments, the line drawn from the point of division parallel to the other side will make congruent segments on third side.

Exercise 11.5

1. In the given figure. $\overleftrightarrow{AX} \parallel \overleftrightarrow{BY} \parallel \overleftrightarrow{CZ} \parallel \overleftrightarrow{DU} \parallel \overleftrightarrow{EV}$ and $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE}$ if $m\overline{MN} = 1cm$ then find the length of $\overline{LN}$ and $\overline{LQ}$



Given

In given figure $\overleftrightarrow{AX} \parallel \overleftrightarrow{BY} \parallel \overleftrightarrow{CZ} \parallel \overleftrightarrow{DU} \parallel \overleftrightarrow{EV}$,

$\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE}$, $m\overline{MN} = 1cm$

Required:

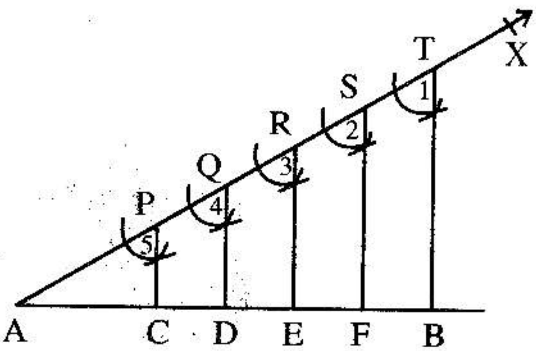
To find $m\overline{LN}$ and $m\overline{LQ}$

Statement	Reasons
$\overleftrightarrow{AX} \parallel \overleftrightarrow{BY} \parallel \overleftrightarrow{CZ} \parallel \overleftrightarrow{DU} \parallel \overleftrightarrow{EV}$	Given
$\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE}$	Given
$\overline{BC} \cong \overline{MN}$ $\overline{NP} \cong \overline{PQ}$	$\because \parallel$ lines through A, B, C, D, E cut $\overline{LQ}$ in points L, M, N, P, Q.
$m\overline{MN} = 1cm$	Given
$\overline{LN} = 2\overline{MN}$ $= 2(1)$ $= 2cm$	$\because \overline{MN} = 1cm$
$\overline{LQ} = 4\overline{MN}$ $= 4 \times 1$ $= 4cm$	

2. Take a line segment of length 5cm and divide it into five congruent parts.

[Hint: Draw an acute angle $\angle BAX$. On $\overrightarrow{AX}$ take $\overline{AP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{ST}$.

Joint T to B. Draw line parallel to $\overline{TB}$ from the points P, Q, R and S.]



Construction:

(i) Take a line segment AB of 5cm long.

(ii) Draw an acute angle $\angle BAX$.

(iii) Mark 5 points on $\overrightarrow{AX}$ at equal distance starting from point A.

(iv) Join the last point (mark)T to B.

(v) Draw $\overline{SF}, \overline{RE}, \overline{QD}, \overline{PC}$ parallel to TB these line segments meet AB at F,E,D,C points.

Result: AB has been divided into five equal points

$\overline{AC} \cong \overline{CD} \cong \overline{DE} \cong \overline{FB}$

3. Fill in the blanks.

(i) In a parallelogram opposite sides are….. (Parallel / Congruent)

(ii) In a parallelogram opposite angles are ……. (Equal / Congruent)

(iii) Diagonals of a parallelogram …….. each other at a point. (Intersect)

(iv) Medians of a triangle are ………… (Concurrent)

(v) Diagonal of a parallelogram divides the parallelogram into two ……….. triangles. (Congruent)

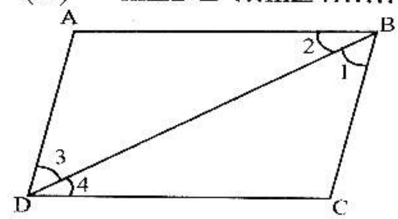
4. In parallelogram ABCD

(i) $m\overline{AB}$ …≅….. $m\overline{DC}$

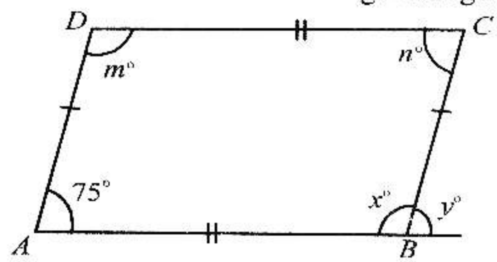
(ii) $m\overline{BC}$…≅…$m\overline{AD}$

(iii) $m\angle 1 \cong$ …$m\angle 3$……

(iv) $m\angle 2 \cong$ …$m\angle 4$……



5. Find the unknowns in the given figure.



Given: Let ABCD be the given figure with

$\overline{AB} \cong \overline{CD}$

$\overline{BC} \cong \overline{AD}$

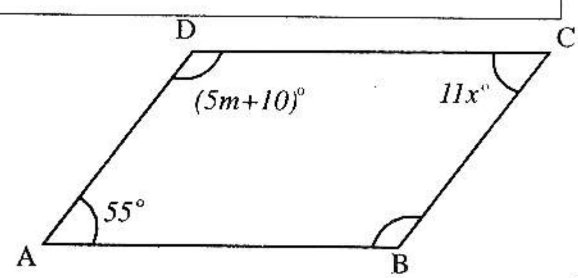
To Find: m°, n°, x°, y°

Proof:

Statement	Reasons
ABCD is a Parallelogram	$\overline{AB} \cong \overline{CD}$ $\overline{AD} \cong \overline{BC}$
$\angle n = 75^\circ$	Opposite interior angles
$m^\circ + 75^\circ = 180^\circ$	supplementary angles
$m^\circ = 180^\circ - 75^\circ = 105^\circ$	
$x^\circ = m^\circ$	
$x^\circ = 105^\circ$	
$x^\circ + y^\circ = 180^\circ$	supplementary angles
$y^\circ = 180^\circ - x^\circ$	
$y^\circ = 180^\circ - 105^\circ$	
$y^\circ = 75^\circ$	

6. If the given figure ABCD is a parallelogram, then find *x*, *m*.

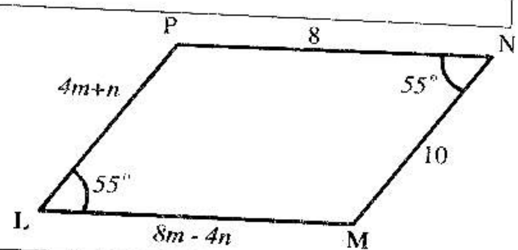
Given: ABCD is a parallelogram with angles as shown To Find x° and m°



Proof:

Statement	Reasons
$11x^\circ = 55^\circ$	Opposite angles of parallelogram
$x^\circ = \frac{55^\circ}{11} = 5^\circ$	
$x^\circ = 5^\circ$	
$(5m + 10)^\circ + 55^\circ = 180^\circ$	Int. supplementary angles
$(5m + 10)^\circ = 180^\circ - 55^\circ$	
$5m^\circ + 10^\circ = 125^\circ$	
$5m^\circ = 125^\circ - 10^\circ$	
$5m^\circ = 115^\circ$	
$m^\circ = 23^\circ$	

7. The given figure LMNP is a parallelogram. Find the value of *m, n.*



Given: The parallelogram LMNP with lengths and angles as shown to find: m° and n°

Proof:

Statement	**Reasons**
$4m + n = 10$.......(i)	Opposite sides of ||gm
$8m - 4n = 8$ ....(ii)	Opposite side of ||gm
Multiplying (i) by 4	
$16m + 4n = 40$ (iii)	
Adding (i) and (iii)	

$$8m \cancel{-4n} = 8$$
$$16m \cancel{+4n} = 40$$
$$24m = 48$$

$$m = \frac{48}{24} = 2$$

Put in (i)

$4(2) + n = 10$

$8 + n = 10$

$n = 10 - 8 \Rightarrow n = 2$

8. In the question 7, sum of the opposite angles of the parallelogram is 110°, find the remaining angles.

Given: LMNP is a parallelogram with angles 55°, 55° as shown

To Find: All angles

Proof:

Statement	Reasons
$\angle LPN + 55^\circ = 180^\circ$	Interior angles
$\angle LPN = 125^\circ$	
Also	
$\angle m = \angle P$	Opposite angles
$\angle m = 125^\circ$	$\because \angle P = 125^\circ$