

EXERCISE 2.7

Solve the following simultaneous equations.

Q. 1 $x + y = 5$
 $x^2 - 2y - 14 = 0$

Solution:

$$x + y = 5 \dots\dots\dots (i)$$

$$x^2 - 2y - 14 = 0 \dots\dots\dots (ii)$$

From equation (i)

$$x + y = 5$$

$$x = 5 - y$$

Put it in equation (ii)

$$(5 - y)^2 - 2y - 14 = 0$$

$$25 + y^2 - 10y - 2y - 14 = 0$$

$$y^2 - 12y + 11 = 0$$

$$y^2 - 11y - y + 11 = 0$$

$$y(y - 11) - 1(y - 11) = 0$$

$$(y - 11)(y - 1) = 0$$

Either $y - 11 = 0$ or $y - 1 = 0$

$y = 11$ or $y = 1$

Putting the values of y in eq(i)

When $y = 11$

$$x + y = 5$$

$$x + 11 = 5$$

$$x = 5 - 11$$

$$x = -6$$

When $y = 1$

$$x + y = 5$$

$$x + 1 = 5$$

$$x = 5 - 1$$

$$x = 4$$

Solution Set is $\{(-6, 11), (4, 1)\}$

Q. 2 $3x - 2y = 1$

$$x^2 + xy - y^2 = 1$$

Solution:

$$3x - 2y = 1 \dots\dots\dots (i)$$

$$x^2 + xy - y^2 = 1 \dots\dots\dots (ii)$$

From equation (i)

$$3x = 1 + 2y$$

$$x = \frac{1 + 2y}{3} \dots\dots\dots (iii)$$

Put it in equation (ii)

$$\left(\frac{1 + 2y}{3}\right)^2 + \left(\frac{1 + 2y}{3}\right)y - y^2 = 1$$

$$\frac{1 + 4y^2 + 4y}{9} + \frac{y + 2y^2}{3} - y^2 = 1$$

Multiplying by '9' on both sides we have

$$\cancel{9}(1 + 4y^2 + 4y) + \cancel{3}(y + 2y^2) - 9(y^2) = 9 \times 1$$

$$1 + 4y^2 + 4y + 3y + 6y^2 - 9y^2 = 9$$

$$y^2 + 7y - 8 = 0$$

$$y^2 + 8y - y - 8 = 0$$

$$y(y + 8) - 1(y + 8) = 0$$

$$(y + 8)(y - 1) = 0$$

Either

$$y + 8 = 0$$

or $y - 1 = 0$

$$y = -8$$

or $y = 1$

Putting these values in equation (iii)

when $y = -8$

$$x = \frac{1 + 2y}{3}$$

$$x = \frac{1 + 2(-8)}{3}$$

$$x = \frac{1 - 16}{3}$$

$$x = \frac{-15}{3} = -5$$

when $y = 1$

$$x = \frac{1 + 2y}{3}$$

$$x = \frac{1 + 2(1)}{3}$$

$$x = \frac{1 + 2}{3}$$

$$x = \frac{3}{3} = 1$$

Solution Set is $\{(-5, -8), (1, 1)\}$

Q. 3 $x - y = 7$

$$\frac{2}{x} - \frac{5}{y} = 2$$

Solution:

$$x - y = 7 \dots\dots\dots (i)$$

$$\frac{2}{x} - \frac{5}{y} = 2 \dots\dots\dots (ii)$$

Multiplying equation (ii) by "xy" we have

$$2y - 5x = 2xy \dots\dots\dots (iii)$$

From equation (i)

$$x = 7 + y$$

Put it in equation (iii)

$$2y - 5(7 + y) = 2(7 + y)y$$

$$2y - 35 - 5y = 14y + 2y^2$$

$$\Rightarrow 2y^2 + 17y + 35 = 0$$

$$2y^2 + 10y + 7y + 35 = 0$$

$$2y(y + 5) + 7(y + 5) = 0$$

$$(y + 5)(2y + 7) = 0$$

Either

$$y + 5 = 0$$

or $2y + 7 = 0$

$$y = -5$$

or $y = \frac{-7}{2}$

Now putting these values of y in equation (i)

When $y = -5$

$$x = 7 + y$$

$$x = 7 + (-5)$$

$$x = 7 - 5$$

$$x = 2$$

When $y = \frac{-7}{2}$

$$x = 7 + y$$

$$x = 7 + \left(\frac{-7}{2}\right)$$

$$x = 7 - \frac{7}{2}$$

$$x = \frac{14-7}{2}$$

$$x = \frac{7}{2}$$

Solution Set is $\left\{(2, -5), \left(\frac{7}{2}, \frac{-7}{2}\right)\right\}$

Q. 4 $x + y = a - b$

$$\frac{a}{x} - \frac{b}{y} = 2$$

Solution:

$$x + y = a - b \dots\dots\dots(i)$$

$$\frac{a}{x} - \frac{b}{y} = 2 \Rightarrow \frac{ay - bx}{xy} = 2$$

$$ay - bx = 2xy \dots\dots\dots(ii)$$

From equation (i)

$$x = a - b - y \dots\dots\dots(iii)$$

Put it in equation (ii)

$$ay - bx = 2xy$$

$$ay - b(a - b - y) = 2(a - b - y)y$$

$$ay - ba + b^2 + by = 2ay - 2by - 2y^2$$

$$2y^2 - 2ay + ay + 2by + by + b^2 - ab = 0$$

$$2y^2 - ay + 3by + b^2 - ab = 0$$

$$2y^2 - y(a - 3b) + (b^2 - ab) = 0$$

By using quadratic formula

$$a = 2, \quad b = -(a - 3b), \quad c = (b^2 - ab)$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-[-(a - 3b)] \pm \sqrt{[-(a - 3b)]^2 - 4(2)(b^2 - ab)}}{2(2)}$$

$$y = \frac{(a - 3b) \pm \sqrt{a^2 + 9b^2 - 6ab - 8b^2 + 8ab}}{4}$$

$$y = \frac{(a - 3b) \pm \sqrt{a^2 + b^2 + 2ab}}{4}$$

$$y = \frac{(a - 3b) \pm \sqrt{(a + b)^2}}{4}$$

$$y = \frac{(a - 3b) \pm (a + b)}{4}$$

$$y = \frac{\cancel{a} - 3b - \cancel{a} - b}{4} \quad \text{or} \quad y = \frac{a - 3b + a + b}{4}$$

$$y = \frac{-4b}{4} \quad \text{or} \quad y = \frac{2a - 2b}{4}$$

$$y = \frac{2(a - b)}{4}$$

$$y = -b \quad \text{or} \quad y = \frac{a - b}{2}$$

Putting these values of y in equation (iii)

when $y = -b$

$$x = a - b - y$$

$$x = a - b - (-b)$$

$$x = a - \cancel{b} + \cancel{b}$$

$$x = a$$

when $y = \frac{a - b}{2}$

$$x = a - b - y$$

$$x = a - b - \frac{a - b}{2}$$

$$x = \frac{2a - 2b - a + b}{2}$$

$$x = \frac{a - b}{2}$$

Solution Set is $\left\{(a, -b), \left(\frac{a - b}{2}, \frac{a - b}{2}\right)\right\}$

Q. 5 $x^2 + (y - 1)^2 = 10$

$x^2 + y^2 + 4x = 1$

Solution:

$x^2 + (y - 1)^2 = 10$ (i)

$x^2 + y^2 + 4x = 1$ (ii)

Subtracting equation (ii) from (i)

$\cancel{x^2} + \cancel{y^2} + 1 - 2y = 10$

$\pm \cancel{x^2} \pm \cancel{y^2} \pm 4x = \pm 1$

$-4x - 2y + 1 = 9$

$-4x - 2y = 9 - 1$

$-4x - 2y = 8$

$-2(2x + y) = 8$

$\Rightarrow 2x + y = \frac{8}{-2}$

$2x + y = -4$

$y = -4 - 2x$ (iii)

Put in equation (ii)

$x^2 + (-4 - 2x)^2 + 4x = 1$

$x^2 + [-(4 + 2x)]^2 + 4x = 1$

$x^2 + [16 + 4x^2 + 16x] + 4x = 1$

$5x^2 + 20x + 16 - 1 = 0$

$5x^2 + 20x + 15 = 0$

$5(x^2 + 4x + 3) = 0$

$\Rightarrow x^2 + 4x + 3 = 0$ ($\because 5 \neq 0$)

$x^2 + 3x + x + 3 = 0$

$x(x + 3) + 1(x + 3) = 0$

$(x + 3)(x + 1) = 0$

Either $x + 3 = 0$ or $x + 1 = 0$

$x = -3$ or $x = -1$

Putting these values of x in equation (iii)

when $x = -3$

$y = -4 - 2x$

$y = -4 - 2(-3)$

$y = -4 + 6$

$y = 2$

when $x = -1$

$y = -4 - 2x$

$y = -4 - 2(-1)$

$y = -4 + 2$

$y = -2$

So, the solution Set is $\{(-3, 2), (-1, -2)\}$

Q.6 $(x+1)^2 + (y+1)^2 = 5, (x+2)^2 + y^2 = 5$

Solution: $(x+1)^2 + (y+1)^2 = 5$ (i)

$(x+2)^2 + y^2 = 5$ (ii)

From equation (i)

$x^2 + 1 + 2x + y^2 + 1 + 2y = 5$

$x^2 + y^2 + 2x + 2y + 2 = 5$

$x^2 + y^2 + 2x + 2y = 3$ (iii)

From equation (ii)

$(x+2)^2 + y^2 = 5$

$x^2 + 4 + 4x + y^2 = 5$

$x^2 + y^2 + 4x = 5 - 4$

$x^2 + y^2 + 4x = 1$ (iv)

Subtracting equation (iv) from (iii)

$\cancel{x^2} + \cancel{y^2} + 2x + 2y = 3$

$\pm \cancel{x^2} \pm \cancel{y^2} \pm 4x = \pm 1$

$-2x + 2y = 2$

$-2(x - y) = 2$

$x - y = \frac{2}{-2}$

$x - y = -1$

$x = y - 1$ (v)

Put it in equation (iv)

$(y-1)^2 + y^2 + 4(y-1) = 1$

$y^2 + 1 - 2y + y^2 + 4y - 4 = 1$

$2y^2 + 2y - 4 + \cancel{1} - \cancel{1} = 0$

$2y^2 + 2y - 4 = 0$

$2(y^2 + y - 2) = 0$

$\Rightarrow y^2 + y - 2 = 0$ ($\because 2 \neq 0$)

$y^2 + y - 2 = 0$

$y^2 + 2y - y - 2 = 0$

$y(y + 2) - 1(y + 2) = 0$

$(y + 2)(y - 1) = 0$

Either $y + 2 = 0$ or $y - 1 = 0$

$y = -2$

$y = 1$

Putting these values of y in equation (v)

when $y = -2$

$x = y - 1$

$x = -2 - 1$

$x = -3$

when $y = 1$

$x = y - 1$

$x = 1 - 1$

$x = 0$

So, the solution Set is $\{(-3, -2), (0, 1)\}$

Q. 7 $x^2 + 2y^2 = 22$

$5x^2 + y^2 = 29$

Solution:

$x^2 + 2y^2 = 22$ (i)

$5x^2 + y^2 = 29$ (ii)

Multiplying equation (ii) by "2" we have

$10x^2 + 2y^2 = 58$ (iii)

Subtracting equation (i) from (iii)

$10x^2 + \cancel{2y^2} = 58$

$\pm x^2 \quad \pm \cancel{2y^2} = \pm 22$

$9x^2 = 36$

$x^2 = \frac{36}{9}$

$x^2 = 4$

Taking square root, we have

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

$\Rightarrow x = -2 \quad \text{or} \quad x = 2$

Now putting these values of x in equation (i)

When $x = -2$

$x^2 + 2y^2 = 22$

$(-2)^2 + 2y^2 = 22$

$4 + 2y^2 = 22$

$2y^2 = 22 - 4$

$2y^2 = 18$

$y^2 = \frac{18}{2}$

$y^2 = 9$

$\Rightarrow y = \pm 3$

When $x = 2$

$x^2 + 2y^2 = 22$

$(2)^2 + 2y^2 = 22$

$4 + 2y^2 = 22$

$2y^2 = 22 - 4$

$2y^2 = 18$

$y^2 = \frac{18}{2}$

$y^2 = 9$

$\Rightarrow y = \pm 3$

So, the solution set is $\{(\pm 2, \pm 3)\}$

Q.8 $4x^2 - 5y^2 = 6$

$3x^2 + y^2 = 14$

Solution:

$4x^2 - 5y^2 = 6$ (i)

$3x^2 + y^2 = 14$ (ii)

Multiplying equation (ii) by 5 and add in equation (i)

$4x^2 - \cancel{5y^2} = 6$

$15x^2 + \cancel{5y^2} = 70$

$19x^2 = 76$

$x = \frac{76}{19}$

$x^2 = 4$

$\Rightarrow x = \pm 2$

Either $x = 2$

or $x = -2$

Putting these values of x in equation (ii)

When $x = 2$,

$3(2)^2 + y^2 = 14$

$3(4) + y^2 = 14$

$12 + y^2 = 14$

$y^2 = 14 - 12$

$y^2 = 2$

$y = \pm\sqrt{2}$

When $x = -2$

$3(-2)^2 + y^2 = 14$

$3(4) + y^2 = 14$

$12 + y^2 = 14$

$y^2 = 14 - 12$

$y^2 = 2$

$y = \pm\sqrt{2}$

So the solution set is $\{(\pm 2, \pm\sqrt{2})\}$

Q. 9 $7x^2 - 3y^2 = 4$

$2x^2 + 5y^2 = 7$

Solution:

$7x^2 - 3y^2 = 4$ (i)

$2x^2 + 5y^2 = 7$ (ii)

Multiply equation (i) by 5 and equation (ii) by 3 and add them

$35x^2 - \cancel{15y^2} = 20$

$6x^2 + \cancel{15y^2} = 21$

$41x^2 = 41$

$x^2 = \frac{41}{41}$

$x^2 = 1$

$x = \pm\sqrt{1}$

$x = \pm 1$

Either $x = 1$

or $x = -1$

Putting these values of x in equation (i)

When $x = 1$

$$7(1)^2 - 3y^2 = 4$$

$$7 - 3y^2 = 4$$

$$-3y^2 = 4 - 7$$

$$-3y^2 = -3$$

$$y^2 = \frac{-3}{-3}$$

$$y^2 = 1$$

$$y = \pm\sqrt{1}$$

$$y = \pm 1$$

When $x = -1$

$$7(-1)^2 - 3y^2 = 4$$

$$7(1) - 3y^2 = 4$$

$$7 - 3y^2 = 4$$

$$-3y^2 = 4 - 7$$

$$-3y^2 = -3$$

$$y^2 = \frac{-3}{-3}$$

$$y^2 = 1$$

$$y = \pm 1$$

So the solution set is $\{(\pm 1, \pm 1)\}$

Q. 10 $x^2 + 2y^2 = 3$

$$x^2 + 4xy - 5y^2 = 0$$

Solution:

$$x^2 + 2y^2 = 3 \quad \text{————— (i)}$$

$$x^2 + 4xy - 5y^2 = 0 \quad \text{————— (ii)}$$

Factorizing equation (ii) we get

$$x^2 + 4xy - 5y^2 = 0$$

$$x^2 + 5xy - xy - 5y^2 = 0$$

$$x(x + 5y) - y(x + 5y) = 0$$

$$(x + 5y)(x - y) = 0$$

Either $x + 5y = 0$ or $x - y = 0$

$$x = -5y \quad \text{———— (iii)} \quad x = y \quad \text{———— (iv)}$$

Putting these values of x in equation (i)

When $x = -5y$

$$(-5y)^2 + 2y^2 = 3$$

$$25y^2 + 2y^2 = 3$$

$$27y^2 = 3$$

$$y^2 = \frac{3}{27}$$

$$y^2 = \frac{1}{9}$$

$$y = \pm\sqrt{\frac{1}{9}}$$

$$y = \pm\frac{1}{3}$$

$$y = \frac{1}{3} \text{ or } y = -\frac{1}{3}$$

When $x = y$

$$y^2 + 2(y^2) = 3$$

$$3y^2 = 3$$

$$y^2 = \frac{3}{3}$$

$$y^2 = 1$$

$$y = \pm\sqrt{1}$$

$$y = -1, \text{ or } y = 1$$

Putting the value of $y = \pm\frac{1}{3}$ in equation (iii)

When $y = \frac{1}{3}$

$$x = -5y$$

$$x = -5\left(\frac{1}{3}\right)$$

$$x = -\frac{5}{3}$$

When $y = -\frac{1}{3}$

$$x = -5y$$

$$x = -5\left(-\frac{1}{3}\right)$$

$$x = \frac{5}{3}$$

Now putting the values of $y = \pm 1$ in equation (iv)

When $y = 1$ then $x = 1$

When $y = -1$ then $x = -1$

Solution Set is $\{(-1, -1), (1, 1), \left(\frac{5}{3}, -\frac{1}{3}\right), \left(-\frac{5}{3}, \frac{1}{3}\right)\}$

Q. 11 $3x^2 - y^2 = 26$

$$3x^2 - 5xy - 12y^2 = 0$$

Solution:

$$3x^2 - y^2 = 26 \quad \text{————— (i)}$$

$$3x^2 - 5xy - 12y^2 = 0 \quad \text{————— (ii)}$$

Factorizing equation (ii)

$$3x^2 - 5xy - 12y^2 = 0$$

$$3x^2 - 9xy + 4xy - 12y^2 = 0$$

$$3x(x - 3y) + 4y(x - 3y) = 0$$

$$(x - 3y)(3x + 4y) = 0$$

Either

$$x - 3y = 0 \quad \text{or} \quad 3x + 4y = 0$$

$$\boxed{x = 3y} \quad \text{———— (iii)} \quad \boxed{x = -\frac{4y}{3}} \quad \text{———— (iv)}$$

From equation (iii) putting the value of x in equation (i)

$$3(3y)^2 - y^2 = 26$$

$$3(9y)^2 - y^2 = 26$$

$$27y^2 - y^2 = 26$$

$$26y^2 = 26$$

$$\boxed{y^2 = 1} \Rightarrow y = \pm 1$$

$$y = 1 \text{ or } y = -1$$

Putting these value of y in equation (iii)

When $y = 1$

$$x = 3y$$

$$x = 3(1)$$

$$x = 3$$

$$(x, y) = (3, 1)$$

When $y = -1$

$$x = 3y$$

$$x = 3(-1)$$

$$x = -3$$

$$(x, y) = (-3, -1)$$

From equation (iv) putting the values of x in equation (i)

$$3\left(\frac{-4y}{3}\right)^2 - y^2 = 26$$

$$3 \times \frac{16y^2}{9} - y^2 = 26$$

$$\frac{48y^2 - 9y^2}{9} = 26$$

$$39y^2 = 26 \times 9$$

$$y^2 = \frac{234}{39}$$

$$y^2 = 6$$

$\Rightarrow$

$$y = \pm\sqrt{6}$$

$$y = \sqrt{6} \text{ or } y = -\sqrt{6}$$

Putting these values of y in equation (iv)

When $y = \sqrt{6}$

$$x = \frac{-4y}{3}$$

$$x = \frac{-4\sqrt{6}}{3}$$

$$(x, y) = \left(\frac{-4\sqrt{6}}{3}, \sqrt{6}\right)$$

When $y = -\sqrt{6}$

$$x = \frac{-4y}{3}$$

$$x = \frac{-4(-\sqrt{6})}{3}$$

$$x = \frac{4\sqrt{6}}{3}$$

$$(x, y) = \left(\frac{4\sqrt{6}}{3}, -\sqrt{6}\right)$$

So, the Solution set is

$$\left\{(3, 1), (-3, -1), \left(\frac{-4\sqrt{6}}{3}, \sqrt{6}\right), \left(\frac{4\sqrt{6}}{3}, -\sqrt{6}\right)\right\}$$

$$\text{Q. 12 } x^2 + xy = 5 \text{ ————— (i)}$$

$$y^2 + xy = 3 \text{ ————— (ii)}$$

Multiply equation (i) by 3 and equation (ii) by 5 and subtract them

$$3x^2 + 3xy = 15$$

$$\underline{\quad \quad \quad \pm 5xy \pm 5y^2 = -15 \quad \quad \quad}$$

$$3x^2 - 2xy - 5y^2 = 0$$

$$3x^2 - 5xy + 3xy - 5y^2 = 0$$

$$x(3x - 5y) + y(3x - 5y) = 0$$

Either

$$(3x - 5y)(x + y) = 0$$

$$3x - 5y = 0$$

$$\text{or } x + y = 0$$

$$3x = 5y$$

$$\text{or } \boxed{x = -y} \dots \text{(iv)}$$

$$\boxed{x = \frac{5y}{3}} \text{ ————— (iii)}$$

From equation (iv) put $y = -x$ in equation (i)

$$(-y)^2 + (-y)y = 5$$

$$y^2 - y^2 = 5$$

$$0 \neq 5$$

Impossible

Now from equation (iii) put $x = \frac{5y}{3}$ in equation (i)

$$\left(\frac{5y}{3}\right)^2 + \frac{5y}{3} \times y = 5$$

$$\frac{25y^2}{9} + \frac{5y^2}{3} = 5$$

Multiply by 9

$$9 \times \frac{25y^2}{9} + 9 \times \frac{5y^2}{3} = 9 \times 5$$

$$25y^2 + 15y^2 = 45$$

$$40y^2 = 45$$

$$y^2 = \frac{45}{40}$$

$$y^2 = \frac{9}{8}$$

$$y = \pm\sqrt{\frac{9}{8}}$$

$$= \pm \sqrt{\frac{3^2}{4 \times 2}}$$

$$y = \pm \frac{3}{2\sqrt{2}}$$

$$y = \frac{3}{2\sqrt{2}} \text{ or } y = \frac{-3}{2\sqrt{2}}$$

Now putting the value of y in equation (iii)

$$\text{When } y = \frac{3}{2\sqrt{2}}$$

$$\text{When } y = \frac{-3}{2\sqrt{2}}$$

$$\text{Then } x = \frac{5}{\cancel{\beta}} \times \frac{\cancel{\beta}}{2\sqrt{2}}$$

$$\text{Then } x = \frac{5}{\cancel{\beta}} \times \left(\frac{-\cancel{\beta}}{2\sqrt{2}} \right)$$

$$x = \frac{5}{2\sqrt{2}}$$

$$x = \frac{-5}{2\sqrt{2}}$$

$$\left(\frac{5}{2\sqrt{2}}, \frac{3}{2\sqrt{2}} \right)$$

$$\left(\frac{-5}{2\sqrt{2}}, \frac{-3}{2\sqrt{2}} \right)$$

$$\text{Solution set is } \left\{ \left(\frac{5}{2\sqrt{2}}, \frac{3}{2\sqrt{2}} \right), \left(\frac{-5}{2\sqrt{2}}, \frac{-3}{2\sqrt{2}} \right) \right\}$$

$$\text{Q. 13 } x^2 - 2xy = 7$$

$$xy + 3y^2 = 2$$

Solution:

$$x^2 - 2xy = 7 \text{ ————— (i)}$$

$$xy + 3y^2 = 2 \text{ ————— (ii)}$$

Multiplying equation (i) by 2 and equation (ii) by 7 and subtracting them, we get

$$2x^2 - 4xy = 14 \quad \checkmark$$

$$\pm 7xy \pm 21y^2 = 14 \quad \checkmark$$

$$2x^2 - 11xy - 21y^2 = 0$$

$$2x^2 - 14xy + 3xy - 21y^2 = 0$$

$$2x(x - 7y) + 3y(x - 7y) = 0$$

$$(x - 7y)(2x + 3y) = 0 \quad \checkmark$$

$$\text{Either } x - 7y = 0 \quad \text{or} \quad 2x + 3y = 0$$

$$\boxed{x = 7y} \text{ ————— (iii) or } 2x = -3y$$

$$\text{or } \boxed{x = \frac{-3}{2}y} \text{ ————— (iv)}$$

From equation (iii) Put $x = 7y$ in equation (i)

$$(7y)^2 - 2(7y)y = 7$$

$$49y^2 - 14y^2 = 7$$

$$35y^2 = 7$$

$$y^2 = \frac{7}{35}$$

$$y^2 = \frac{1}{5}$$

$$\boxed{y = \pm \frac{1}{\sqrt{5}}}$$

$$\text{Either } y = \frac{1}{\sqrt{5}} \text{ or } y = \frac{-1}{\sqrt{5}}$$

Putting these values of y in equation (iii)

$$\text{When } y = \frac{1}{\sqrt{5}}$$

$$\text{When } y = \frac{-1}{\sqrt{5}}$$

$$x = 7y$$

$$x = 7y$$

$$\text{Then } x = 7 \left(\frac{1}{\sqrt{5}} \right)$$

$$\text{Then } x = 7 \left(\frac{-1}{\sqrt{5}} \right)$$

$$\boxed{x = \frac{7}{\sqrt{5}}}$$

$$\boxed{x = \frac{-7}{\sqrt{5}}}$$

$$(x, y) = \left(\frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right)$$

$$(x, y) = \left(\frac{-7}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right)$$

From equation (iv) putting the value of x in equation (i)

$$\left(\frac{-3}{2}y \right)^2 - 2 \left(\frac{-3}{2}y \right)y = 7$$

$$\frac{9}{4}y^2 + 3y^2 = 7$$

$$9y^2 + 12y^2 = 28$$

$$21y^2 = 28$$

$$y^2 = \frac{28}{21}$$

$$y^2 = \frac{4}{3}$$

$$\sqrt{y^2} = \pm \sqrt{\frac{4}{3}}$$

$$\boxed{y = \pm \frac{2}{\sqrt{3}}}$$

$$\text{Either } y = \frac{2}{\sqrt{3}} \text{ or } y = \frac{-2}{\sqrt{3}}$$

Putting these values of y in equation (iv)

$$\text{When } y = \frac{2}{\sqrt{3}}$$

$$\text{Then } x = \frac{-3}{2} \left(\frac{2}{\sqrt{3}} \right)$$

$$x = -\sqrt{3}$$

$$(x, y) = \left(-\sqrt{3}, \frac{2}{\sqrt{3}} \right)$$

$$\text{When } y = \frac{-2}{\sqrt{3}}$$

$$\text{Then } x = -\frac{3}{2} \left(\frac{-2}{\sqrt{3}} \right)$$

$$x = \sqrt{3}$$

$$(x, y) = \left(\sqrt{3}, \frac{-2}{\sqrt{3}} \right)$$

So, the Solution set is

$$\left\{ \left(\frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right), \left(\frac{-7}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right), \left(-\sqrt{3}, \frac{2}{\sqrt{3}} \right), \left(\sqrt{3}, \frac{-2}{\sqrt{3}} \right) \right\}$$

Problems Leading to Quadratic Equations

Example 1: Three less than a certain number multiplied by 9 less than twice the number is 104: Find the number.

Solution: Let the required number be x . Then
three less than the number $= x - 3$
and 9 less than twice the number $= 2x - 9$
According to the given condition, we have

$$(x - 3)(2x - 9) = 104$$

$$2x^2 - 9x - 6x + 27 = 104$$

$$2x^2 - 15x + 27 - 104 = 0$$

$$2x^2 - 15x - 77 = 0$$

Factorizing, we get

$$2x^2 - 22x + 7x - 77 = 0$$

$$2x(x - 11) + 7(x - 11) = 0$$

$$(2x + 7)(x - 11) = 0$$

$$\text{Either } (2x + 7) = 0 \text{ or } (x - 11) = 0$$

$$\Rightarrow x = -\frac{7}{2} \text{ or } x = 11$$

i.e., $x = -\frac{7}{2}$ and 11 are the required numbers.

Example 2: The length of a rectangle is 4cm more than its breadth. If the area of the rectangle is 45cm^2 . Find its sides.

Solution: Let the breadth in cm be x .

Then the length in cm will be $x + 4$.

By the given condition

Rectangular area in $\text{cm}^2 = 45$, that is,

$$x(x + 4) = 45$$

$$x^2 + 4x - 45 = 0$$

$$x^2 + 9x - 5x - 45 = 0$$

$$x(x + 9) - 5(x + 9) = 0$$

$$(x + 9)(x - 5) = 0$$

$$x + 9 = 0 \text{ or } x - 5 = 0$$

$$x = -9 \text{ or } x = 5$$

If $x = 5$, then $x + 4 = 5 + 4 = 9$

(neglecting -ve value)

Thus the breadth is 5cm and length is 9cm.

Example 3: The sum of the co-ordinates of a point is 6 and the sum of their squares is 20. Find the co-ordinates of the point.

Solutions: Let (x, y) be the co-ordinates of the point. Then by the given conditions, we have

$$x + y = 6 \quad \dots\dots\dots(i)$$

$$x^2 + y^2 = 20 \quad \dots\dots\dots(ii)$$

From equation (i)

$$y = 6 - x$$

Putting $y = 6 - x$ in equation (ii) we get

$$x^2 + (6 - x)^2 = 20$$

$$x^2 + 36 + x^2 - 12x - 20 = 0$$

$$2x^2 - 12x + 16 = 0 \quad \text{or}$$

$$2(x^2 - 6x + 8) = 0$$

$$\therefore x^2 - 6x + 8 = 0 \quad (\because 2 \neq 0)$$

$$x^2 - 4x - 2x + 8 = 0$$

$$x(x - 4) - 2(x - 4) = 0$$

$$(x - 4)(x - 2) = 0$$

$$\text{Either } x - 4 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = 4 \quad \text{or} \quad x = 2$$

Putting these values of x in equation (iii)

$$\text{When } x = 4 \quad \text{When } x = 2$$

$$y = 6 - 4 = 2 \quad \text{or} \quad y = 6 - 2 = 4$$

$$y = 2 \quad \text{or} \quad y = 4$$

$\therefore$ the co-ordinates of the point are $(4, 2)$ or $(2, 4)$.