

MISCELLANEOUS EXERCISE - 2

Q.1 Multiple Choice Questions.

Four possible answers are given for the following questions. Tick (✓) the correct answer.

1. $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to

- (a) $\frac{1}{\alpha}$ (b) $\frac{1}{\alpha} - \frac{1}{\beta}$
(c) $\frac{\alpha - \beta}{\alpha\beta}$ (d) $\frac{\alpha + \beta}{\alpha\beta}$

2. Product of cube roots of unity is

- (a) 0 (b) 1
(c) -1 (d) 3

3. Roots of the equation $4x^2 - 5x + 2 = 0$ are

- (a) irrational (b) imaginary
(c) rational (d) none of these

4. Two square roots of unity are

- (a) 1, -1 (b) 1, ω
(c) 1, $-\omega$ (d) ω, ω^2

5. Roots of the equation $4x^2 - 4x + 1 = 0$ are

- (a) real, equal (b) real, unequal
(c) imaginary (d) irrational

6. If α, β are the roots of $7x^2 - x + 4 = 0$ then $\alpha\beta$ is.....

- (a) $-\frac{1}{7}$ (b) $\frac{4}{7}$
(c) $\frac{7}{4}$ (d) $-\frac{4}{7}$

7. If $b^2 - 4ac < 0$ then the roots of $ax^2 + bx + c = 0$ are

- (a) irrational (b) rational
(c) imaginary (d) None of these

8. If α, β are the roots of $px^2 + qx + r = 0$, then sum of the roots 2α and 2β is

- (a) $-\frac{q}{p}$ (b) $\frac{r}{p}$
(c) $-\frac{2q}{p}$ (d) $-\frac{q}{2p}$

9. If α, β are the roots of $3x^2 + 5x - 2 = 0$ then $\alpha + \beta$ is.....

- (a) $\frac{5}{3}$ (b) $\frac{3}{5}$
(c) $-\frac{5}{3}$ (d) $-\frac{2}{3}$

10. If $b^2 - 4ac > 0$ and is a perfect square, then roots of $ax^2 + bx + c = 0$ are

- (a) irrational, equal
(b) rational, equal
(c) rational, unequal
(d) irrational, unequal

11. Cube roots of -1 are

- (a) -1, $-\omega, -\omega^2$ (b) -1, $\omega, -\omega^2$
(c) -1, $-\omega, \omega^2$ (d) 1, $-\omega, -\omega^2$

12. Sum of the cube roots of unity is

- (a) 0 (b) 1
(c) -1 (d) 3

13. If $b^2 - 4ac > 0$, but not a perfect square then roots of $ax^2 + bx + c = 0$ are

- (a) imaginary (b) rational
(c) irrational (d) None of these

14. If α, β are the roots of $x^2 - x - 1 = 0$, then product of the roots 2α and 2β is

- (a) -2 (b) 2
(c) 4 (d) -4

15. Disc. of $x^2 - 3x + 3 = 0$ is

- (a) 6 (b) 12
(c) 21 (d) -3

16. The discriminant of $ax^2 + bx + c = 0$ is

- (a) $b^2 - 4ac$ (b) $b^2 + 4ac$
(c) $-b^2 + 4ac$ (d) $-b^2 - 4ac$

17. $\alpha^2 + \beta^2$ is equal to

- (a) $\alpha^2 - \beta^2$ (b) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$
(c) $(\alpha + \beta)^2 - 2\alpha\beta$ (d) $\alpha + \beta$

18. If roots of a quadratic equation are irrational and distinct, then Disc. is

- (a) perfect square
(b) not perfect square
(c) zero
(d) negative

19. Disc. of $2x^2 - 7x + 1 = 0$ is

- (a) 47 (b) 41
(c) 40 (d) 51

20. The nature of the roots of equation $ax^2 + bx + c = 0$ is determined by.

- (a) Sum of the roots
(b) Product of the roots
(c) Synthetic division
(d) Discriminant

21. If for a quadratic equation

$b^2 - 4ac = 205$, then roots are...

- (a) complex (b) irrational
(c) rational (d) equal

22. If roots of a quadratic equation are equal, then Disc. is

- (a) positive (b) negative
(c) zero (d) irrational

23. If roots of a quadratic equation are real, rational and unequal then possible value of Disc. is

- (a) 0 (b) 36
(c) 40 (d) -25

24. If roots of an quadratic equation are real and distinct then Disc. is

- (a) positive (b) negative
(c) zero (d) imaginary

25. If ω and ω^2 are complex cube root of unity, then $\omega \cdot \omega^2 = \dots\dots\dots$

- (a) 1 (b) -1
(c) 0 (d) 2

26. If $b^2 - 4ac = 0$, then roots of $ax^2 + bx + c = 0$ are

- (a) irrational, equal
(b) rational, equal
(c) rational, unequal
(d) irrational, unequal

27. If for a quadratic equation $b^2 - 4ac = 49$, then roots are real and

- (a) equal (b) unequal
(c) irrational (d) imaginary

28. If roots of a quadratic equation are imaginary, and unequal, the possible value of Disc. is

- (a) 0 (b) 9
(c) 8 (d) -9

29. If for a quadratic equation $b^2 - 4ac = 0$, then roots are .

- (a) complex (b) irrational
(c) repeated (d) distinct

30. The roots of $x^2 + 8x + 16 = 0$ are...

- (a) imaginary (b) equal
(c) unequal (d) irrational

31. If $\omega = \frac{-1 - \sqrt{-3}}{2}$, then $\omega^2 = \dots\dots\dots$

- (a) $\frac{-1 \pm \sqrt{3}}{2}$ (b) $\frac{-1 + \sqrt{3}}{2}$
(c) $\frac{-1 + \sqrt{-3}}{2}$ (d) $\frac{-1 \pm \sqrt{-3}}{2}$

32. If 1, ω , ω^2 are cube root of unity, then $1 + \omega + \omega^2 = \dots\dots\dots$
 (a) 0 (b) ω^3
 (c) 1 (d) -1

33. If roots of a quadratic equation are imaginary, then Disc. is.....
 (a) positive (b) negative
 (c) zero (d) irrational

34. Cube root of 64 are.....
 (a) $-4, -4\omega, -4\omega^2$ (b) $4, 16\omega$
 (c) $4, 4\omega, 4\omega^2$ (d) $(4)^3$

35. If for a quadratic equation $b^2 - 4ac = -47$, then roots are.....
 (a) real (b) rational
 (c) irrational (d) complex

36. Which of the following is true description of nature of roots of a quadratic equation?
 (a) real, irrational, equal
 (b) real, imaginary, unequal
 (c) real, irrational, unequal
 (d) complex, repeated, rational

37. If roots of a quadratic equation are rational and distinct, then Disc. is.....
 (a) perfect square
 (b) not perfect square
 (c) zero
 (d) negative

38. If ω is complex cube root of unity, then $\omega^{63} = \dots\dots\dots$
 (a) ω (b) 1
 (c) $-\omega$ (d) $-\omega^2$

39. If roots of a quadratic equation are real, rational and equal, then possible value of Disc. is.....
 (a) 0 (b) 36
 (c) 40 (d) -49

40. If ω is complex cube root of unity, then $\omega^{-16} = \dots\dots\dots$
 (a) ω (b) $-\omega$
 (c) $-\omega^2$ (d) ω^2

41. $(-1 + \sqrt{-3})^3 = \dots\dots\dots$
 (a) 8 (b) 1
 (c) -4 (d) -28

42. Cube roots of -27 are.....
 (a) $3, -3\omega, 3\omega^2$
 (b) $-3, -3\omega, -3\omega^2$
 (c) $-3, 3\omega, 3\omega^2$
 (d) $3, 3\omega, -3\omega^2$

43. If ω is complex cube root of unity, then $\omega^{-27} = \dots\dots\dots$
 (a) 1 (b) -1
 (c) ω (d) ω^2

44. $(9 + 4\omega + 4\omega^2)^3 = \dots\dots\dots$
 (a) 15 (b) 25
 (c) 125 (d) $(17)^3$

45. $\omega^4 = \dots\dots\dots$
 (a) ω^2 (b) ω
 (c) 1 (d) 0

46. Which of the following shows "the product of two consecutive positive numbers."
 (a) $x(x+1)$ (b) $x(x+2)$
 (c) $x(x+3)$ (d) $x(x+4)$

47. If 1, ω , ω^2 are cube root of unity, then $1 + \omega = \dots\dots\dots$
 (a) 0 (b) ω
 (c) ω^2 (d) $-\omega^2$

48. If ω is complex cube root of unity, then $\omega^7 = \dots\dots\dots$
 (a) ω (b) $-\omega$
 (c) ω^2 (d) $-\omega^2$

49. If 1, ω , ω^2 are cube root of unity, then $1 + \omega^2 = \dots\dots\dots$
 (a) $-\omega$ (b) ω
 (c) ω^2 (d) $-\omega^2$

50. "Five less than three times a certain number" is
 (a) $3x - 5$ (b) $3x + 5$
 (c) $5x + 3$ (d) $5x - 3$

51. If 1, ω , ω^2 are cube root of unity, then $\omega + \omega^2 = \dots\dots\dots$
 (a) 1 (b) -1
 (c) ω^3 (d) $2\omega^2$

52. If roots of a quadratic equation are real, irrational and unequal then possible value of Disc. is.....

- (a) 0 (b) 9
(c) 5 (d) -7

53. $(1 - \omega - \omega^2)^5 = \dots\dots\dots$

- (a) 6 (b) 16
(c) 32 (d) 64

54. If length and width of a rectangle are x and y respectively then which of the following shows perimeter?

- (a) $(x + y)^2$ (b) $2x - 2y$
(c) $2xy$ (d) $2(x + y)$

55. If ω is complex cube root of unity, then $\omega^{23} = \dots\dots\dots$

- (a) ω (b) $-\omega$
(c) ω^2 (d) $-\omega^2$

56. Which of the following are symmetric functions of the roots of a quadratic equation?

- (a) $\alpha^2 + \beta^2$ (b) $\alpha^3 + \beta^3$
(c) $\frac{1}{\alpha} + \frac{1}{\beta}$ (d) all of these

57. If ω is complex cube root of unity, then $\omega^{-5} = \dots\dots\dots$

- (a) ω (b) 1
(c) $-\omega$ (d) $-\omega^2$

58. The equation $x^4 - 49x^2 + 36x + 252 = 0$ is called equation.

- (a) quadratic (b) quartic
(c) linear (d) cubic

59. $(1 - 3\omega - 3\omega^2)^3 = \dots\dots\dots$

- (a) 12 (b) 16
(c) -125 (d) 64

60. Cube roots of 8 are.....

- (a) $2, 2\omega, 2\omega^2$
(b) $-2, -2\omega, -2\omega^2$
(c) $2, -2\omega, -2\omega^2$
(d) $2, -2\omega, 2\omega^2$

61. The sum of five times a number and the square of the number is

- (a) $5x^2 + x$ (b) $5x + x^2$
(c) $(5x + x)^2$ (d) $5(x + x^2)$

ANSWER KEY

1.	d	2.	b	3.	b	4.	a	5.	a
6.	b	7.	c	8.	c	9.	c	10.	c
11.	a	12.	a	13.	c	14.	d	15.	d
16.	a	17.	c	18.	b	19.	b	20.	d
21.	b	22.	c	23.	b	24.	a	25.	a
26.	b	27.	b	28.	d	29.	c	30.	b
31.	c	32.	a	33.	b	34.	c	35.	d
36.	c	37.	a	38.	b	39.	a	40.	d
41.	a	42.	b	43.	a	44.	c	45.	b
46.	a	47.	d	48.	a	49.	a	50.	a
51.	b	52.	c	53.	c	54.	d	55.	c
56.	d	57.	a	58.	b	59.	d	60.	a
61.	b								

Q.2. Write short answers of the following questions.

(i) Discuss the nature of roots of the following equations.

(a) $x^2 + 3x + 5 = 0$

Solution:

$$x^2 + 3x + 5 = 0$$

$$ax^2 + bx + c = 0$$

$$\Rightarrow a=1, b=3, c=5$$

$$\text{Disc.} = b^2 - 4ac$$

$$= (3)^2 - 4(1)(5)$$

$$= 9 - 20$$

$$= -11 < 0$$

So roots of equation are imaginary

(b) $2x^2 - 7x + 3 = 0$

Solution:

$$2x^2 - 7x + 3 = 0$$

$$ax^2 + bx + c = 0$$

$$\Rightarrow a=2, b=-7, c=3$$

$$\text{Disc.} = b^2 - 4ac$$

$$\begin{aligned}
 &= (-7)^2 - 4(2)(3) \\
 &= 49 - 24 \\
 &= 25 \\
 &= (5)^2 > 0
 \end{aligned}$$

As Disc. is positive and perfect square so roots of equation are real, rational and distinct.

(c) $x^2 + 6x - 1 = 0$

Solution:

$$x^2 + 6x - 1 = 0$$

$$ax^2 + bx + c = 0$$

$$\Rightarrow a = 1, b = 6, c = -1$$

$$\begin{aligned}
 \text{Disc.} &= b^2 - 4ac \\
 &= (6)^2 - 4(1)(-1) \\
 &= 36 + 4 \\
 &= 40 > 0
 \end{aligned}$$

As Disc. is positive and not perfect square, so roots of equations are real, irrational and unequal.

(d) $16x^2 - 8x + 1 = 0$

Solution: $16x^2 - 8x + 1 = 0$

$$ax^2 + bx + c = 0$$

$$\Rightarrow a = 16, b = -8, c = 1$$

$$\begin{aligned}
 \text{Disc.} &= b^2 - 4ac \\
 &= (-8)^2 - 4(16)(1) \\
 &= 64 - 64 \\
 &= 0
 \end{aligned}$$

So roots of equation are real rational and equal.

(ii) Find ω^2 , if $\omega = \frac{-1 + \sqrt{-3}}{2}$

Solution: $\omega = \frac{-1 + \sqrt{-3}}{2}$

Taking square of both sides, we get

$$\omega^2 = \left(\frac{-1 + \sqrt{-3}}{2} \right)^2$$

$$\omega^2 = \frac{(-1 + \sqrt{-3})^2}{(2)^2}$$

$$\omega^2 = \frac{(-1)^2 + (\sqrt{-3})^2 + 2(-1)\sqrt{-3}}{4}$$

$$\omega^2 = \frac{1 + (-3) - 2\sqrt{-3}}{4}$$

$$\omega^2 = \frac{-2 - 2\sqrt{-3}}{4}$$

$$\omega^2 = \frac{-2(-1 - \sqrt{-3})}{4}$$

$$\omega^2 = \frac{-1 - \sqrt{-3}}{2}$$

(iii) Prove that the sum of all the cube roots of unity is zero.

Solution: We have cube roots of unity

$$1, \omega = \frac{-1 + \sqrt{-3}}{2}, \omega^2 = \frac{-1 - \sqrt{-3}}{2}$$

Taking sum, we get

$$1 + \omega + \omega^2 = 1 + \frac{(-1 + \sqrt{-3})}{2} + \frac{(-1 - \sqrt{-3})}{2}$$

$$1 + \omega + \omega^2 = 1 + \frac{(-1 + \sqrt{-3}) + (-1 - \sqrt{-3})}{2}$$

$$1 + \omega + \omega^2 = \frac{2 + (-1 + \sqrt{-3} - 1 - \sqrt{-3})}{2}$$

$$1 + \omega + \omega^2 = \frac{2 - 2}{2}$$

$$1 + \omega + \omega^2 = \frac{0}{2}$$

$$1 + \omega + \omega^2 = 0$$

It is proved that sum of all the cube roots of unity is zero.

(iv) Find the product of complex cube roots of unity.

Solution: Following are the complex cube roots of unity

$$\omega = \frac{-1 + \sqrt{-3}}{2}, \omega^2 = \frac{-1 - \sqrt{-3}}{2}$$

Taking product, we get

$$\omega \cdot \omega^2 = \left(\frac{-1 + \sqrt{-3}}{2} \right) \left(\frac{-1 - \sqrt{-3}}{2} \right)$$

$$\omega^3 = \frac{(-1)^2 - (\sqrt{-3})^2}{4}$$

$$\omega^3 = \frac{1 - (-3)}{4}$$

$$\therefore (a+b)(a-b) = a^2 - b^2$$

$$\omega^3 = \frac{1+3}{4}$$

$$\omega^3 = \frac{4}{4}$$

$$\omega^3 = 1$$

Product of complex cube roots of unity is 1.

(v) Show that $x^3 + y^3 = (x + y)(x + \omega y)(x + \omega^2 y)$

Solution: Let

$$\text{R.H.S.} = (x + y)(x + \omega y)(x + \omega^2 y)$$

$$= (x + y)[x^2 + \omega^2 xy + \omega xy + \omega^3 y^2]$$

$$= (x + y)[x^2 + xy(\omega^2 + \omega)(1)y^2] \quad \boxed{\because \omega^3 = 1}$$

$$= (x + y)[x + xy(-1) + y^2] \quad \boxed{\because \omega + \omega^2 = -1}$$

$$= (x + y)(x^2 - xy + y^2)$$

$$= x^3 + y^3 = \text{L.H.S.}$$

(vi) Evaluate $\omega^{37} + \omega^{38} + 1$

Solution:

$$\begin{aligned} \omega^{37} + \omega^{38} + 1 &= \omega \cdot \omega^{36} + \omega^2 \cdot \omega^{36} + 1 \\ &= \omega (\omega^3)^{12} + \omega^2 (\omega^3)^{12} + 1 \\ &= \omega (1)^{12} + \omega^2 (1)^{12} + 1 \\ &= \omega + \omega^2 + 1 \quad (\because \omega^3 = 1) \\ &= 0 \quad (\because 1 + \omega + \omega^2 = 0) \end{aligned}$$

(vii) Evaluate $(1 - \omega + \omega^2)^6$

Solution:

$$\begin{aligned} (1 + \omega^2 - \omega)^6 &= (-\omega - \omega)^6 \quad (\because \omega^2 = -\omega) \\ &= (-2\omega)^6 \\ &= (-2)^6 (\omega)^6 \\ &= 64\omega^6 \\ &= 64(\omega^3)^2 \quad (\because \omega^3 = 1) \\ &= 64(1)^2 \\ &= 64(1) \\ &= 64 \end{aligned}$$

(viii) If ω is cube root of unity, form an equation whose roots are 3ω and $3\omega^2$.

Solution:

Since 3ω and $3\omega^2$ are the roots of the required equation.

Sum of roots

$$\begin{aligned} S &= 3\omega + 3\omega^2 \\ S &= 3(\omega + \omega^2) \\ S &= 3(-1) \\ S &= -3 \end{aligned}$$

Product of roots

$$P = (3\omega)(3\omega^2)$$

$$P = 9\omega^3$$

$$P = 9(1) \quad (\because \omega^3 = 1)$$

$$P = 9$$

Hence the required equation is

$$x^2 - Sx + P = 0$$

$$x^2 - (-3)x + 9 = 0$$

$$x^2 + 3x + 9 = 0$$

(ix) Using synthetic division, find the remainder and quotient when

$$(x^3 + 3x^2 + 2) \div (x - 2)$$

Solution: As $x - 2 = 0 \Rightarrow x = 2$

Now write the coefficients of dividend in a row and $x = 2$ on the left side.

2	1	3	0	2
	↓	2	10	20
	1	5	10	22

$$\text{Quotient } Q(x) = (x^2 + 5x + 10)$$

$$\text{Remainder} = 22$$

(x) Using synthetic division, show that $x - 2$ is the factor of $x^3 + x^2 - 7x + 2$.

Solution: Write the coefficients of dividend in a row and $x = 2$ on the left side.

$x^3 + x^2 - 7x + 2$				
2	1	1	-7	2
	↓	2	6	-2
	1	3	-1	0

As remainder is zero, so by synthetic division $x - 2$ is factor of $x^3 + x^2 - 7x + 2$

(xi) Find the sum and product of the roots of the equation $2px^2 + 3qx - 4r = 0$

Solution: $2px^2 + 3qx - 4r = 0$

$$ax^2 + bx + c = 0$$

$$a = 2p, b = 3q, c = -4r$$

$$\text{Sum of roots} = S = \frac{-b}{a} = \frac{-3q}{2p}$$

$$\text{Product of roots} = P = \frac{c}{a} = \frac{-4r}{2p}$$

$$P = \frac{-2r}{p}$$

(xii) Find $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ of the roots of the equation $x^2 - 4x + 3 = 0$

Solution: $x^2 - 4x + 3 = 0$

$$ax^2 + bx + c = 0$$

$$\Rightarrow a = 1, b = -4, c = 3$$

If α and β are roots of the given equation then

$$\text{Sum of roots} = \alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{1} = 4$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$$

$$\begin{aligned} \text{Given that } \frac{1}{\alpha^2} + \frac{1}{\beta^2} &= \frac{\beta^2 + \alpha^2}{\beta^2 \alpha^2} \\ &= \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} \\ &= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} \\ &= \frac{(4)^2 - 2(3)}{(3)^2} \\ &= \frac{16 - 6}{9} \\ &= \frac{10}{9} \end{aligned}$$

(xiii) If α, β are the roots of $4x^2 - 3x + 6 = 0$ find

(a) $\alpha^2 + \beta^2$ (b) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (c) $\alpha - \beta$

Solution:

$$4x^2 - 3x + 6 = 0$$

$$ax^2 + bx + c = 0$$

$$\Rightarrow a = 4, b = -3, c = 6$$

As α and β are roots of the equation

$$\text{Sum of roots} = \alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{4} = \frac{3}{4}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a} = \frac{6}{4} = \frac{3}{2}$$

Now

(a) $\alpha^2 + \beta^2$

$$\begin{aligned} \alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2(\alpha\beta) \\ &= \left(\frac{3}{4}\right)^2 - 2\left(\frac{3}{2}\right) \\ &= \frac{9}{16} - 3 \\ &= \frac{9 - 48}{16} \\ &= \frac{-39}{16} \end{aligned}$$

(b) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$\begin{aligned} \frac{\alpha}{\beta} + \frac{\beta}{\alpha} &= \frac{\alpha^2 + \beta^2}{\alpha\beta} \\ &= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} \\ &= \left[(a + b)^2 - 2ab \right] \div \alpha\beta \\ &= \left[\left(\frac{3}{4}\right)^2 - 2\left(\frac{3}{2}\right) \right] \div \frac{3}{2} \\ &= \left(\frac{9}{16} - 3 \right) \times \frac{2}{3} \\ &= \left(\frac{9 - 48}{16} \right) \times \frac{2}{3} \\ &= \frac{-39}{16} \times \frac{2}{3} \\ &= \frac{-13}{8} \end{aligned}$$

(c) $\alpha - \beta$

$$\begin{aligned} (\alpha - \beta)^2 &= \alpha^2 + \beta^2 - 2\alpha\beta \\ \sqrt{(\alpha - \beta)^2} &= \sqrt{\alpha^2 + \beta^2 - 2\alpha\beta} \\ \alpha - \beta &= \sqrt{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta - 2\alpha\beta} \\ \alpha - \beta &= \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} \\ &= \sqrt{\left(\frac{3}{4}\right)^2 - 4\left(\frac{3}{2}\right)} \end{aligned}$$

$$\begin{aligned}
&= \sqrt{\frac{9}{16} - 6} \\
&= \sqrt{\frac{9-96}{16}} \\
&= \sqrt{\frac{-87}{16}} \\
&= \frac{\sqrt{-87}}{4}
\end{aligned}$$

(xiv) If α, β are the roots of $x^2 - 5x + 7 = 0$, find an equation whose roots are

(a) $-\alpha, -\beta$ (b) $2\alpha, 2\beta$

Solution: $x^2 - 5x + 7 = 0$

$$ax^2 + bx + c = 0$$

$$\Rightarrow a = 1, b = -5, c = 7$$

$$\text{Sum of roots} = S = \frac{-b}{a} = \frac{-(-5)}{1} = 5$$

$$\text{Product of roots} = P = \alpha\beta = \frac{c}{a} = \frac{7}{1} = 7$$

Now if roots are $-\alpha, -\beta$

(a) Equation with the roots $-\alpha, -\beta$

$$\text{Sum of roots} = S = (-\alpha) + (-\beta)$$

$$S = -(\alpha + \beta)$$

$$S = -(+5)$$

$$S = -5$$

$$\text{Product of roots} = P = (-\alpha)(-\beta)$$

$$P = \alpha\beta$$

$$P = 7$$

So required equation

$$x^2 - Sx + P = 0$$

$$x^2 - (-5)x + 7 = 0$$

$$x^2 + 5x + 7 = 0$$

(b) The equation with the roots $2\alpha, 2\beta$

$$\text{Sum of roots} = S = 2\alpha + 2\beta$$

$$= 2(\alpha + \beta)$$

$$= 2(5)$$

$$= 10$$

$$\text{Product of roots} = P = 2\alpha \times 2\beta$$

$$= 4\alpha\beta$$

$$= 4(7)$$

$$= 28$$

So required equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - 10x + 28 = 0$$

Q.3: Fill in the blanks

i. The discriminant of $ax^2 + bx + c = 0$ is _____.

ii. If $b^2 - 4ac = 0$ then roots of $ax^2 + bx + c = 0$ are _____.

iii. If $b^2 - 4ac > 0$ then roots of $ax^2 + bx + c = 0$ are _____.

iv. If $b^2 - 4ac < 0$ then the root of $ax^2 + bx + c = 0$ are _____.

v. If $b^2 - 4ac > 0$ and perfect square, then roots of $ax^2 + bx + c = 0$ are _____.

vi. If $b^2 - 4ac > 0$ and not perfect square, then roots of $ax^2 + bx + c = 0$ are _____.

vii. If α, β are the roots of $ax^2 + bx + c = 0$, then sum of the roots is _____.

viii. If α, β are the roots of $ax^2 + bx + c = 0$, then product of the roots is _____.

ix. If α, β are the roots of $7x^2 - 5x + 3 = 0$, then sum of the roots is _____.

x. If α, β are the roots of $5x^2 + 3x - 9 = 0$, then product of the roots is _____.

xi. For a quadratic equation $ax^2 + bx + c = 0$ $\frac{1}{\alpha\beta}$ is equal to _____.

xii. Cube roots of unity are _____.

xiii. Under usual notation sum of the cube roots of unity is _____.

xiv. If $1, \omega, \omega^2$ are the cube roots of unity, then ω^{-7} is equal to _____.

xv. If α, β are the roots of the quadratic equation, then the quadratic equation is written as _____.

xvi. If 2ω and $2\omega^2$ are the roots of an equation, then equation is _____.

$$\begin{aligned}
&= \sqrt{\frac{9}{16} - 6} \\
&= \sqrt{\frac{9-96}{16}} \\
&= \sqrt{\frac{-87}{16}} \\
&= \frac{\sqrt{-87}}{4}
\end{aligned}$$

(xiv) If α, β are the roots of $x^2 - 5x + 7 = 0$, find an equation whose roots are

(a) $-\alpha, -\beta$ (b) $2\alpha, 2\beta$

Solution: $x^2 - 5x + 7 = 0$

$$ax^2 + bx + c = 0$$

$$\Rightarrow a = 1, b = -5, c = 7$$

$$\text{Sum of roots} = S = \frac{-b}{a} = \frac{-(-5)}{1} = 5$$

$$\text{Product of roots} = P = \alpha\beta = \frac{c}{a} = \frac{7}{1} = 7$$

Now if roots are $-\alpha, -\beta$

(a) Equation with the roots $-\alpha, -\beta$

$$\text{Sum of roots} = S = (-\alpha) + (-\beta)$$

$$S = -(\alpha + \beta)$$

$$S = -(+5)$$

$$S = -5$$

$$\text{Product of roots} = P = (-\alpha)(-\beta)$$

$$P = \alpha\beta$$

$$P = 7$$

So required equation

$$x^2 - Sx + P = 0$$

$$x^2 - (-5)x + 7 = 0$$

$$x^2 + 5x + 7 = 0$$

(b) The equation with the roots $2\alpha, 2\beta$

$$\text{Sum of roots} = S = 2\alpha + 2\beta$$

$$= 2(\alpha + \beta)$$

$$= 2(5)$$

$$= 10$$

$$\text{Product of roots} = P = 2\alpha \times 2\beta$$

$$= 4\alpha\beta$$

$$= 4(7)$$

$$= 28$$

So required equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - 10x + 28 = 0$$

Q.3: Fill in the blanks

i. The discriminant of $ax^2 + bx + c = 0$ is _____.

ii. If $b^2 - 4ac = 0$ then roots of $ax^2 + bx + c = 0$ are _____.

iii. If $b^2 - 4ac > 0$ then roots of $ax^2 + bx + c = 0$ are _____.

iv. If $b^2 - 4ac < 0$ then the root of $ax^2 + bx + c = 0$ are _____.

v. If $b^2 - 4ac > 0$ and perfect square, then roots of $ax^2 + bx + c = 0$ are _____.

vi. If $b^2 - 4ac > 0$ and not perfect square, then roots of $ax^2 + bx + c = 0$ are _____.

vii. If α, β are the roots of $ax^2 + bx + c = 0$, then sum of the roots is _____.

viii. If α, β are the roots of $ax^2 + bx + c = 0$, then product of the roots is _____.

ix. If α, β are the roots of $7x^2 - 5x + 3 = 0$, then sum of the roots is _____.

x. If α, β are the roots of $5x^2 + 3x - 9 = 0$, then product of the roots is _____.

xi. For a quadratic equation $ax^2 + bx + c = 0$ $\frac{1}{\alpha\beta}$ is equal to _____.

xii. Cube roots of unity are _____.

xiii. Under usual notation sum of the cube roots of unity is _____.

xiv. If $1, \omega, \omega^2$ are the cube roots of unity, then ω^{-7} is equal to _____.

xv. If α, β are the roots of the quadratic equation, then the quadratic equation is written as _____.

xvi. If 2ω and $2\omega^2$ are the roots of an equation, then equation is _____.

ANSWER Key

i.	$b^2 - 4ac$	ix.	$\frac{5}{7}$
ii.	Equal	x.	$-\frac{9}{5}$
iii.	Real	xi.	$\frac{a}{c}$
iv.	Imaginary	xii.	$1, \omega, \omega^2$
v.	Rational	xiii.	Zero
vi.	Irrational	xiv.	ω^2
vii.	$-\frac{b}{a}$	xv.	$x^2 - (\alpha + \beta)x + \alpha\beta = 0$
viii.	$\frac{c}{a}$	xvi.	$x^2 + 2x + 4 = 0$