

EXERCISE 3.5

Q.1 If s varies directly as u^2 and inversely as v and $s = 7$, when $u = 3$, $v = 2$. Find the value of s when $u = 6$ and $v = 10$.

Solution: $s \propto \frac{u^2}{v}$ $s = 7, u = 3, v = 2$

$s = ?, u = 6, v = 10$

$$s \propto \frac{u^2}{v}$$

$$s = \frac{ku^2}{v} \dots\dots\dots (i)$$

To find k ,

Put $s = 7$, $u = 3$, $v = 2$ in equation (i)

$$7 = \frac{k(3)^2}{2}$$

$$7 \times 2 = k(9)$$

$$\frac{14}{9} = k \quad \Rightarrow \quad k = \frac{14}{9}$$

Put $k = \frac{14}{9}$ in equation (i)

$$s = \frac{14u^2}{9v} \dots\dots\dots (ii)$$

To find s ,

Now put, $u=6$ and $v=10$ in equation (i)

$$s = \frac{14u^2}{9v}$$

$$s = \frac{14(6)^2}{9(10)}$$

$$s = \frac{14 \times \cancel{36}^2}{\cancel{90}_5} \quad (\text{dividing by } 18)$$

$$s = \frac{14 \times 2}{5}$$

$s = \frac{28}{5}$

Q.2 If w varies jointly as x , y^2 and z and $w = 5$ when $x = 2$, $y = 3$, $z = 10$. Find w when $x = 4$, $y = 7$ and $z = 3$.

Solution:

$w \propto xy^2z$

$w = 5, x = 2$

$y = 3, z = 10$

$w = ?, x = 4$

$y = 7, z = 3$

$$w \propto xy^2z$$

$$w = kxy^2z \dots\dots\dots (i)$$

To find k ,

Put $w=5$, $x = 2$, $y = 3$ and $z = 10$ in equation (i)

$$5 = k(2)(3)^2(10)$$

$$5 = k(20)(9)$$

$$5 = k(180)$$

$$\frac{5}{180} = k$$

$$\Rightarrow k = \frac{\cancel{5}^1}{\cancel{180}_{36}}$$

$$k = \frac{1}{36}$$

Put $k = \frac{1}{36}$ in equation (i)

$$w = \frac{1}{36}xy^2z \dots\dots\dots (ii)$$

To find w ,

Now put $x = 4$, $y = 7$, $z = 3$ in equation (ii)

$$w = \frac{1}{36}xy^2z$$

$$w = \frac{1}{36}(4)(7)^2(3)$$

$$w = \frac{1}{36}(4)(49)(3)$$

$$w = \frac{1}{\cancel{36}_3}(49) \times \cancel{12}^1$$

$$w = \frac{1}{3}(49)$$

$w = \frac{49}{3}$

Q.3 If y varies directly as x^3 and inversely as z^2 and t , and $y = 16$ when $x = 4$, $z = 2$, $t = 3$. Find the value of y when $x = 2$, $z = 3$ and $t = 4$.

Solution:

$y \propto \frac{x^3}{z^2t}$	$y = 16, x = 4$ $z = 2, t = 3$	$y = ?, x = 2$ $z = 3, t = 4$
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$$y \propto \frac{x^3}{z^2t}$$

$$y = \frac{kx^3}{z^2t} \dots\dots\dots(i)$$

To find k,

Put $y = 16, x = 4, z = 2$ and $t = 3$ in equation (i)

$$16 = \frac{k(4)^3}{(2)^2(3)}$$

$$16 = \frac{k(64)}{4 \times 3}$$

$$\frac{16 \times 4 \times 3}{64} = k$$

$$\frac{\cancel{64}}{\cancel{64}} \times 3 = k \Rightarrow k = 3$$

Put $k = 3$, in equation (i)

$$y = \frac{3x^3}{z^2t} \dots\dots\dots(ii)$$

To find y,

Now, put $x = 2$, $z = 3$ and $t = 4$ in equation (ii)

$$y = \frac{3 \times (2)^3}{(3)^2(4)}$$

$$y = \frac{3 \times 8}{9 \times 4}$$

$$y = \frac{24}{36}$$

$$y = \frac{\cancel{24}}{\cancel{3} \times \cancel{36}} \text{ (dividing by 12)}$$

$y = \frac{2}{3}$

Q.4 If u varies directly as x^2 and inversely as product yz^3 , and $u = 2$, when $x = 8$, $y = 7$, $z = 2$. Find the value of u when $x = 6, y = 3, z = 2$.

Solution:

$u \propto \frac{x^2}{yz^3}$	$u = 2, x = 8$ $y = 7, z = 2$	$u = ?, x = 6$ $y = 3, z = 2$
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$$u \propto \frac{x^2}{yz^3}$$

$$u = \frac{kx^2}{yz^3} \dots\dots\dots(i)$$

To find k,

Put $u = 2$, $x = 8$, $y = 7$ and $z = 2$ in equation(i)

$$2 = \frac{k(8)^2}{7(2)^3}$$

$$2 = \frac{k64}{7(8)}$$

$$\frac{2(8)(7)}{64} = k$$

$$\frac{\cancel{16}^1(7)}{\cancel{64}^4_4} = k$$

$$\frac{7}{4} = k \Rightarrow k = \frac{7}{4}$$

Put $k = \frac{7}{4}$ in equation (i)

$$u = \frac{7x^2}{4yz^3} \dots\dots\dots(ii)$$

To find u,

Now put $x = 6$, $y = 3$, $z = 2$ in equation (ii)

$$u = \frac{7x^2}{4yz^3} = \frac{7(6)^2}{4(3)(2)^3}$$

$$u = \frac{7(\cancel{36}^3)}{\cancel{12}^4 \times 8}$$

$$u = \frac{7(3)}{8}$$

$u = \frac{21}{8}$

Q.5 If v varies directly as the product xy^3 and inversely as z^2 and $v = 27$ when $x = 7$, $y = 6$, $z = 7$. Find the value of V when $x = 6$; $y = 2$, $z = 3$.

Solution: $v \propto \frac{xy^3}{z^2}$ $v = 27, x = 7$ $v = ?, x = 6$
 $y = 6, z = 7$ $y = 2, z = 3$

$$v \propto \frac{xy^3}{z^2}$$

$$v = \frac{kxy^3}{z^2} \dots\dots\dots (i)$$

To find k ,
 Put $v = 27$, $x = 7$, $y = 6$, $z = 7$ in equation (i)

$$27 = \frac{k(7)(6)^3}{(7)^2}$$

$$27 = \frac{k(216)}{7}$$

$$\frac{127 \times 7}{8216} = k$$

$$\Rightarrow k = \frac{7}{8}$$

Put $k = \frac{7}{8}$ in equation (i)

$$v = \frac{7xy^3}{8z^2} \dots\dots\dots (ii)$$

To find v ,
 Now, put $x = 6$, $y = 2$, $z = 3$ in equation (ii).

$$v = \frac{7(6)(2)^3}{8(3)^2}$$

$$v = \frac{7(6)(8)}{8 \times 9}$$

$$v = \frac{42^{14}}{8 \times 3}$$

$$\Rightarrow v = \frac{14}{3}$$

Q.6 If w varies inversely as the cube of U , and $w = 5$ when $U = 3$. Find w , when $U = 6$.

Solution: $w \propto \frac{1}{u^3}$ $w = 5$ $w = ?$
 $u = 3$ $u = 6$

$$w \propto \frac{1}{u^3}$$

$$w = \frac{k}{u^3} \dots\dots\dots (i)$$

To find k ,

Put $w = 5$ and $u = 3$ in equation (i)

$$5 = \frac{k}{(3)^3}$$

$$5 = \frac{k}{27}$$

$$\Rightarrow k = 27 \times 5$$

$$k = 135$$

Put $k = 135$ in equation (i)

$$w = \frac{k}{u^3}$$

$$w = \frac{135}{u^3} \dots\dots\dots (ii)$$

To find w ,

Now, Put $u = 6$ in equation (ii)

$$w = \frac{135}{(6)^3}$$

$$w = \frac{135}{216}$$

$$w = \frac{5 \times 27}{8 \times 27}$$

$$w = \frac{5}{8}$$

K-Method

If $a : b :: c : d$ is a proportion, then putting each ratio equal to k .

$$\text{i.e. } \frac{a}{b} = \frac{c}{d} = k$$

$$\frac{a}{b} = k \text{ and } \frac{c}{d} = k$$

$$a = bk \text{ and } c = dk$$

Using the above equations, we can solve certain problems relating to proportions more easily. This method is known as k -method.

Example 1: If $a : b = c : d$, then show that

$$\frac{3a + 2b}{3a - 2b} = \frac{3c + 2d}{3c - 2d}$$

Solution: $a : b = c : d$

$$\text{Let } \frac{a}{b} = \frac{c}{d} = k$$

$$\frac{a}{b} = k \text{ and } \frac{c}{d} = k$$

Then $a = bk$ and $c = dk$

$$\begin{aligned} \text{L.H.S} &= \frac{3a + 2b}{3a - 2b} = \frac{3bk + 2b}{3bk - 2b} = \frac{b(3k + 2)}{b(3k - 2)} \\ &= \frac{3k + 2}{3k - 2} \dots\dots\dots(i) \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= \frac{3c + 2d}{3c - 2d} = \frac{3dk + 2d}{3dk - 2d} = \frac{d(3k + 2)}{d(3k - 2)} \\ &= \frac{3k + 2}{3k - 2} \dots\dots\dots(ii) \end{aligned}$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{i.e., } \frac{3a + 2b}{3a - 2b} = \frac{3c + 2d}{3c - 2d}$$

Example 2: If $a : b = c : d$, then show that

$$pa + qb : ma - nb = pc + qd : mc - nd$$

Solution: Let $\frac{a}{b} = \frac{c}{d} = k$, then

$$\frac{a}{b} = k \text{ and } \frac{c}{d} = k$$

$$a = bk \text{ and } c = dk$$

$$\begin{aligned} \text{L.H.S} &= pa + qb : ma - nb = \frac{pa + qb}{ma - nb} = \frac{pkb + qb}{mkb - nb} \\ &= \frac{b(pk + q)}{b(mk - n)} = \frac{pk + q}{mk - n} \dots\dots\dots(i) \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= pc + qd : mc - nd = \frac{pc + qd}{mc - nd} = \frac{pkd + qd}{mkd - nd} \\ &= \frac{d(pk + q)}{d(mk - n)} = \frac{pk + q}{mk - n} \dots\dots\dots(ii) \end{aligned}$$

From (i) and (ii) $\text{L.H.S} = \text{R.H.S}$

$$pa + qb : ma - nb = pc + qd : mc - nd$$

Example 3:

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then show that

$$\frac{a^3 + c^3 + e^3}{b^3 + d^3 + f^3} = \frac{ace}{bdf}$$

Solution: Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$$\text{When } \frac{a}{b} = k, \frac{c}{d} = k \text{ and } \frac{e}{f} = k$$

$$a = bk, c = dk \text{ and } e = fk$$

Let

$$\begin{aligned} \text{L.H.S} &= \frac{a^3 + c^3 + e^3}{b^3 + d^3 + f^3} = \frac{(bk)^3 + (dk)^3 + (fk)^3}{b^3 + d^3 + f^3} \\ &= \frac{b^3k^3 + d^3k^3 + f^3k^3}{b^3 + d^3 + f^3} = k^3 \left(\frac{(b^3 + d^3 + f^3)}{(b^3 + d^3 + f^3)} \right) \\ &= k^3 \dots\dots\dots(i) \end{aligned}$$

$$\text{R.H.S} = \frac{ace}{bdf} = \frac{(bk)(dk)(fk)}{bdf} = k^3 \frac{\cancel{bdf}}{\cancel{bdf}}$$

$$\text{R.H.S} = k^3 \dots\dots(ii)$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{i.e. } \frac{a^3 + c^3 + e^3}{b^3 + d^3 + f^3} = \frac{ace}{bdf}$$

Example 4:

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then show that

$$\frac{a^2b + c^2d + e^2f}{ab^2 + cd^2 + ef^2} = \frac{a + c + e}{b + d + f}$$

Solution: Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$$\frac{a}{b} = k, \frac{c}{d} = k, \frac{e}{f} = k$$

$$\boxed{a = bk, c = dk \text{ and } e = fk}$$

To prove $a = bk, c = dk, e = fk$

$$\frac{a^2b + c^2d + e^2f}{ab^2 + cd^2 + ef^2} = \frac{a + c + e}{b + d + f}$$

$$\begin{aligned} \text{L.H.S} &= \frac{a^2b + c^2d + e^2f}{ab^2 + cd^2 + ef^2} \\ &= \frac{(bk)^2b + (dk)^2d + (fk)^2f}{(bk)b^2 + (dk)d^2 + (fk)f^2} = \frac{k^2b^3 + k^2d^3 + k^2f^3}{kb^3 + kd^3 + kf^3} \\ &= \frac{k^2(\cancel{b^3} + \cancel{d^3} + f^3)}{k(\cancel{b^3} + \cancel{d^3} + f^3)} = k \dots \dots \dots (i) \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= \frac{a + c + e}{b + d + f} = \frac{bk + dk + fk}{b + d + f} \\ &= \frac{k(\cancel{b} + \cancel{d} + f)}{(\cancel{b} + \cancel{d} + f)} = k \dots \dots \dots (ii) \end{aligned}$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Thus } \frac{a^2b + c^2d + e^2f}{ab^2 + cd^2 + ef^2} = \frac{a + c + e}{b + d + f}$$