

Secant Line:

A secant is a straight line which cuts the circumference of a circle in two distinct points.

In the figure ℓ indicates the secant line to the circle C_1 .

Tangent Line: A tangent to a circle is the straight line which touches the circumference at a single point only and perpendicular at the outer end of radial segment obtained by joining the centre and point of tangency. The point of tangency is also known as the point of contact.

In the figure line AB indicates the tangent line to the circle C_2 .

Length of a tangent segment

The distance between the given point outside the circle and point of tangency is called length of tangent segment.

THEOREM 1

If a line is drawn perpendicular to a radial segment of a circle at its outer end point, it is tangent to the circle at that point.

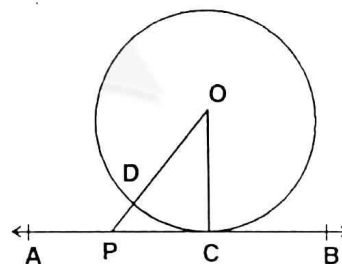
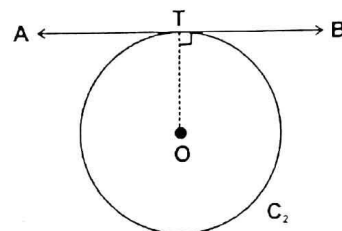
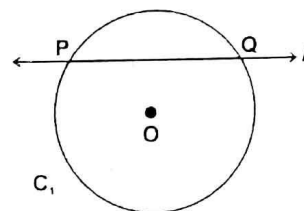
Given: A circle with centre O and $\overline{OC}$ is the radial segment.

$\overleftrightarrow{AB}$ is perpendicular to $\overline{OC}$ at its outer end C.

To Prove: $\overleftrightarrow{AB}$ is a tangent to the circle at C.

Construction: Take any point P other than C on $\overleftrightarrow{AB}$. Join O with P.

Proof:



Statements	Reasons
In $\triangle OCP$, $m\angle OCP = 90^\circ$ and $m\angle OPC < 90^\circ$ $m\overline{OP} > m\overline{OC}$ $\therefore$ P is a point outside the circle. Similarly, every point on $\overleftrightarrow{AB}$ except C lies outside the circle. Hence $\overleftrightarrow{AB}$ intersects the circle at one point C only. i.e., $\overleftrightarrow{AB}$ is a tangent to the circle at one point only.	$\overleftrightarrow{AB} \perp \overline{OC}$ (given) Acute angle of right angled triangle. Greater angle has greater side opposite to it. $\overline{OC}$ is the radial segment.

THEOREM 2

The tangent to a circle and the radial segment joining the point of contact and the centre are perpendicular to each other.

Given:

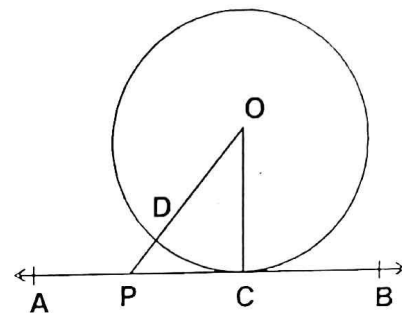
In a circle with centre O and radius $\overline{OC}$.

Also $\overleftrightarrow{AB}$ is the tangent to the circle at point C.

To Prove:

$\overleftrightarrow{AB}$ and radial segment $\overline{OC}$ are perpendicular to each other.

i.e. $\overline{OC} \perp \overleftrightarrow{AB}$.



Construction:

Take any point P other than C on the tangent line $\overleftrightarrow{AB}$.

Join O with P so that $\overline{OP}$ meets the circle at D.

Proof:

Statements	Reasons
$\overleftrightarrow{AB}$ is the tangent to the circle at point C.	Given
$\overline{OP}$ cuts the circle at D.	Construction
$\therefore m\overline{OC} = m\overline{OD}$ (i)	Radii of the same circle
But $m\overline{OD} < m\overline{OP}$ (ii)	Point P is outside the circle.
$\therefore m\overline{OC} < m\overline{OP}$	Using (i) and (ii)
So radius $\overline{OC}$ is shortest of all line segments that can be drawn from O to the tangent line $\overleftrightarrow{AB}$.	
Also $\overline{OC} \perp \overleftrightarrow{AB}$	
Hence, radial segment $\overline{OC}$ is perpendicular to the tangent $\overleftrightarrow{AB}$.	

Corollary:

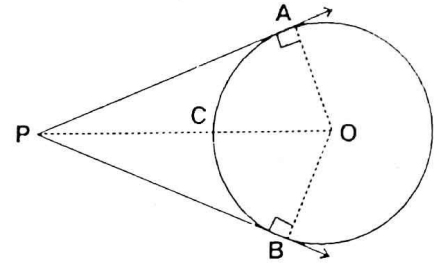
There can only be one perpendicular draw to the radial segment $\overline{OC}$ at the point C of the circle. It follows that one and only one tangent can be drawn to the circle at the given point C on its circumference.

THEOREM 3

Two tangents drawn to a circle from a point outside it, are equal in length.

Given:

Two tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are drawn from an external point P to the circle with centre O.



To prove:

$$m\overline{PA} = m\overline{PB}$$

Construction:

Join O with A, B and P, So that we form $\angle rt \Delta^s$ OAP and OBP.

Proof:

Statements	Reasons
In $\angle rt \Delta^s$ OAP $\leftrightarrow$ OBP	
$m\angle OAP = m\angle OBP = 90^\circ$	Radii $\perp$ to the tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$
$hyp.\overline{OP} = hyp.\overline{OP}$	Common
$m\overline{OA} = m\overline{OB}$	Radii of the same circle.
$\therefore \Delta OAP \cong \Delta OBP$	In $\angle rt \Delta^s$ H.S $\cong$ H.S
Hence, $m\overline{PA} = m\overline{PB}$	Corresponding sides of congruent triangles.

Note:

The length of a tangent to a circle is measured from the given point to the point of contact.

Corollary:

If O is the centre of a circle and two tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are drawn from an external point P then $\overline{OP}$ is the right bisector of the chord of contact $\overline{AB}$.

Example 1:

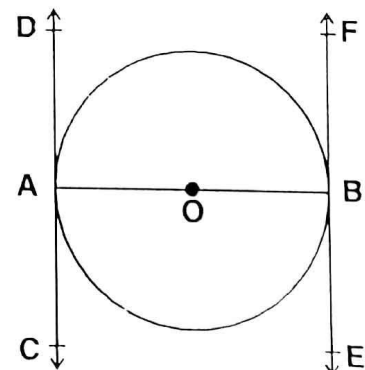
$\overline{AB}$ is a diameter of a given circle with centre O. Tangents are drawn at the end points A and B. Show that the two tangents are parallel.

Given:

$\overline{AB}$ is a diameter of a given circle with centre O. $\overleftrightarrow{CD}$ is the tangent to the circle at point A and $\overleftrightarrow{EF}$ is an other tangent at point B.

To Prove:

$$\overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$$



Proof:

Statements	Reasons
$\overline{AB}$ is the diameter of a circle with centre O. $\therefore \overline{OA}$ and $\overline{OB}$ are radii of the same circle. Moreover $\overleftrightarrow{CD}$ is a tangent to the circle at A.	Given Given By Theorem 1
$\therefore \overline{OA} \perp \overleftrightarrow{CD}$ $\overline{AB} \perp \overleftrightarrow{CD}$(i)	Given By Theorem 1
Similarly $\overleftrightarrow{EF}$ is tangent at point B. So $\overline{OB} \perp \overleftrightarrow{EF}$ $\Rightarrow \overline{AB} \perp \overleftrightarrow{EF}$ (ii)	From (i) and (ii) ($\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ are perpendicular to same $\overline{AB}$)
Hence $\overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$	

Example 2:

In a circle, the tangents drawn at the ends of a chord, make equal angles with that chord.

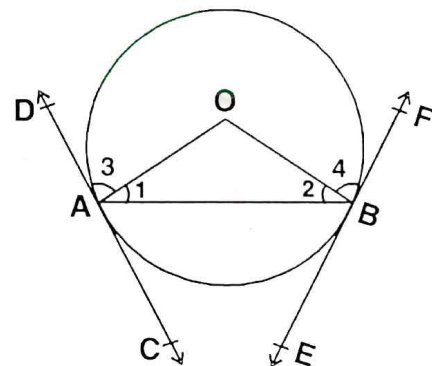
Given: $\overline{AB}$ is the chord of a circle with centre O.

$\overleftrightarrow{CAD}$ is the tangent at point A and $\overleftrightarrow{EBF}$ is another tangent at point B.

To Prove: $m\angle BAD = m\angle ABF$

Construction: Join O with A and B so that we form a $\triangle OAB$ then write $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$ as shown in the figure.

Proof:



Statements	Reasons
In $\triangle OAB$ $\therefore m\overline{OA} = m\overline{OB}$ $\therefore m\angle 1 = m\angle 2$(i)	Construction Radii of the same circle. Angles opposite to equal sides of $\triangle OAB$
Also $\therefore \overline{OA} \perp \overleftrightarrow{CD}$ $\therefore m\angle 3 = m\angle OAD = 90^\circ$(ii)	Radial segment $\perp$ to the tangent line
Similarly $\overline{OB} \perp \overleftrightarrow{EF}$ $\therefore m\angle 4 = m\angle OBF = 90^\circ$(iii)	Radial segment $\perp$ to the tangent
Hence $m\angle 3 = m\angle 4$ (iv)	Using (ii) and (iii)
$\Rightarrow m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$ i.e., $m\angle BAD = m\angle ABF$	Adding (i) and (iv)

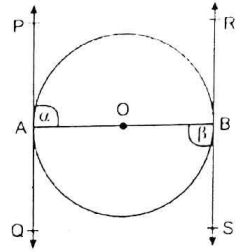
Exercise 10.1

Q.1 Prove that the tangents drawn at the ends of a diameter in a given circle must be parallel.

Given: A circle with centre O, has $\overline{AB}$ as diameter. $\overleftrightarrow{PAQ}$ is tangent at point A.
 $\overleftrightarrow{RBS}$ is tangent at point B

To Prove: $\overleftrightarrow{PAQ} \parallel \overleftrightarrow{RBS}$

Proof:



Statements	Reasons
$\overleftrightarrow{PAQ} \perp \overline{OA}$	Tangent is $\perp$ at the outer end of radial segment.
$\Rightarrow \overleftrightarrow{PAQ} \perp \overline{AB}$	
$\therefore m\angle\alpha = 90^\circ \dots\dots\dots (i)$	
$\overleftrightarrow{RBS} \perp \overline{OB}$	
$\Rightarrow \overleftrightarrow{RBS} \perp \overline{AB}$	If alternate Angles are equal in measurement, then lines are parallel.
$\therefore m\angle\beta = 90^\circ \dots\dots\dots (ii)$	
Thus, $m\angle\alpha = m\angle\beta$	
Therefore $\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$	

Q.2 The diameters of two concentric circles are 10cm and 5cm respectively. Look for the length of any chord of the outer circle which touches the inner one.

Solution: Let $\overline{AB}$ be any chord of the outer circle that touches the inner circle.

Diameter of outer circle = 10 cm

$$\text{Radius of outer circle} = m\overline{OB} = \frac{10\text{cm}}{2} = 5\text{cm}$$

Diameter of inner circle = 5 cm

$$\text{Radius of inner circle} = m\overline{OC} = \frac{5\text{cm}}{2} = 2.5\text{cm}$$

$\triangle OCB$ is right angled triangle with right angle at C. ($\because \overline{OC} \perp \overline{AB}$)

By Pythagoras theorem

$$\begin{aligned} (m\overline{OB})^2 &= (m\overline{BC})^2 + (m\overline{OC})^2 \\ (5\text{ cm})^2 &= (x)^2 + (2.5\text{cm})^2 \\ \Rightarrow x^2 &= (5\text{cm})^2 - (2.5\text{cm})^2 \\ x^2 &= 25\text{cm}^2 - 6.25\text{ cm}^2 \\ x^2 &= 18.75\text{cm}^2 \end{aligned}$$

Taking Square root of both sides.

$$\begin{aligned} \sqrt{x^2} &= \sqrt{18.75\text{cm}^2} \\ x &= \sqrt{18.75}\text{cm} \end{aligned}$$

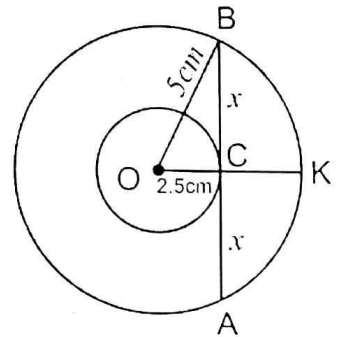
Length of Chord = $m\overline{AB} = 2x$

$$m\overline{AB} = 2(\sqrt{18.75}\text{cm})$$

$$m\overline{AB} = 8.66\text{cm}$$

$\Rightarrow$

$$\boxed{m\overline{AB} \approx 8.7\text{cm}}$$



Q.3 $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are the common tangents drawn to the pair of circles.

If A and C are the points of tangency of 1st circle where B and D are the points of tangency of 2nd circle, then prove that $\overline{AC} \parallel \overline{BD}$.

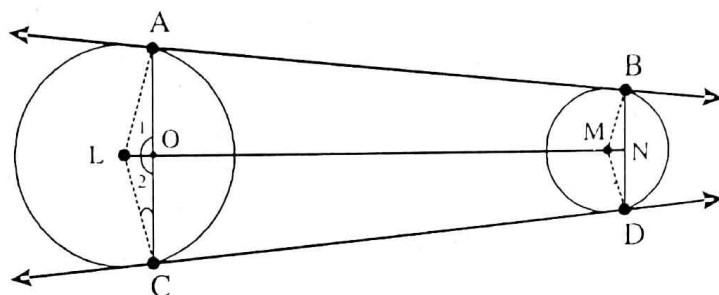
Given: Two circles with centre L and M. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are their common tangents. A is joined with C and B is joined with D.

To prove:

$$\overline{AC} \parallel \overline{BD}$$

Construction:

Join L to A and C. Join M to B and D. Join L to M and produce it to meet the $\overline{BD}$ at N.



Proof:

Statements	Reasons
In $\triangle AOL \leftrightarrow \triangle COL$	
$\overline{AL} \cong \overline{CL}$	Radii of the same circle
$\angle A \cong \angle C$	angles opposite to congruent sides
$\overline{LO} \cong \overline{LO}$	common side.
$\therefore \triangle AOL \cong \triangle COL$	S.A.S $\cong$ S.A.S
$m\angle 1 = m\angle 2$(i)	Corresponding angles of congruent triangle.
$m\angle 1 + m\angle 2 = 180^\circ$(ii)	O is the point on line segment $\overline{AC}$.
$\Rightarrow m\angle 1 = m\angle 2 = 90^\circ$	
$\overline{LO} \perp \overline{AO}$	
or $\overline{LO} \perp \overline{AC}$	
or $\overline{AC} \perp \overline{LOMN}$(iii)	
Similarly in the circle with centre M, it can be proved that	
$\overline{BD} \perp \overline{MN}$	
or $\overline{BD} \perp \overline{LOMN}$(iv)	
Both $\overline{AC}$ and $\overline{BD}$ are $\perp$ to the same line segment	
$\therefore \overline{AC} \parallel \overline{BD}$	Two line segments making same angle with a line are parallel to each other.

THEOREM 4 (A)

If two circles touch externally then the distance between their centres is equal to the sum of their radii.

Given:

Two circles with centres D and F respectively touch each other externally at point C. So that $\overline{CD}$ and $\overline{CF}$ are respectively the radii of the two circles.

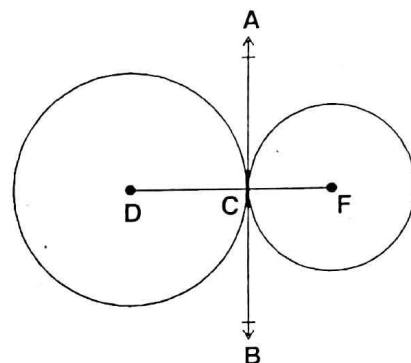
To Prove:

Point C lies on the join of centres D and F and
 $m\overline{DF} = m\overline{DC} + m\overline{CF}$

Construction:

Draw $\overleftrightarrow{ACB}$ as a common tangent to the pair of circles at C.

Proof:



Statements	Reasons
Both circles touch externally at C whereas $\overline{CD}$ is radial segment and $\overleftrightarrow{ACB}$ is the common tangent.	
$\therefore m\angle ACD = 90^\circ$(i)	Radial segment $\overline{CD} \perp$ the tangent line $\overleftrightarrow{AB}$
Similarly $\overline{CF}$ is radial segment and $\overleftrightarrow{ACB}$ is the common tangent	
$\therefore m\angle ACF = 90^\circ$(ii)	Radial segment $\overline{CF} \perp$ the tangent line $\overleftrightarrow{AB}$
$m\angle ACD + m\angle ACF = 90^\circ + 90^\circ$	Adding (i) and (ii)
$m\angle DCF = 180^\circ$(iii)	Sum of supplementary adjacent angles
Hence DCF is a line segment with point C between D and F	
and $m\overline{DF} = m\overline{DC} + m\overline{CF}$	