

## THEOREMS RELATED WITH AREA

### Area of a Figure

The region enclosed by the bounding lines of a closed figure is called the area of the figure.

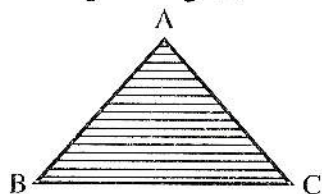
The area of a closed region is expressed in square units (say, sq. m or  $m^2$ ) i.e., a positive real number.

### Triangular Region

The interior of a triangle is the part of the plane enclosed by the triangle.

A triangular region is the union of a triangle and its interior i.e., the three line segments forming the triangle and its interior.

By area of a triangle, we mean the area of its triangular region.



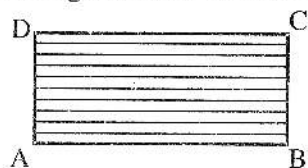
### Congruent Area Axiom

If  $\triangle ABC \cong \triangle PQR$ , then area of (region  $\triangle ABC$ ) = area of (region  $\triangle PQR$ )

### Define Rectangular Region

The interior of a rectangle is the part of the plane enclosed by the rectangle.

A rectangular region is the union of a rectangle and its interior.



A rectangular region can be divided into two or more than two triangular regions in many ways.

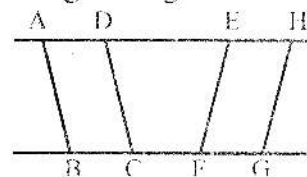
### Note

If the length and width of a rectangle are  $a$  units and  $b$  units respectively, then the area of the rectangle is equal to  $a \times b$  square units.

If  $a$  is the side of a square, its area =  $a^2$ , square units.

### Between the same Parallels

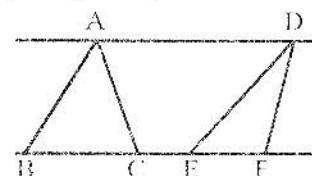
(i) Two parallelograms are said to be between the same parallels, when their bases are in the same straight line and their sides opposite to these bases are also in a straight line; as the parallelograms ABCD, EFGH in the given figure.



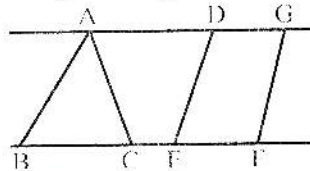
(ii) Two triangles are said to be between the same parallels,

when their bases are in the same straight line and the line joining their vertices is parallel to their bases; as the  $\triangle ABC$ ,  $\triangle DEF$  in the given figure.

(iii) A triangle and a parallelogram are said to be between the same parallels,



when their bases are in the same straight line, and the side of the parallelogram opposite the base, produced if necessary, passes through the vertex of the triangle as are the  $\triangle ABC$  and the parallelogram  $DEFG$  in the given figure.



### Altitude of Parallelogram

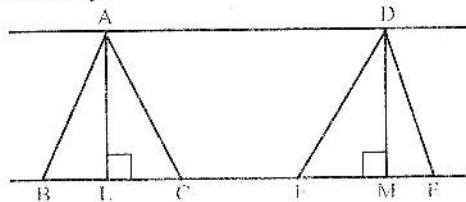
If one side of a parallelogram is taken as its base, the perpendicular distance between that side and the side parallel to it, is called the Altitude or Height of the parallelogram.

### Altitude of the triangle

If one side of a triangle is taken as its base, the perpendicular to that side, from the opposite vertex is called the Altitude or Height of the triangle.

### Example

"Triangles or parallelograms having the same or equal altitudes can be placed between the same parallels and conversely."



Place the triangles  $ABC$ ,  $DEF$  so that their bases  $BC$ ,  $EF$  are in the same

### Proof

Statements	Reasons
Area of (parallelogram $ABCD$ ) = Area of (quad. $ABED$ ) + area of ( $\triangle CBE$ )..(i)	[Area addition axiom]

straight line and the vertices on the same side of it and suppose  $\overline{AL}$ ,  $\overline{DM}$  are the equal altitudes. We have to show that  $\overline{AD}$  is parallel to  $BCEF$ .

### Proof

$\overline{AL}$  and  $\overline{DM}$  are parallel, for they are both perpendicular to  $\overline{BF}$ . Also  $m\overline{AL} = m\overline{DM}$ . (given)

$\therefore \overline{AD}$  is parallel to  $\overline{LM}$ . A similar proof may be given in the case of parallelograms.

### Note:

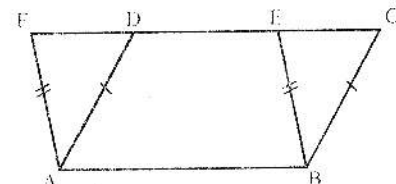
A diagonal of a parallelogram divides it into two congruent triangles (SSS) and hence of equal area.

### Theorem

Parallelograms on the same base and between the same parallel lines (or of the same altitude) are equal in area.

### Given

Two parallelograms  $ABCD$  and  $ABEF$  having the same base  $\overline{AB}$  and  $\overline{DE}$  between the same parallel lines  $\overline{AB}$  and  $\overline{DE}$ .



### To Prove

Area of parallelogram  $ABCD$  = area of parallelogram  $ABEF$

<p>Area of (parallelogram ABEF)          = area of (quad. ABED) + area of (<math>\triangle DAF</math>)...(ii)          In <math>\triangle</math>s CBE and DAF  <math>\overline{mCB} = \overline{mDA}</math>  <math>\overline{mBE} = \overline{mAF}</math>  <math>\angle CBE = \angle DAF</math>  <math>\therefore \triangle CBE \cong \triangle DAF</math>  <math>\therefore</math> area of (<math>\triangle CBE</math>) = area of (<math>\triangle DAF</math>).....(iii)          Hence area of (parallelogram ABCD) = area          of (parallelogram ABEF)</p>	<p>[Area addition axiom]           [opposite sides of a parallelogram]          [opposite sides of a parallelogram]  <math>[\because \overline{BC} \parallel \overline{AD}, \overline{BE} \parallel \overline{AF}]</math>          [S.A.S. cong. Axiom]          [cong. Area axiom]          From (i), (ii) and (iii)</p>
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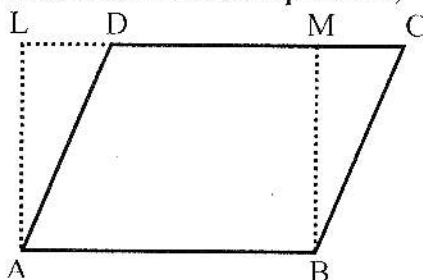
### Example

- (i) The area of a parallelogram is equal to that of a rectangle on the same base and having the same altitude.  
 (ii) Hence area of parallelogram = base  $\times$  altitude

### Proof

Let ABCD be a parallelogram.  $\overline{AL}$  is an altitude corresponding to side  $\overline{AB}$ .

- (i) Since parallelogram ABCD and rectangle ALMB are on the same base  $\overline{AB}$  and between the same parallels,



### To Prove

Area of (parallelogram ABCD) = area of (parallelogram EFGH)

### Construction

Place the parallelograms ABCD and EFGH so that their equal bases  $\overline{BC}$ ,  $\overline{FG}$  are in the straight line BCFG. Join  $\overline{BE}$  and  $\overline{CH}$ .

$\therefore$  by above theorem it follows that  
 area of (parallelogram ABCD) = area of (rect. ALMB)

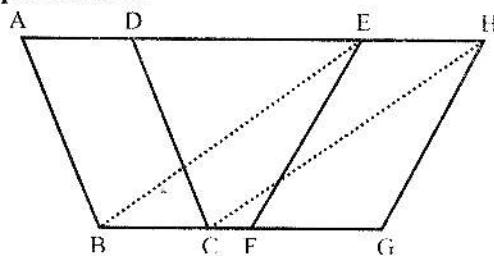
(ii) But area of (rect. ALMB) =  $\overline{AB} \times \overline{AL}$

Hence

Area of (parallelogram ABCD) =  $\overline{AB} \times \overline{AL}$

### Theorem

Parallelograms on equal bases and having the same (or equal) altitude are equal in area.



### Given

Parallelograms ABCD, EFGH are on the equal bases  $\overline{BC}$ ,  $\overline{FG}$ , having equal altitudes.

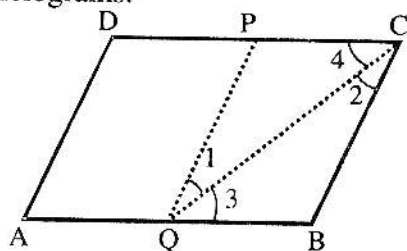


### Proof

Statements	Reasons
The given $\parallel^{\text{gms}}$ ABCD and EFGH are between the same parallels	Their altitudes are equal (given)
Hence ADEH is a straight line $\parallel$ to $\overline{BC}$	Given
$\therefore m\overline{BC} = m\overline{FG}$	EFGH is a parallelogram
$= m\overline{EH}$	
Now $m\overline{BC} = m\overline{EH}$ and they are $\parallel$	
$\therefore \overline{BE}$ and $\overline{CH}$ are both equal and $\parallel$	A quadrilateral with two opposite sides congruent and parallel is a parallelogram
Hence EBCH is a parallelogram	Being on the same base $\overline{BC}$ and between the same parallels
Now $\parallel^{\text{gm}}$ ABCD = $\parallel^{\text{gm}}$ EBCH ... (i)	Being on the same base $\overline{EH}$ and between the same parallels
But $\parallel^{\text{gm}}$ EBCH = $\parallel^{\text{gm}}$ EFGH ... (ii)	From (i) and (ii)
Hence area ( $\parallel^{\text{gm}}$ ABCD) = area ( $\parallel^{\text{gm}}$ EFGH)	

### Exercise 16.1

(1) Show that the line segment joining the mid-points of opposite sides of a parallelogram, divides it into two equal parallelograms.



**Given** ABCD is parallelogram. point p is midpoint of side  $\overline{DC}$  i.e.  $\overline{DP} \cong \overline{PC}$  and point Q is midpoint of side  $\overline{AB}$  i.e.  $\overline{AQ} \cong \overline{QB}$ .

#### To Prove

Parallelogram AQPQ  $\cong$  parallelogram PBCP

#### Construction

Join P to Q and Q to C.

### Proof

Statements	Reasons
$m\overline{AB} = m\overline{DC}$	
$\frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{DC}$	Dividing by 2
$m\overline{QB} = m\overline{PC}$	

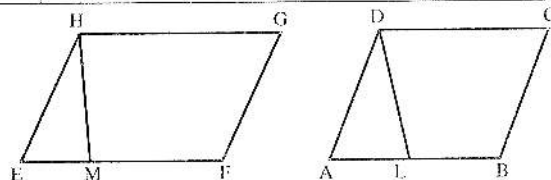
Statements	Reasons
$\therefore$ Area of parallelogram ABCD = $10 \times 7$ .....(i) Also area of the llgm ABCD = $\overline{AD} \times 8$ .....(ii)	
$\therefore m \overline{AD} \times 8 = 10 \times 7$	

$m\overline{AD} = \frac{10 \times 7}{8}$ $m\overline{AD} = \frac{35}{4} = 8\frac{3}{4} \text{ cm}$	
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(3) If two parallelograms of equal areas have the same or equal bases, their altitudes are equal.

**Given** Two parallelograms of same or equal bases and same areas.

**To Prove** Their altitudes are equal.



**Construction** Make the  $\parallel\text{gm}$  ABCD and EFGH. Draw  $\overline{DL} \perp \overline{AB}$  and  $\overline{HM} \perp \overline{EF}$

**Proof**

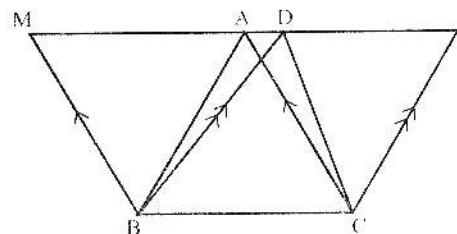
Statements	Reasons
Area of the $\parallel\text{gm}$ ABCD = area of the $\parallel\text{gm}$ EFGH base x altitude = base x altitude $m\overline{AB} \times m\overline{DL} = m\overline{EF} \times m\overline{HM}$	Area = base x altitude
But $m\overline{AB} = m\overline{EF}$	
$\therefore m\overline{EF} \times m\overline{DL} = m\overline{EF} \times m\overline{HM}$ $m\overline{DL} = m\overline{HM}$ so altitudes are equal	Dividing by $m\overline{EF}$ we get

**Theorem** Triangles on the same base and of the same (i.e., equal) altitudes are equal in area.

**Given**  $\Delta$ s ABC, DBC on the same base  $\overline{BC}$  and having equal altitudes.

**To Prove** Area of  $(\Delta ABC) = \text{area of } (\Delta DBC)$

**Construction** Draw  $\overline{BM} \parallel \text{to } \overline{CA}$ ,  $\overline{CN} \parallel \text{to } \overline{BD}$  meeting  $\overline{AD}$  produced in M, N.



**Proof**

Statements	Reasons
$\Delta ABC$ and $\Delta DBC$ are between the same $\parallel^s$ Hence $\overline{MADN}$ is parallel to $\overline{BC}$ $\therefore \text{Area}(\parallel^{\text{gm}} \text{BCAM}) = \text{Area}(\parallel^{\text{gm}} \text{BCND}) \dots (i)$	Their altitudes are equal  These $\parallel^{\text{gm}}$ s are on the same base $\overline{BC}$ and between the same $\parallel^s$
But $\Delta ABC = \frac{1}{2}(\parallel^{\text{gm}} \text{BCAM}) \dots (ii)$	Each diagonal of a $\parallel^{\text{gm}}$ bisects it into two congruent triangles

and $\Delta BDC = \frac{1}{2} (\text{ll}^{\text{gm}} \text{BCND}) \dots\dots(\text{iii})$	
Hence area ( $\Delta \text{ABC}$ ) = Area ( $\Delta \text{DBC}$ )	From (i), (ii) and (iii)

**Theorem** Triangles on equal bases and of equal altitudes are equal in area.

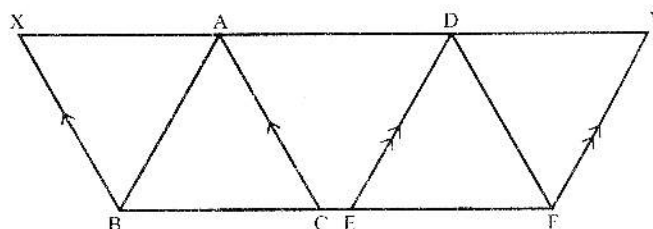
**Given**

$\Delta \text{s ABC, DEF}$  on equal bases

$\overline{\text{BC}}, \overline{\text{EF}}$  and having altitudes equal.

**To Prove**

Area ( $\Delta \text{ABC}$ ) = Area ( $\Delta \text{DEF}$ )



**Construction**

Place the  $\Delta \text{s ABC}$  and  $\text{DEF}$  so that their equal bases  $\overline{\text{BC}}$  and  $\overline{\text{EF}}$  are in the same straight line  $\text{BCEF}$  and their vertices on the same side of it. Draw  $\text{BX} \parallel$  to  $\text{CA}$  and  $\text{FY} \parallel$  to  $\text{ED}$  meeting  $\text{AD}$  produced in  $\text{X, Y}$  respectively

**Proof**

Statements	Reasons
$\Delta \text{ABC}$ and $\Delta \text{DEF}$ are between the same parallels	Their altitudes are equal (given)
$\therefore \text{XADY}$ is $\parallel$ to $\text{BCEF}$	
$\therefore \text{Area} (\text{ll}^{\text{gm}} \text{BCAX}) = \text{Area} (\text{ll}^{\text{gm}} \text{EFYD}) \dots\dots(\text{i})$	These $\text{ll}^{\text{gm}} \text{s}$ are on equal bases and between the same parallels
But $\Delta \text{ABC} = \frac{1}{2} (\text{ll}^{\text{gm}} \text{BCAX}) \dots\dots(\text{ii})$	Diagonal of a $\text{ll}^{\text{gm}}$ bisects it
and $\Delta \text{DEF} = \frac{1}{2} (\text{ll}^{\text{gm}} \text{EFYD}) \dots\dots(\text{iii})$	
$\therefore \text{area} (\Delta \text{ABC}) = \text{area} (\Delta \text{DEF})$	From (i), (ii) and (iii)

**Corollaries**

- (1) Triangles on equal bases and between the same parallels are equal in area.
- (2) Triangles having a common vertex and equal bases in the same straight line, are equal in area.



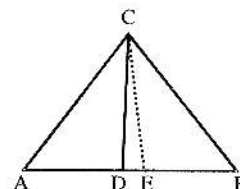
## Exercise 16.2

- (1) Show that a median of a triangle divides it into two triangles of equal area.

**Given** Median of the triangle

**To Prove:** Median divides the triangle into two triangles of equal area.

**Proof** Make  $\triangle ABC$ , with  $\overline{CD}$  as median and  $\overline{CE}$  as altitude



Statements	Reasons
$m\overline{AD} = m\overline{DB}$ .....(i)	D is midpoint of $m\overline{AB}$
Area of the $\triangle ACD = \frac{1}{2} \cdot m\overline{AD} \cdot m\overline{CE}$ ... (ii)	
Area of the $\triangle BCD = \frac{1}{2} \cdot m\overline{BD} \cdot m\overline{CE}$	
$= \frac{1}{2} \cdot m\overline{AD} \cdot m\overline{CE}$ ... (iii)	By (i)
$\triangle ACD = \triangle BCD$	By (ii) and (iii)

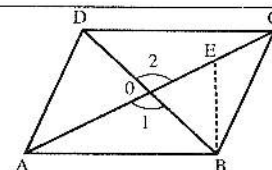
- (2) Prove that a parallelogram is divided by its diagonals into four triangles of equal area.

**Given**

llgm divided by its diagonals into four triangles

**To Prove**

Areas of the four triangles are equal



**Construction** Make the llgm ABCD with diagonals  $m\overline{AC}$ ,  $m\overline{BD}$  intersecting each other at O. Draw  $\overline{BE} \perp \overline{AC}$ .

**Proof**

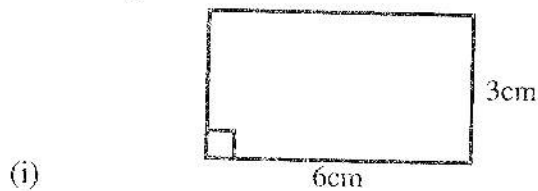
Statements	Reasons
Area of $\triangle OBC = \frac{1}{2} m\overline{OA} \cdot m\overline{BE}$	
$= \frac{1}{2} m\overline{OC} \cdot m\overline{BE}$ .....(i)	
The diagonals of the llgm bisect each other	
$\therefore m\overline{OA} \cong m\overline{OC}$	
In $\triangle OAB \leftrightarrow \triangle OCD$	
$m\overline{OB} \cong m\overline{OD}$	
$m\overline{OA} \cong m\overline{OC}$	
$\angle 1 \cong \angle 2$	opposite angles
$\triangle OAB \cong \triangle OCD$ ..... (ii)	
$\triangle OAD \cong \triangle OBC$ ..... (iii)	
$\therefore \text{Area } \triangle OAB = \text{Area } \triangle OBC = \text{Area } \triangle OCD = \text{Area } \triangle ODA$	By (i), (ii), (iii)



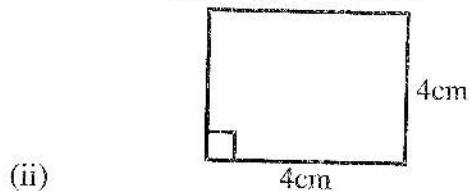
(3) Which of the following are true and which are false?

- |   |       |
|---|-------|
| (i) Area of a figure means region enclosed by bounding lines of closed figure.          | TRUE  |
| (ii) Similar figures have same area.  | FALSE |
| (iii) Congruent figures have same area.   | TRUE  |
| (iv) A diagonal of a parallelogram divides it into two non-congruent triangles.         | FALSE |
| (v) Altitude of a triangle means perpendicular from vertex to the opposite side (base). | TRUE  |
| (vi) Area of a parallelogram is equal to the product of base and height.                | TRUE  |

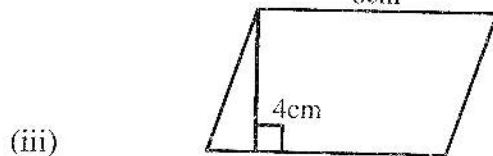
Q.4 Find the area of the following.



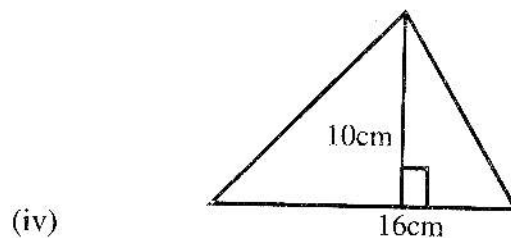
$$\text{Area} = 6 \times 3 = 18\text{cm}^2$$



$$\text{Area} = 4 \times 4 = 16\text{cm}^2$$



$$\text{Area} = 8 \times 4 = 32\text{cm}^2$$



$$\text{Area} = \frac{1}{2} \times 16 \times 10 = 80\text{cm}^2$$

## OBJECTIVE

- |   |   |
|---|---|
| <p>1. The region enclosed by the bounding lines of a closed figure is called the ___ of the figure:</p> <p>(a) Area (b) Circle</p> <p>(c) Boundary (d) None</p> <p>2. Base x altitude =</p> <p>(a) Area of parallelogram</p> <p>(b) Area of square</p> <p>(c) Area of Rectangular</p> <p>(d) None</p> <p>3. The union of a rectangular and its interior is called:</p> <p>(a) Circle region</p> <p>(b) Rectangular region</p> <p>(c) Triangle region</p> <p>(d) None</p> <p>4. If a is the side of a square, its area =</p> | <p>(a) a square unit</p> <p>(b) <math>a^2</math> square units</p> <p>(c) <math>a^3</math> square units</p> <p>(d) <math>a^4</math> square units</p> <p>5. The union of a triangle and its interior is called as:</p> <p>(a) Triangular region</p> <p>(b) Rectangular region</p> <p>(c) Circle region</p> <p>(d) None of these</p> <p>6. Altitude of a triangle means perpendicular distance to base from its opposite:___</p> <p>(a) Vertex (b) Side</p> <p>(c) Midpoint (d) None</p> |
|---|---|

## ANSWER KEY

1.	a	2.	a	3.	b	4.	b	5.	a	6.	a
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