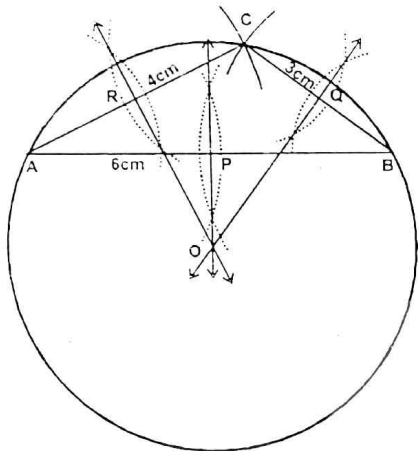


EXERCISE 13.2

Q. 1 Circumscribe a circle about a triangle ABC with sides $|\overline{AB}| = 6\text{cm}$, $|\overline{BC}| = 3\text{cm}$ and $|\overline{CA}| = 4\text{cm}$. Also measure its circum radius.

Solution:

Data: $|\overline{AB}| = 6\text{cm}$, $|\overline{BC}| = 3\text{cm}$, $|\overline{CA}| = 4\text{cm}$



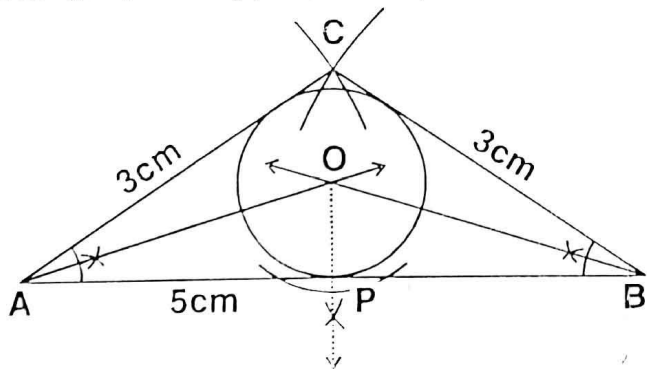
Steps of construction:

- i. We construct triangle ABC according to given condition.
- ii. We draw right bisectors $\overleftrightarrow{OP}$, $\overleftrightarrow{OQ}$ and $\overleftrightarrow{OR}$ of sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively, concurrent at point 'O'.
- iii. Taking 'O' as centre and radius equal to $m\overline{OA}$ or $m\overline{OB}$ or $m\overline{OC}$, we draw a circle passing through the vertices A , B and C .
- iv. This is the required circum circle, whose radius is measured to be 3.3 cm.

Q. 2 Inscribe a circle in a triangle ABC with side $|\overline{AB}| = 5\text{cm}$, $|\overline{BC}| = 3\text{cm}$ and $|\overline{CA}| = 3\text{cm}$. Also measure its in-radius.

Solution:

Data: $|\overline{AB}| = 5\text{cm}$, $|\overline{AB}| = 5\text{cm}$, $|\overline{CA}| = 3\text{cm}$



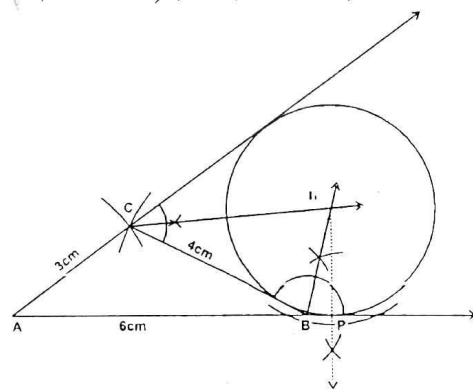
Steps of construction:

- i. We construct triangle ABC according to given condition.
- ii. We draw bisectors of $\angle A$ and $\angle B$ intersecting each other at point 'O'.
- iii. From point O , we draw $\overrightarrow{OP}$ perpendicular to $\overline{AB}$.
- iv. Taking 'O' as centre and radius equal to $m\overline{OP}$, we draw a circle, touching three sides of triangle internally.
- v. This is the required in-circle whose radius is measured to be 0.8 cm.

Q. 3 Escribe a circle opposite to vertex A to a triangle ABC with sides $|\overline{AB}| = 6\text{cm}$, $|\overline{BC}| = 4\text{cm}$ and $|\overline{CA}| = 3\text{cm}$. Find its radius also.

Solution:

Data: $|\overline{AB}| = 6\text{cm}$, $|\overline{BC}| = 4\text{cm}$, $|\overline{CA}| = 3\text{cm}$



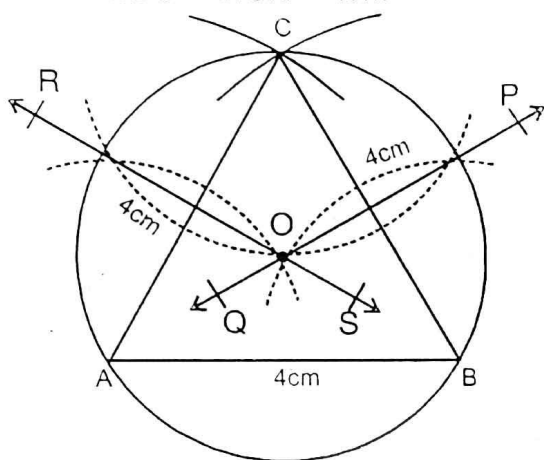
Steps of construction:

- i. We construct a triangle ABC according to given condition.
- ii. We produce the sides $\overline{AB}$ and $\overline{AC}$ beyond B and C respectively.
- iii. We draw, bisectors of exterior angles at points B and C , intersecting each other at point I_1 .
- iv. From point I_1 , we draw $\overrightarrow{I_1P}$ perpendicular to $\overline{AB}$ produced.
- v. Taking I_1 as centre and radius equal to I_1P , we draw a circle, touching one side of $\triangle ABC$ externally and other two produced sides internally.
- vi. This is the required escribed circle, whose radius is measured to be 2.2 cm.

Q. 4 Circumscribe a circle about an equilateral triangle ABC with each side of length 4cm.

Solution:

Data: $m\overline{AB} = m\overline{BC} = m\overline{CA} = 4\text{cm}$



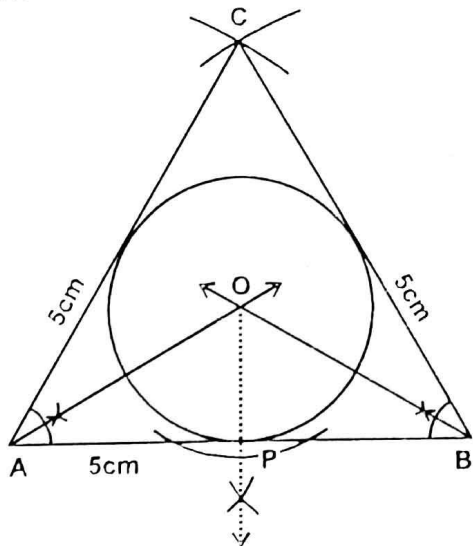
Steps of construction:

- We construct equilateral triangle ABC with each side 4cm long.
- We draw right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ of side $\overline{BC}$ and $\overline{AC}$ respectively intersecting each other at point O.
- Taking O as centre and radius equal to $m\overline{OA}$ or $m\overline{OB}$ or $m\overline{OC}$, we draw a circle passing through the points A, B and C.
- This is our required circum circle whose radius is measured to be 2.3 cm.

Q. 5 Inscribe a circle in an equilateral triangle ABC with each side of length 5cm.

Solution:

Data: $m\overline{AB} = m\overline{BC} = m\overline{CA} = 5\text{cm}$



Steps of construction:

- We construct equilateral triangle ABC with each side 5cm long.
- We draw bisectors of $\angle A$ and $\angle B$ intersecting each other at point 'O'.
- From point O, we draw $\overrightarrow{OP}$ perpendicular to $\overline{AB}$.
- Taking 'O' as centre and radius equal to $\overline{OP}$, we draw a circle, touching three sides of triangle internally.
- This is the required in-circle whose radius is measured to be 1.4 cm.

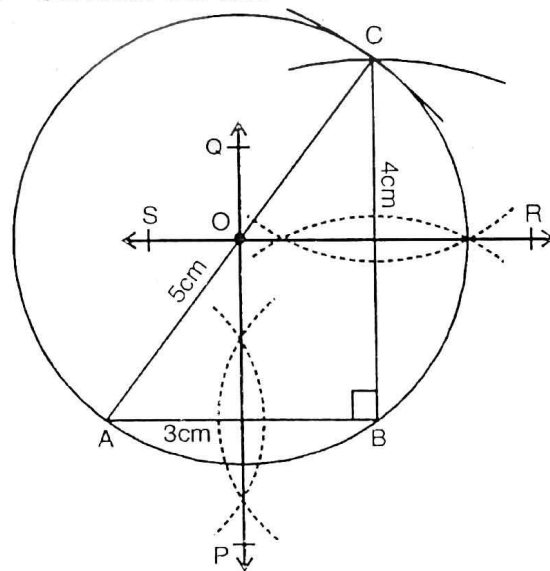
Q. 6 Circumscribe and inscribe circles with regard to a right angle triangle with sides 3cm, 4cm and 5cm.

Solution:

Let

$m\overline{AB} = 3\text{cm}, m\overline{BC} = 4\text{cm}$ and $m\overline{CA} = 5\text{cm}$

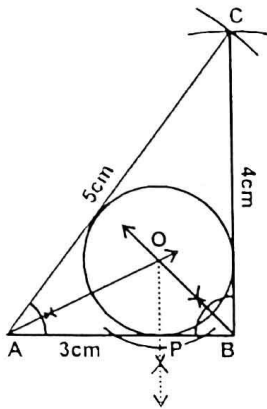
a. Circum Circle:



Steps of construction:

- We construct right angle triangle ABC with sides 3cm, 4cm and 5cm.
- We draw right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ of side $\overline{AB}$ and $\overline{BC}$ respectively intersecting each other at point O.
- Taking O as centre and radius equal to $m\overline{OA}$ or $m\overline{OB}$ or $m\overline{OC}$, we draw a circle passing through the points A, B and C.
- This is our required circum circle whose radius is measured to be 2.5 cm.

b. Inscribed Circle



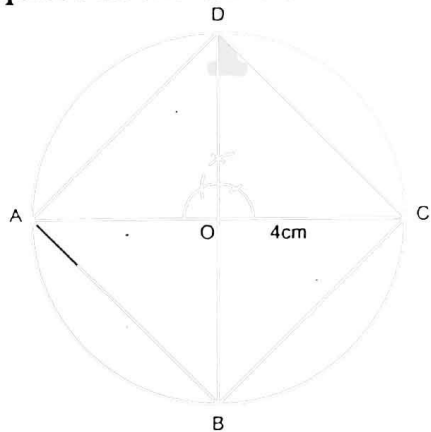
Steps of construction:

- We construct right angle triangle ABC according to given condition.
- We draw bisectors of $\angle A$ and $\angle B$ intersecting each other at point 'O'.
- From point O, we draw $\overline{OP}$ perpendicular to $\overline{AB}$.
- Taking 'O' as centre and radius equal to $\overline{OP}$, we draw a circle, touching three sides of triangle internally.
- This is the required in-circle whose radius is measured to be 1 cm.

Q. 7 In and about a circle of radius 4 cm describe a square.

Solution:

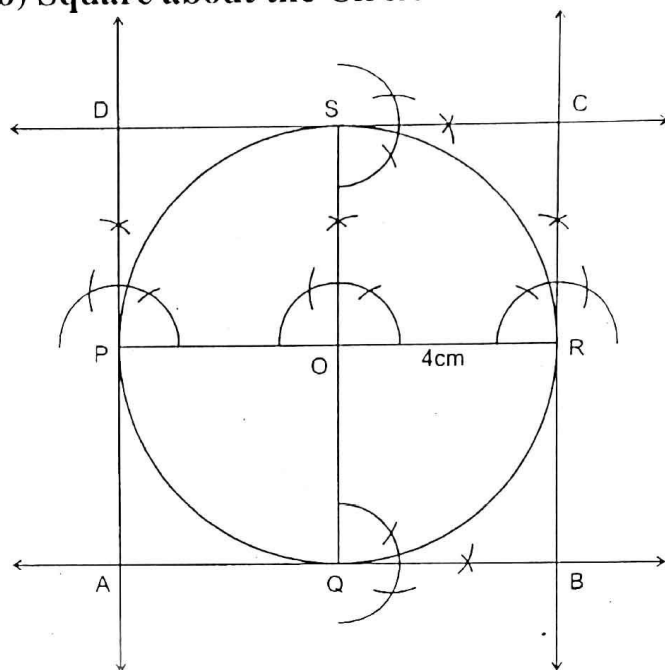
a. Square in the Circle



Steps of construction:

- We draw a circle with centre 'O' of radius 4cm.
- We draw two diameters $\overline{AC}$ and $\overline{BD}$ of circle perpendicular to each other.
- By joining points A with B, B with C, C with D and D with A, we get the required square inscribed in the given circle.

(b) Square about the Circle

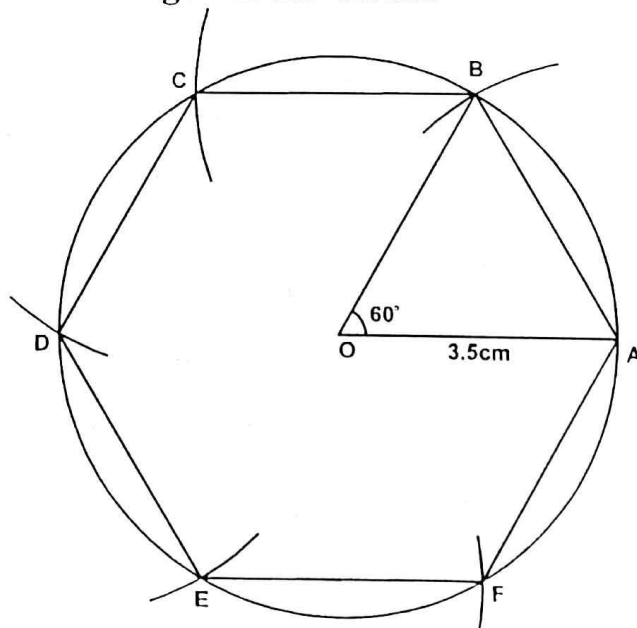


Steps of Construction:

- We draw a circle with centre "O" and a radius 4cm.
- We draw two diameters $\overline{PR}$ and $\overline{QS}$ of circle perpendicular to each other.
- We draw tangents to the circle at points P, Q, R and S.
- We produce the tangents to meet each other at point A, B, C and D.
- ABCD is the required circumscribed square.

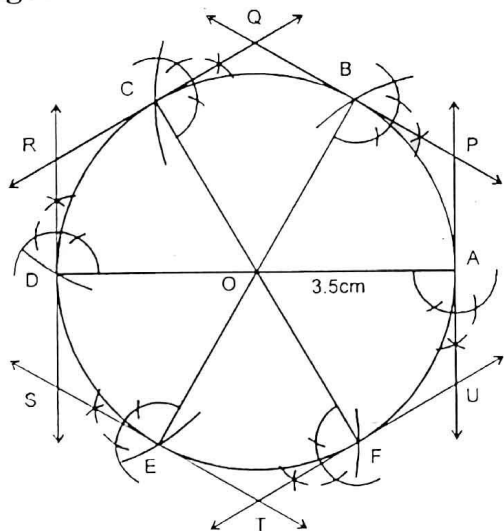
Q. 8 In and about a circle of radius 3.5 cm describe a hexagon.

a. Hexagon in the Circle:



Steps of Construction:

- i. We draw a circle with centre 'O' of radius 3.5 cm.
 - ii. We take a point A anywhere on the circle and draw the radial segment $\overline{OA}$.
 - iii. From point A, we draw an arc of radius $\overline{OA}$ which intersects the circle at point B.
 - iv. By joining 'O' with A and B we get an equilateral triangle OAB, so that the angle subtended by the chord at the centre is 60° .
 - v. From point B, we draw an arc of same radius intersecting the circle at point C, then joining B to C we get another chord $\overline{BC}$.
 - vi. We continue to draw the arcs, which cut the circle at points D, E and F, such that
$$\overline{OA} = \overline{AB} = \overline{BC} = \overline{CD}$$
$$= \overline{DE} = \overline{EF} = \overline{FA}$$
 - vii. We draw end to end on the circle the six chords $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$ and $\overline{FA}$, which completes the required hexagon
- b. Hexagon about the Circle:**

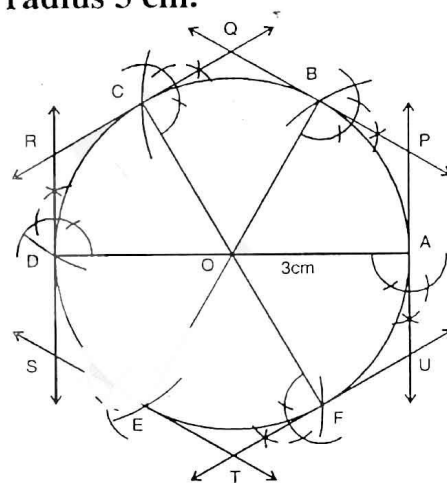


Steps Construction:

- i. We draw a circle with centre 'O' of radius 3.5 cm.
- ii. We take a point A anywhere on the circle and draw the radial segment $\overline{OA}$.
- iii. From point A, we draw an arc of radius $\overline{OA}$, which intersects the circle at point B.

- iv. From point B, we draw an arc of same radius intersecting the circle at point C.
- v. We continue to draw the arcs, which cut the circle at points D, E and F.
- vi. We draw the diameters $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$.
- vii. We draw tangents at points A, B, C, D, E and F to the circle.
- viii. We produce the tangents to meet each other at points P, Q, R, S, T and U.
- ix. PQRSTU is the required circumscribed hexagon.

Q. 9 Circumscribe a regular hexagon about a circle of radius 3 cm.



Steps Construction:

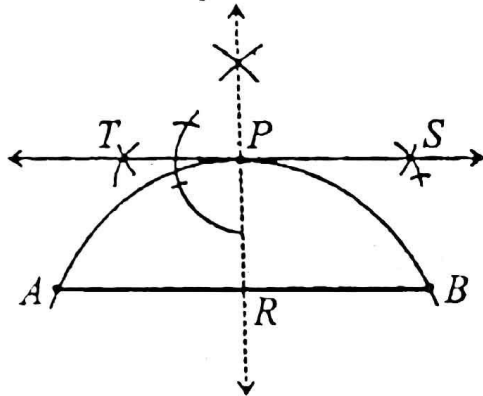
- i. We draw a circle with centre 'O' of radius 3 cm.
- ii. We take a point A anywhere on the circle and draw radial segment $\overline{OA}$.
- iii. From point A, we draw an arc of radius $\overline{OA}$, which intersects the circle at point B.
- iv. From point B, we draw an arc of same radius intersecting the circle at point C.
- v. We continue to draw the arcs, which cut the circle at points D, E and F.
- vi. We draw diameter $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$.
- vii. We draw tangents at points A, B, C, D, E and F to the circle.
- viii. We produce the tangents to meet each other at points P, Q, R, S, T and U.
- ix. PQRSTU is the required circumscribed hexagon.

TANGENT TO THE CIRCLE

(1) Draw a tangent to a given arc without using the centre through a given point P :

Case (i) When P is the middle point of the arc.

Given: P is the mid-point of an arc AB .

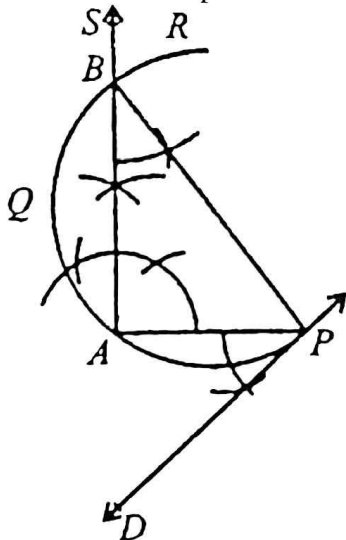


Steps of Construction:

- i. Join A and B , to form chord $\overline{AB}$.
- ii. Draw the perpendicular bisector of chord $\overline{AB}$ which passes through mid point P of arc AB and midpoint R of $\overline{AB}$.
- iii. At point P construct a right angle TPR .
- iv. Produce $\overrightarrow{PT}$ in the direction of P beyond point S . Thus $\overleftrightarrow{TP}$ is the required tangent to the arc AB at point P .

Case (ii) When P is at end point of the arc

Given: P is the end point of arc PQR .



Steps of Construction:

- i. Take a point A on the arc PQR .
- ii. Join the point A and P .
- iii. Draw perpendicular $\overrightarrow{AS}$ at A which intersect the arc PQR at B .
- iv. Join the points B and P .

v. Draw $\angle APD$ of measure equal to that of $\angle ABP$.

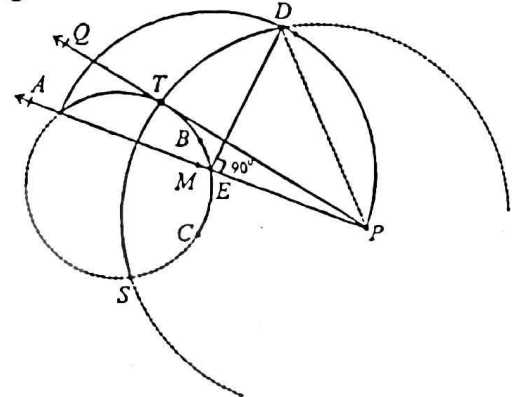
vi. Now

$$\begin{aligned} m\angle BPD &= m\angle BPA + m\angle APD \\ &= m\angle BPA + m\angle ABP \quad [\because m\angle APD = m\angle ABP] \\ &= 90^\circ \end{aligned}$$

vii. $\overleftrightarrow{PD}$ is the required tangent.

Case (iii) When point P is outside the arc.

Given: Point P is outside the arc ABC without knowing its centre.

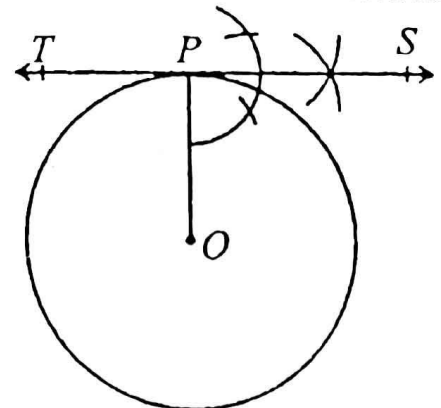


Steps of Construction:

- i. Join A to P . $\overline{AP}$ cuts the arc at E .
- ii. Find mid-point M of $\overline{AP}$.
- iii. Draw a semi circle of radius $|AM| = |MP|$ with center at M .
- iv. Draw perpendicular at point E which meets the semi circle at D .
- v. Draw an arc of radius $\overline{PD}$ with P as its center.
- vi. This arc cuts the given arc ABC at point T .
- vii. Join P with T .
- viii. $\overleftrightarrow{PTQ}$ is the required tangent.

(2) To draw a tangent to a circle at a given point P on the circumference:

Given: A circle with the centre O and some point P lies on the circumference.



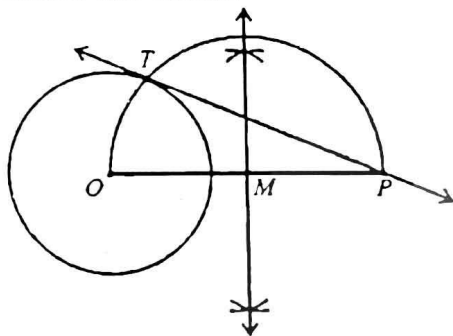
Steps of Construction:

- i. Join point P to the centre O , so that $\overline{OP}$ is the radius of the circle.
- ii. Draw a line TPS which is perpendicular to the radius $\overline{OP}$.

$\therefore$ $\overleftrightarrow{\text{TPS}}$ is the required tangent to the circle at given point P.

(3) To draw a tangent to a circle from a given point P which lies outside the circle.

Given: A circle with centre O and some point P outside the circle.

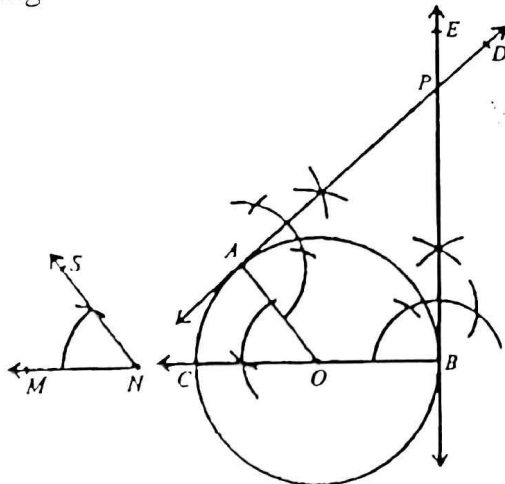


Steps of Construction:

- i. Join point P to the centre O.
- ii. Find M, the midpoint of a $\overline{OP}$ by drawings its right bisector.
- iii. Construct a semi circle of radius $\overline{OM}$, with M as its centre. This semi circle cuts the given circle at T.
- iv. Join P with T and produce $\overline{PT}$ on both sides, then $\overleftrightarrow{PT}$ is the required tangent.

(4) To draw two tangents to a circle meeting each other at a given angle:

Given: A circle with centre O, $\angle MNS$ is a given angle.



Steps of Construction:

- i. Take a point A on the circumference of circle having centre O.
- ii. Join the points O and A.
- iii. Draw $\angle COA$ of measure equal to that of $\angle MNS$.
- iv. Produce $\overline{CO}$ to meet the circle at B.
- v. $m\angle AOB = 180^\circ - m\angle COA$.
- vi. Draw $\overleftrightarrow{AD}$ perpendicular to $\overline{OA}$.
- vii. Draw $\overleftrightarrow{BE}$ perpendicular to $\overline{OB}$.
- viii. $\overleftrightarrow{AD}$ and $\overleftrightarrow{BE}$ intersect at P.
- ix. $m\angle AOB + m\angle APB = 180^\circ$, that is,
 $m\angle AOB = 180^\circ - m\angle APB$.
- x. From step (v) and step (ix), we have.

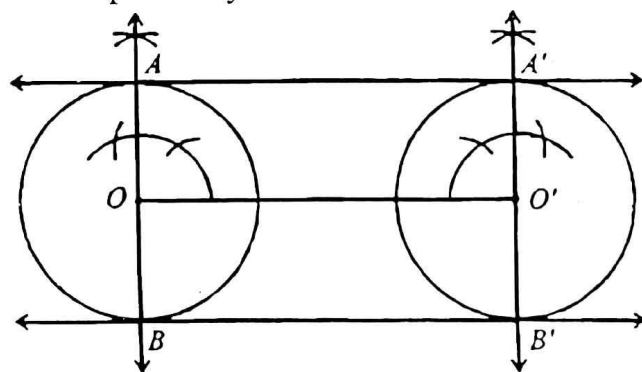
$$180^\circ - m\angle COA = 180^\circ - m\angle APB$$

$$\Rightarrow m\angle COA = m\angle APB$$

$$\Rightarrow m\angle APB = m\angle MNS \quad (\because m\angle COA = m\angle MNS)$$
- xi. $\overleftrightarrow{AP}$ and $\overleftrightarrow{BP}$ are the required tangents meeting at the given $\angle MNS$.

(5) To draw direct (or external) common tangents to equal circles.

Given: Two circles of equal radii with centres O and O' respectively.

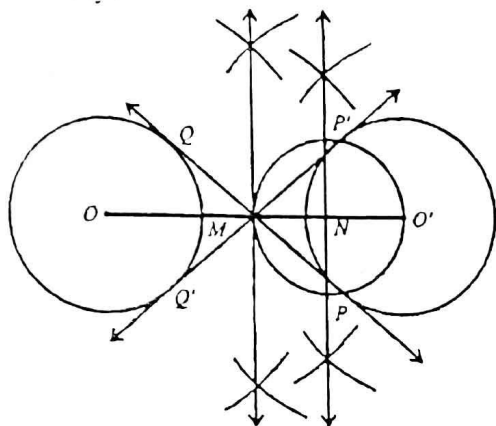


Steps of Construction:

- i.** Join the centre O and O'.
- ii.** Draw diameter AOB of the first circle so that $\overline{AOB} \perp \overline{OO'}$.
- iii.** Draw diameter A'O'B' of the second circle so that $\overline{A'O'B'} \perp \overline{OO'}$.
- iv.** Draw $\overleftrightarrow{AA'}$ and $\overleftrightarrow{BB'}$ which are the required common tangents.

(6) To draw transverse or (internal) common tangents to two equal circles:

Given: Two equal circles with centres O and O' respectively.

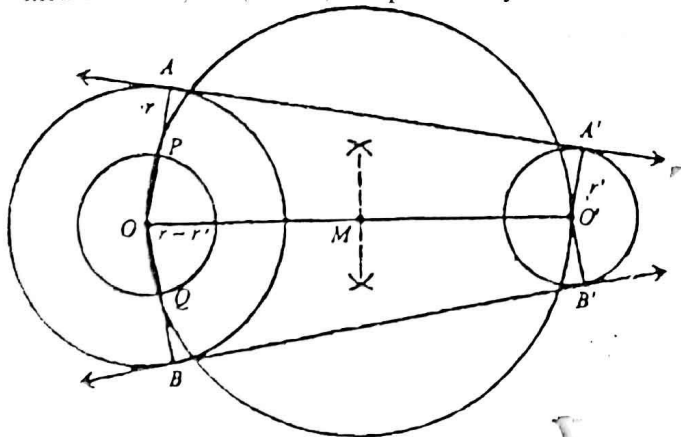


Steps of Construction:

- Join the centre O and O'.
 - Find mid-point M of $\overline{OO'}$.
 - Find mid-point N of $\overline{MO'}$.
 - Taking point N as centre and radius equal to $\overline{MN}$, draw a circle intersecting the circle with centre O' at point P and P'.
 - Draw a line through the points M and P touching the second circle at the point Q.
 - Draw a line through the points M and P' touching the second circle at the point Q'.
- Thus $\overleftrightarrow{PQ}$ and $\overleftrightarrow{P'Q'}$ are the required transverse common tangents to the given circles.

(7) To draw direct (or external) common tangents to (two) unequal circles:

Given: Two unequal circles with centres O, O' and radii r, r' ($r > r'$) respectively.

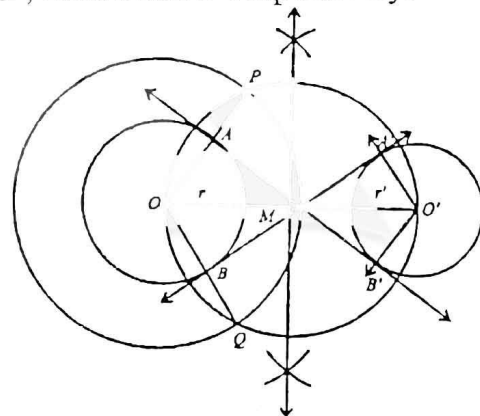


Steps of Construction:

- Join the points O and O'.
- On diameter $\overline{OO'}$, construct a new circle with centre M, the mid-point of $\overline{OO'}$.
- Draw another circle with centre at O and radius $= r - r'$, cutting the circle with diameter $\overline{OO'}$ at P and Q.
- Produce $\overline{OP}$ and $\overline{OQ}$ to meet the first circle at A and B respectively.
- Draw $\overrightarrow{O'A'} \parallel \overrightarrow{OA}$ and $\overrightarrow{O'B'} \parallel \overrightarrow{OB}$.
- Join AA' and BB' which are the required direct common tangents.

(8) To draw transverse or internal common tangents to two unequal circles:

Given: Two unequal circles with centres O and O', radii r and r' respectively.



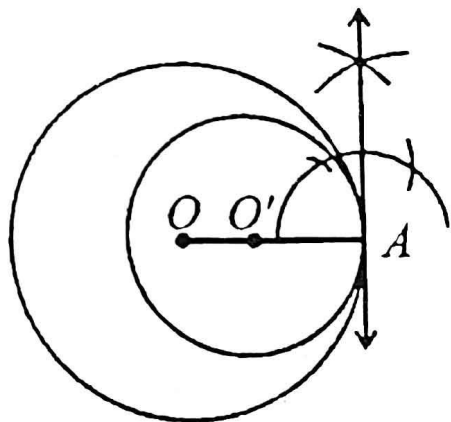
Steps of Construction:

- Join the centres O and O' of the given circles.
- Find the midpoint M of $\overline{OO'}$.
- On diameter $\overline{OO'}$, construct a new circle of radius $\overline{OM}$ with centre M.
- Draw another circle with centre at O and radius $= r + r'$ intersecting the circle of diameter $\overline{OO'}$ at P and Q.
- Join O with P and Q. $\overline{OP}$ and $\overline{OQ}$ meet the circle with radius r at A and at B respectively.
- Draw $\overrightarrow{O'B'} \parallel \overrightarrow{OA}$ and $\overrightarrow{O'A'} \parallel \overrightarrow{OB}$.
- Join A with B' and A' with B. Thus $\overleftrightarrow{AB}$ and $\overleftrightarrow{A'B'}$ are the required transverse common tangents.

(9) To draw a tangent to two unequal touching circles:

Case I:

Given: Two unequal touching circles with centres O and O' .

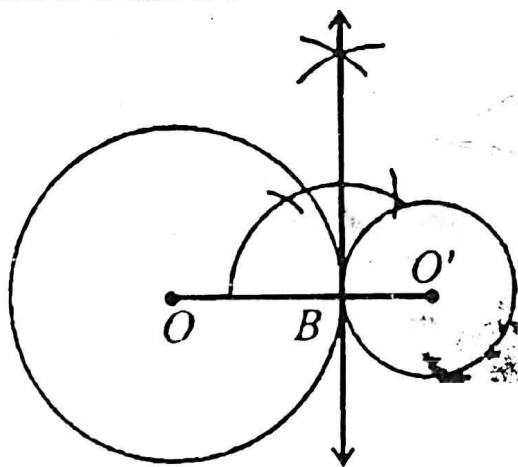


Steps of Construction:

- Join O with O' and produce OO' to meet the circles at the point A where these circles touch each other.
- Tangent is perpendicular to the line segment OA .
- Draw perpendicular to OA at the point A which is the required tangent.

Case II :

Given: Two unequal touching circles with centres O and O' .

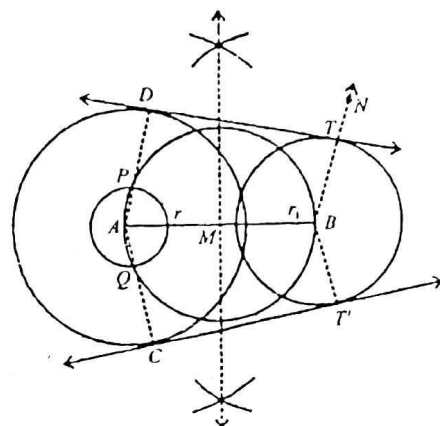


Steps of Construction:

- Join O with O' . OO' intersects the circles at the point B where these circles touch each other.
- Tangent is perpendicular to line segment containing the centre of the circles.
- Draw perpendicular to OO' at the point B which is the required tangent.

(10) To draw a tangent to two unequal intersecting circles:

Given: Two intersecting circles with centres A and B .



Steps of Construction:

- Take a line segment AB .
- Draw two circles of radii r and r_1 (where $r > r_1$) with centre at A and B respectively.
- Taking centre at A , draw a circle of radius $r - r_1$.
- Bisect the line segment AB at point M .
- Taking centre at M and radius $= \frac{1}{2}AB$, draw a circle intersecting the circle of radius $r - r_1$ at P and Q .
- Join the point A with P and produce it to meet the circle with centre A at D . Also join A with Q and produce it to meet the circle with centre A at C .
- Draw BN parallel to AD , which intersects the circle with centre B at T .
- Draw a line joining the points D and T . DT is a common tangent to the given two circles.
- Repeat the same process on the other side of AB . CT' is also a common tangent to the given two circles.

(11) To Draw a circle which touches both the arms of a given angle:

Given: An angle $\angle BAC$.

