

EXERCISE 2.8

Q. 1 The product of two positive consecutive numbers is 182. Find the numbers.

Solution:

Suppose first positive number = x

Very next positive number = $x + 1$

By given condition

$$x(x+1) = 182$$

$$x^2 + x - 182 = 0$$

$$x^2 + 14x - 13x - 182 = 0$$

$$x(x+14) - 13(x+14) = 0$$

$$(x+14)(x-13) = 0$$

$$\begin{array}{ll} \text{Either } x+14=0 & \text{or } x-13=0 \\ x=-14 & \text{or } x=13 \end{array}$$

As x is positive number therefore we neglect the negative value, So $x = 13$

Thus first positive number = $x = 13$

Very next positive number = $x + 1$
 $= 13 + 1 = 14$

Thus 13 and 14 are two required consecutive positive numbers.

Q. 2 The sum of the squares of three positive consecutive numbers is 77. Find them.

Solution:

Let $x, (x+1)$ and $(x+2)$ be the three consecutive positive number

By Give condition

$$x^2 + (x+1)^2 + (x+2)^2 = 77$$

$$x^2 + [x^2 + (1)^2 + 2(1)(x)] + [(x)^2 + (2)^2 + 2(x)(2)] = 77$$

$$x^2 + x^2 + 1 + 2x + x^2 + 4 + 4x = 77$$

$$3x^2 + 6x + 5 - 77 = 0$$

$$3x^2 + 6x - 72 = 0$$

$$3[x^2 + 2x - 24] = 0$$

$$\therefore x^2 + 2x - 24 = 0 \quad (\because 3 \neq 0)$$

$$x^2 + 6x - 4x - 24 = 0$$

$$x(x+6) - 4(x+6) = 0$$

$$(x+6)(x-4) = 0$$

$$\begin{array}{ll} \text{Either } x+6=0 & \text{or } x-4=0 \\ x=-6 & \text{or } x=4 \end{array}$$

As x is a positive number therefore we neglect the negative value and we take positive value of x i.e. $x = 4$

$$1^{\text{st}} \text{ Number} = x = 4$$

$$2^{\text{nd}} \text{ Number} = x + 1 = 4 + 1 = 5$$

$$3^{\text{rd}} \text{ Number} = x + 2 = 4 + 2 = 6$$

Thus 4, 5 and 6 are three required positive numbers.

Q. 3 The sum of five times a number and the square of the number is 204. Find the number.

Solution:

Let required number = x

Five times the number = $5x$

Square of the number = x^2

By given condition

$$x^2 + 5x = 204$$

$$x^2 + 5x - 204 = 0$$

$$x^2 + 17x - 12x - 204 = 0$$

$$x(x+17) - 12(x+17) = 0$$

$$(x+17)(x-12) = 0$$

Either

$$x+17=0 \quad \text{or} \quad x-12=0$$

$$\boxed{x=-17} \quad \text{or} \quad \boxed{x=12}$$

Thus required number is -17 or 12 .

Q. 4 The product of five less than three times a certain number and one less than four times the number is 7. Find the number.

Solution:

Let required number = x

Five less than three times the number = $3x-5$

One less than four times the number = $4x-1$

By given condition

$$(3x-5)(4x-1) = 7$$

$$12x^2 - 3x - 20x + 5 - 7 = 0$$

$$12x^2 - 23x - 2 = 0$$

$$12x^2 - 24x + x - 2 = 0$$

$$12x(x-2) + 1(x-2) = 0$$

$$(x-2)(12x+1) = 0$$

$$\begin{array}{ll}
 x-2=0 & \text{or} \\
 x=2 & \text{or} \\
 x=2 & \text{or}
 \end{array}
 \begin{array}{l}
 12x+1=0 \\
 12x=-1 \\
 x=\frac{-1}{12}
 \end{array}$$

Thus required number is 2 or $\frac{-1}{12}$

Q. 5 The difference of a number and its reciprocal is $\frac{15}{4}$. Find the number.

Solution:

Let required number = x

Reciprocal of the number = $\frac{1}{x}$

By given condition

$$x - \frac{1}{x} = \frac{15}{4}$$

$$\frac{x^2 - 1}{x} = \frac{15}{4}$$

$$4(x^2 - 1) = 15x$$

$$4x^2 - 4 - 15x = 0$$

$$4x^2 - 15x - 4 = 0$$

$$4x^2 - 16x + 1x - 4 = 0$$

$$4x(x - 4) + 1(x - 4) = 0$$

$$(x - 4)(4x + 1) = 0$$

$$\begin{array}{ll}
 \text{Either } x - 4 = 0 & \text{or} \\
 x = 0 + 4 & \text{or}
 \end{array}
 \begin{array}{l}
 4x + 1 = 0 \\
 4x = -1
 \end{array}$$

$$\boxed{x = 4} \quad \text{or} \quad \boxed{x = \frac{-1}{4}}$$

Thus required numbers is 4 or $\frac{-1}{4}$.

Q. 6 The sum of the squares of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits. Find the number.

Solution:

Let

Digits at unit's place of a number = x

Digit at ten's place of a number = y

Required number = $10y + x$

By 1st condition

$$x^2 + y^2 = 65 \quad \text{.....(i)}$$

By 2nd condition

$$10y + x = 9(x + y)$$

$$10y + x = 9x + 9y$$

$$10y - 9y = 9x - x$$

$$y = 8x \quad \text{.....(ii)}$$

Put value of y in equation (i)

$$x^2 + (8x)^2 = 65$$

$$x^2 + 64x^2 = 65$$

$$65x^2 = 65$$

$$x^2 = 1$$

$$\sqrt{x^2} = \pm\sqrt{1}$$

$$x = \pm 1$$

$$x = 1 \text{ or } x = -1$$

As x is a digit at unit's place which is always positive therefore we neglect the negative value and take the positive value i.e. $x = 1$

Put $x = 1$ in equation (ii)

$$y = 8(1)$$

$$y = 8$$

So, required number = $10y + x$

$$= 10(8) + 1$$

$$= 80 + 1$$

$$= 81$$

Q. 7 The sum of the co-ordinates of a point is 9 and sum of their squares is 45. Find the co-ordinates of the point.

Solution:

Let (x, y) are co-ordinates of required point.

By given conditions

$$x + y = 9 \quad \text{.....(i)}$$

$$x^2 + y^2 = 45 \quad \text{.....(ii)}$$

From equation (i)

$$x + y = 9$$

$$x = 9 - y \quad \text{.....(iii)}$$

Putting this in equation (ii), we get

$$(9 - y)^2 + y^2 = 45$$

$$(9)^2 - 2(9)(y) + (y)^2 + (y)^2 = 45$$

$$81 - 18y + y^2 + y^2 = 45$$

$$2y^2 - 18y + 81 - 45 = 0$$

$$2y^2 - 18y + 36 = 0$$

$$2(y^2 - 9y + 18) = 0$$

$$\therefore y^2 - 9y + 18 = 0 \quad (\because 2 \neq 0)$$

$$y^2 - 6y - 3y + 18 = 0$$

$$y(y - 6) - 3(y - 6) = 0$$

$$(y - 6)(y - 3) = 0$$

$$\text{Either } y - 6 = 0 \quad \text{or} \quad y - 3 = 0$$

$$y = 6 \quad \text{or} \quad y = 3$$

Putting the values of y in equation (iii), we get

$$\text{When } y = 6$$

$$x = 9 - 6$$

$$x = 3$$

$$\text{When } y = 3$$

$$x = 9 - 3$$

$$x = 6$$

Thus co-ordinates of the point are either (3, 6) or (6, 3)

Q. 8 Find two integers whose sum is 9 and the difference of their squares is also 9.

Solution:

02(105)

Suppose x and y are two integer

By given conditions

$$x + y = 9 \quad \dots\dots\dots (i)$$

$$x^2 - y^2 = 9 \quad \dots\dots\dots (ii)$$

From equation (i)

$$x + y = 9$$

$$x = 9 - y \quad \dots\dots\dots (iii)$$

Putting the value of x in equation (ii), we get

$$(9 - y)^2 - y^2 = 9$$

$$(9)^2 + (y)^2 - 2(9)(y) - y^2 = 9$$

$$81 + \cancel{y^2} - 18y - \cancel{y^2} - 9 = 0$$

$$72 - 18y = 0$$

$$-18y = -72$$

$$y = \frac{-72}{-18}$$

$$y = 4$$

Putting the value of y in equation (iii), we get

$$x = 9 - y$$

$$x = 9 - 4$$

$$x = 5$$

So 4 and 5 are required integers.

Q. 9 Find two integers whose difference is 4 and whose squares differ by 72.

Solution:

Let x and y are two integers

By given conditions

$$x - y = 4 \quad \dots\dots\dots (i)$$

$$x^2 - y^2 = 72 \quad \dots\dots\dots (ii)$$

From equation (i)

$$x = 4 + y \quad \dots\dots\dots (iii)$$

Putting the value of x in equation (ii), we get

$$(4 + y)^2 - y^2 = 72$$

$$[(4)^2 + (y)^2 + 2(4)(y)] - y^2 = 72$$

$$16 + \cancel{y^2} + 8y - \cancel{y^2} = 72$$

$$16 + 8y = 72$$

$$8y = 72 - 16$$

$$8y = 56$$

$$y = \frac{56}{8}$$

$$y = 7$$

Putting the value of y in equation (iii)

$$x = 4 + y$$

$$x = 4 + 7$$

$$x = 11$$

So 7 and 11 are required integers

Q. 10 Find the dimensions of a rectangle, whose perimeter is 80cm and its area is 375cm².

Solution:

Let width of rectangle = x cm

Length of rectangle = y cm

Perimeter of rectangle = 80cm

Area of rectangle = 375 cm²

We know that

$$2(L + W) = P$$

$$2(x + y) = 80$$

$$x + y = \frac{80}{2}$$

$$x + y = 40 \quad \dots\dots\dots (i)$$

$$\text{Area} = \text{Length} \times \text{Width}$$

$$375 = x \times y$$

$$\Rightarrow xy = 375 \quad \dots\dots\dots (ii)$$

From equation (i)

$$x + y = 40$$

$$y = 40 - x$$

Put it in equation (ii)

$$x(40 - x) = 375$$

$$40x - x^2 = 375$$

$$0 = x^2 - 40x + 375$$

$$\Rightarrow x^2 - 40x + 375 = 0$$

$$x^2 - 25x - 15x + 375 = 0$$

$$x(x - 25) - 15(x - 25) = 0$$

$$(x - 25)(x - 15) = 0$$

Putting these values of x in equation (i), we get

When $x = 25$

Then $25 + y = 40$

$$y = 40 - 25$$

$$y = 15$$

If $x = 25$ then $y = 15$

When $x = 15$

Then $15 + y = 40$

$$y = 40 - 15$$

$$y = 25$$

If $x = 15$ then $y = 25$

Thus dimensions of rectangle is either 25cm by 15cm or 15cm by 25cm.