

EXERCISE 2.6

Q.1 Use synthetic division to find the quotient and the remainder, when

(i) $(x^2 + 7x - 1) \div (x + 1)$

As $x + 1 = x - (-1)$ So $a = -1$

Now write the co-efficient of dividend in a row and $a = -1$ on the left side

	1	7	-1
-1	↓	-1	-6
	1	6	-7

Quotient $Q(x) = x + 6$

Remainder $R = -7$

(ii) $(4x^3 - 5x + 15) \div (x + 3)$

$(4x^3 + 0x^2 - 5x + 15) \div (x + 3)$

or As $x + 3 = x - (-3)$, So $a = -3$

Now write the co-efficient of dividend in a row and $a = -3$ on the left side

	4	0	-5	15
-3	↓	-12	36	-93
	4	-12	31	-78

Quotient = $Q(x) = 4x^2 - 12x + 31$

Remainder = $R = -78$

(iii) $(x^3 + x^2 - 3x + 2) \div (x - 2)$

As $(x - 2)$ So $a = 2$

Now write the co-efficient of dividend in a row and $a = 2$ on the left side.

	1	1	-3	2
2	↓	2	6	6
	1	3	3	8

Quotient $Q(x) = x^2 + 3x + 3$

Remainder $R = 8$

Q.2 Find the value of h using synthetic division, if 3 is the zero of the polynomial $2x^3 - 3hx^2 + 9$

(i) **Solution:** Let $P(x) = 2x^3 - 3hx^2 + 0x + 9$ and its zero is 3. Then by synthetic division.

	2	-3h	0	9
3	↓	6	18-9h	54-27h
	2	6-3h	18-9h	63-27h

Remainder = $63 - 27h$

Since 3 is the zero of the polynomial, therefore Remainder = 0

$63 - 27h = 0$

$63 = 27h$

$\Rightarrow h = \frac{63}{27} \Rightarrow \boxed{h = \frac{7}{3}}$

(ii) Find the value of h using synthetic division, if 1 is the zero of the polynomial $x^3 - 2hx^2 + 11$

Solution:

Let $P(x) = x^3 - 2hx^2 + 0x + 11$ and its zero is 1.

Then by synthetic division

	1	-2h	0	11
1	↓	1	1-2h	1-2h
	1	1-2h	1-2h	12-2h

Remainder = $12 - 2h$

Since 1 is the zero of the polynomial

So, Remainder = 0 that is

$12 - 2h = 0$

$12 = 2h$

$\Rightarrow h = \frac{12}{2}$

$\boxed{h = 6}$

(iii) Find the value of h using synthetic division, if -1 is the zero of the Polynomial $2x^3 + 5hx - 23$

Solution:

Let $P(x) = 2x^3 + 5hx - 23$

$$P(x) = 2x^3 + 0x^2 + 5hx - 23$$

If -1 is zero of $p(x)$ then by Synthetic division

	2	0	5h	-23
-1	↓	-2	2	-5h-2
	2	-2	5h+2	-5h-25

Remainder = $-5h - 25$

Since -1 is the zero of the Polynomial

So Remainder = 0

$$-5h - 25 = 0$$

$$-5h = 25$$

$$h = \frac{25}{-5} \Rightarrow \boxed{h = -5}$$

Q.3 Use synthetic division to find the values of l and m ,

(i). if $(x+3)$ and $(x-2)$ are the factors of the polynomial $x^3 + 4x^2 + 2lx + m$

Solution: Since $(x+3)$ and $(x-2)$ are the factors of $P(x) = x^3 + 4x^2 + 2lx + m$

Therefore -3 and 2 are the zeros of polynomial $P(x)$. Now by synthetic division.

	1	4	2l	m
-3	↓	-3	-3	-6l+9
	1	1	2l-3	m+(-6l+9)

Since -3 is the zero of polynomial, therefore remainder is zero that is

$$m - 6l + 9 = 0$$

$$\Rightarrow m - 6l = -9 \dots\dots\dots(i)$$

And

	1	4	2l	m
2	↓	2	12	4l+24
	1	6	2l+12	m+4l+24

Since 2 is the zero of polynomial, therefore remainder is zero that is

$$m + 4l + 24 = 0$$

$$m + 4l = -24 \dots\dots\dots(ii)$$

Subtracting equations (ii) from (i)

$$\cancel{m} - 6l = -9$$

$$\pm \cancel{m} \pm 4l = \mp 24$$

$$-10l = 15$$

$$l = \frac{15}{-10} = \frac{3}{-2}$$

$$\Rightarrow \boxed{l = -\frac{3}{2}}$$

Put it in equations (i), we get

$$m - 6\left(\frac{-3}{2}\right) = -9$$

$$m + \frac{18}{2} = -9$$

$$m + 9 = -9$$

$$m = -9 - 9$$

$$\boxed{m = -18}$$

(ii). Find the values of l and m if $(x-1)$ and $(x+1)$ are the factors of the polynomial $x^3 - 3lx^2 + 2mx + 6$

Solution: Since $(x-1)$ and $(x+1)$ are the factors of $P(x) = x^3 - 3lx^2 + 2mx + 6$

Therefore 1 and -1 are zeros of polynomial $P(x)$. Now by synthetic division

	1	-3l	2m	6
1	↓	1	1-3l	1-3l+2m
	1	1-3l	1-3l+2m	7-3l+2m

Since 1 is the zero of polynomial, therefore remainder is zero that is

$$7 - 3l + 2m = 0$$

$$2m - 3l = -7 \dots\dots\dots(i)$$

And

	1	-3l	2m	6
-1	↓	-1	1+3l	-1-3l-2m
	1	-1-3l	1+3l+2m	5-3l-2m

Since -1 is the zero of polynomial therefore remainder is zero that is

$$5 - 3\ell - 2m = 0$$

$$\Rightarrow 2m + 3\ell = 5 \dots\dots\dots(ii)$$

Adding equations (i) and (ii)

$$2m - 3\ell = -7$$

$$2m + 3\ell = 5$$

$$\hline 4m = -2$$

$$m = \frac{-2}{4}$$

$$\boxed{m = \frac{-1}{2}}$$

Put it in equation (i),

$$2\left(-\frac{1}{2}\right) - 3\ell = -7$$

$$-1 - 3\ell = -7$$

$$-3\ell = -7 + 1$$

$$-3\ell = -6$$

$$\Rightarrow \ell = \frac{-6}{-3}$$

$$\Rightarrow \boxed{\ell = 2}$$

Q.4 Solve by using synthetic division,

(i) If 2 is the root of the equation $x^3 - 28x + 48 = 0$

Solution: Let $P(x) = x^3 + 0x^2 - 28x + 48$

Since 2 is the root of the equation

$x^3 - 28x + 48 = 0$ then by synthetic division.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -28 & 48 \\ & \downarrow & 2 & 4 & -48 \\ \hline & 1 & 2 & -24 & 0 \end{array} \quad \text{02(080)}$$

The depressed equation is

$$x^2 + 2x - 24 = 0$$

$$x^2 + 6x - 4x - 24 = 0$$

$$x(x + 6) - 4(x + 6) = 0$$

$$(x + 6)(x - 4) = 0$$

$$\begin{array}{lll} \text{Either} & x + 6 = 0 & \text{or} & x - 4 = 0 \\ & x = -6 & \text{or} & x = 4 \end{array}$$

Thus 2, -6 and 4 are the roots of the given equation

(ii). If 3 is the root of the equation

$$2x^3 - 3x^2 - 11x + 6 = 0$$

Solution: Since 3 is the root of the equation

$$2x^3 - 3x^2 - 11x + 6 = 0$$

Then by synthetic division

$$\begin{array}{r|rrrr} & 2 & -3 & -11 & 6 \\ 3 & \downarrow & 6 & 9 & -6 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$

The depressed equation is

$$2x^2 + 3x - 2 = 0$$

$$2x^2 + 4x - x - 2 = 0$$

$$2x(x + 2) - 1(x + 2) = 0$$

$$(x + 2)(2x - 1) = 0$$

$$\text{Either } x + 2 = 0 \quad \text{or} \quad 2x - 1 = 0$$

$$\boxed{x = -2} \quad \text{or} \quad 2x = 1$$

$$\boxed{x = \frac{1}{2}}$$

Thus, 3, -2 and $\frac{1}{2}$ are the roots of the given equation.

(iii). If -1 is the root of the equation

$$4x^3 - x^2 - 11x - 6 = 0$$

Solution: Since -1 is the root of the equation.

$$4x^3 - x^2 - 11x - 6 = 0$$

Then by synthetic division

$$\begin{array}{r|rrrr} & 4 & -1 & -11 & -6 \\ -1 & \downarrow & -4 & 5 & 6 \\ \hline & 4 & -5 & -6 & 0 \end{array}$$

The depressed equation is

$$4x^2 - 5x - 6 = 0$$

$$4x^2 - 8x + 3x - 6 = 0$$

$$4x(x - 2) + 3(x - 2) = 0$$

$$(x - 2)(4x + 3) = 0$$

$$\text{Either } x - 2 = 0 \quad \text{or} \quad 4x + 3 = 0$$

$$\boxed{x = 2} \quad \text{or} \quad 4x = -3$$

$$\boxed{x = \frac{-3}{4}}$$

Thus -1, 2 and $\frac{-3}{4}$ are the roots of the given equation

Q.5

- (i) Solve by using synthetic division, if 1 and 3 are the roots of the equation $x^4 - 10x^2 + 9 = 0$

Solution: Since 1 and 3 are the roots of the equation $x^4 - 10x^2 + 9 = 0$

Then by synthetic division, we get

	1	0	-10	0	9
1	↓	1	1	-9	-9
	1	1	-9	-9	0
3	↓	3	12	9	
	1	4	3	0	

Thus the depressed equation is

$$x^2 + 4x + 3 = 0$$

$$x + 3x + x + 3 = 0$$

$$x(x+3) + 1(x+3) = 0$$

$$(x+3)(x+1) = 0$$

Either $x+3=0$ or $x+1=0$

$$\boxed{x = -3}$$

$$\boxed{x = -1}$$

Hence 1, 3, -3 and -1 are the roots of the given equation

- (ii) Solve by using synthetic division, if 3 and -4 are the roots of the equation

$$x^4 + 2x^3 - 13x^2 - 14x + 24 = 0 \quad 02(084)$$

Solution: Since 3 and -4 are the roots of the given equation $x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$ then by synthetic division, we get

	1	2	-13	-14	24
3	↓	3	15	6	-24
	1	5	2	-8	0
-4	↓	-4	-4	8	
	1	1	-2	0	

The depressed equation is

$$x^2 + x - 2 = 0$$

$$x^2 + 2x - x - 2 = 0$$

$$x(x+2) - 1(x+2) = 0$$

$$(x+2)(x-1) = 0$$

Either $x+2=0$ or $x-1=0$

$$\boxed{x = -2}$$

$$\boxed{x = 1}$$

Thus 3, -4, -2 and 1 are the roots of the given equation.

Simultaneous equations

A system of equations having a common solution is called a system of **simultaneous equations**.

The set of all the ordered pairs (x, y), which satisfies the system of equations is called the **solution set** of the system.

(i) Solve a system of two equations in two variables

(a) When one equation is linear and the other is quadratic.

To solve a system of equations in two variables x and y. Find 'y' in terms of x from the given linear equation. Substitute the value of y in the quadratic equation we get an other quadratic equation in one variable x. Solve this equation for x and then find the values of y.

Example 1:

Solve the system of equations

$$3x + y = 4 \quad \text{and} \quad 3x^2 + y^2 = 52.$$

Solution:

The given equations are

$$3x + y = 4 \quad \dots\dots\dots(i)$$

$$\text{and} \quad 3x^2 + y^2 = 52 \quad \dots\dots\dots(ii)$$

From equation (i)

$$y = 4 - 3x \quad \dots\dots\dots(iii)$$

Put value of y in equation (ii)

$$3x^2 + (4 - 3x)^2 = 52$$

$$3x^2 + [(4)^2 - 2(4)(3x) + (3x)^2] - 52 = 0$$

$$3x^2 + [16 - 24x + 9x^2] - 52 = 0$$

$$3x^2 + 16 - 24x + 9x^2 - 52 = 0$$

$$12x^2 - 24x - 36 = 0$$

$$12(x^2 - 2x - 3) = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0 \quad (\because 12 \neq 0)$$

By factorization

$$x^2 - 3x + x - 3 = 0$$

$$x(x-3) + 1(x-3) = 0$$

$$\Rightarrow (x-3)(x+1) = 0$$

$$\text{Either } x-3=0 \quad \text{or} \quad x+1=0$$

$$x=3 \quad \text{or} \quad x=-1$$

Put the values of x in equation (iii)

$$\text{When } x = 3$$

$$y = 4 - 3x$$

$$y = 4 - 3(3)$$

$$y = 4 - 9$$

$$y = -5$$

$$\text{When } x = -1$$

$$y = 4 - 3x$$

$$y = 4 - 3(-1)$$

$$y = 4 + 3$$

$$y = 7$$

$\therefore$ The ordered pairs are $(3, -5)$ and $(-1, 7)$

Thus, the solution set is $\{(3, -5), (-1, 7)\}$

(b) When both the equations are quadratic.

Example 2: Solve the equations

$$x^2 + y^2 + 2x = 8 \text{ and } (x-1)^2 + (y+1)^2 = 8$$

Solution: The given equations are

$$x^2 + y^2 + 2x = 8 \dots\dots\dots(i)$$

$$(x-1)^2 + (y+1)^2 = 8 \dots\dots\dots(ii)$$

From equation (ii), we get

$$x^2 - 2x + 1 + y^2 + 2y + 1 = 8$$

$$x^2 + y^2 - 2x + 2y = 8 - 2$$

$$x^2 + y^2 - 2x + 2y = 6 \dots\dots\dots(iii)$$

Subtracting equation (iii) from equation (i), we have

$$4x - 2y = 2$$

$$2(2x - y) = 2$$

$$2x - y = 1$$

$$\Rightarrow y = 2x - 1 \dots\dots\dots(iv)$$

Put the value of y in equation (ii)

$$(x-1)^2 + (2x-1+1)^2 = 8$$

$$(x)^2 - 2(x)(1) + (1)^2 + (2x)^2 = 8$$

$$x^2 - 2x + 1 + 4x^2 - 8 = 0$$

$$5x^2 - 2x - 7 = 0$$

$$5x^2 - 7x + 5x - 7 = 0$$

$$\text{or } x(5x-7) + 1(5x-7) = 0$$

$$\Rightarrow (5x-7)(x+1) = 0$$

$$\text{Either } 5x-7=0 \quad \text{or} \quad x+1=0$$

$$5x=7 \quad \text{or} \quad x=-1$$

$$\Rightarrow x = \frac{7}{5}$$

Now putting the values of x in equation (iv), we have

$$\text{When } x = \frac{7}{5}$$

$$y = 2\left(\frac{7}{5}\right) - 1$$

$$y = \frac{14}{5} - 1$$

$$y = \frac{14-5}{5} = \frac{9}{5}$$

$$\text{When } x = -1$$

$$y = 2(-1) - 1$$

$$y = -2 - 1$$

$$y = -3$$

Thus, the solution set is $\left\{(-1, -3), \left(\frac{7}{5}, \frac{9}{5}\right)\right\}$

Example 3: Solve the equations

$$x^2 + y^2 = 7 \text{ and } 2x^2 + 3y^2 = 18$$

Solution: Given equations are

$$x^2 + y^2 = 7 \dots\dots\dots(i)$$

$$2x^2 + 3y^2 = 18 \dots\dots\dots(ii)$$

Multiply equation (i) by 3

$$3x^2 + 3y^2 = 21 \dots\dots\dots(iii)$$

Subtracting equations (ii) from (iii)

$$3x^2 + 3y^2 = 21$$

$$\underline{\pm 2x^2 \pm 3y^2 = \pm 18}$$

$$\underline{x^2} = 3$$

$$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$$

Now putting the values of $x = \pm\sqrt{3}$, in equation (i)

We have

$$x^2 + y^2 = 7$$

$$3 + y^2 = 7$$

$$y^2 = 7 - 3$$

$$\Rightarrow y^2 = 4$$

$$y = \pm 2$$

Thus, the required solution set is $\{(\pm\sqrt{3}, \pm 2)\}$

Example 4: Solve the equations

$$x^2 + y^2 = 20 \text{ and } 6x^2 + xy - y^2 = 0$$

Solution: Given equations are:

$$x^2 + y^2 = 20 \dots\dots\dots(i)$$

$$6x^2 + xy - y^2 = 0 \dots\dots\dots(ii)$$

The equation (ii) can be written as

$$y^2 - xy - 6x^2 = 0$$

Here we take "x" as constant and "y" as variable

$$a = 1, b = -x, c = -6x^2$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow y = \frac{-(-x) \pm \sqrt{(-x)^2 - 4 \times 1 \times (-6x^2)}}{2 \times 1}$$

$$y = \frac{x \pm \sqrt{x^2 + 24x^2}}{2} = \frac{x \pm \sqrt{25x^2}}{2}$$

$$y = \frac{x \pm 5x}{2}$$

Either,

$$y = \frac{x + 5x}{2} \quad \text{or} \quad y = \frac{x - 5x}{2}$$

$$y = \frac{6x}{2} \quad \text{or} \quad y = \frac{-4x}{2}$$

$$\boxed{y = 3x} \dots \text{(iii)} \quad \text{or} \quad \boxed{y = -2x} \dots \text{(iv)}$$

Putting the values of y in equation (i), we get

When $y = 3x$

$$x^2 + (3x)^2 = 20$$

$$x^2 + 9x^2 = 20$$

$$10x^2 = 20$$

$$\Rightarrow x^2 = 2$$

$$x = \pm\sqrt{2}$$

Put $x = \sqrt{2}$ in eq. (iii)

$$y = 3(\sqrt{2}) = 3\sqrt{2}$$

Now put $x = -\sqrt{2}$ in eq.iii

$$y = 3(-\sqrt{2}) = -3\sqrt{2}$$

When $y = -2x$

$$x^2 + (-2x)^2 = 20$$

$$x^2 + 4x^2 = 20$$

$$5x^2 = 20$$

$$x^2 = 4$$

$$x = \pm 2$$

Put $x = 2$ in eq. (iv)

$$y = -2(2) = -4$$

Now put $x = -2$ in eq. (iv)

$$y = -2(-2) = 4$$

Thus, the solution is

$$\{(\sqrt{2}, 3\sqrt{2}), (-\sqrt{2}, -3\sqrt{2}), (2, -4), (-2, 4)\}$$

Example 5: Solve the equations

$$x^2 + y^2 = 40 \quad \text{and} \quad 3x^2 - 2xy - y^2 = 80$$

Solution: Given equations are

$$x^2 + y^2 = 40 \dots \text{(i)}$$

$$3x^2 - 2xy - y^2 = 80 \dots \text{(ii)}$$

Multiplying equation (i) by 2, we have

$$2x^2 + 2y^2 = 80 \dots \text{(iii)}$$

Subtracting the equation (iii) from equation (ii), we get

$$3x^2 - 2xy - y^2 = 80$$

$$\pm 2x^2 \quad \pm 2y^2 = \pm 80$$

$$x^2 - 2xy - 3y^2 = 0 \dots \text{(iv)}$$

The equation (iv) can be written as

$$x^2 - 3xy + xy - 3y^2 = 0$$

$$\text{or} \quad x(x - 3y) + y(x - 3y) = 0$$

$$\Rightarrow (x - 3y)(x + y) = 0$$

$$\text{Either } x - 3y = 0 \quad \text{or} \quad x + y = 0$$

$$\boxed{x = 3y}$$

$$\text{or} \quad \boxed{x = -y}$$

Putting the values of x in equation (i)

When $x = 3y$

$$(3y)^2 + y^2 = 40$$

$$10y^2 = 40$$

$$y^2 = 4$$

$$y = \pm 2$$

Now putting these values of y in $x = 3y$

When $y = 2$

$$x = 3y$$

$$x = 3(2)$$

$$x = 6$$

$$(x, y) = (6, 2)$$

When $y = -2$

$$x = 3y$$

$$x = 3(-2)$$

$$x = -6$$

$$(x, y) = (-6, -2)$$

Now putting $x = -y$ in equation (i) we get

$$(-y)^2 + y^2 = 40$$

$$2y^2 = 40$$

$$y^2 = 20$$

$$y = \pm 2\sqrt{5}$$

Now putting these values of y in $x = -y$

When $y = 2\sqrt{5}$

$$x = -y$$

$$x = -(2\sqrt{5})$$

$$x = -2\sqrt{5}$$

When $y = -2\sqrt{5}$

$$x = -y$$

$$x = -(-2\sqrt{5})$$

$$x = 2\sqrt{5}$$

$\therefore$ The solution set is

$$\{(6, 2), (-6, -2), (2\sqrt{5}, -2\sqrt{5}), (-2\sqrt{5}, 2\sqrt{5})\}$$