

EXERCISE 3.6

Q.1 If $a:b=c:d$, ($a, b, c, d \neq 0$), then show that

$$(i) \quad \frac{4a-9b}{4a+9b} = \frac{4c-9d}{4c+9d}$$

$$(ii) \quad \frac{6a-5b}{6a+5b} = \frac{6c-5d}{6c+5d}$$

$$(iii) \quad \frac{a}{b} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$$

$$(iv) \quad a^6+c^6 : b^6+d^6 = a^3c^3 : b^3d^3$$

$$(v) \quad p(a+b)+qb : p(c+d)+qd = a : c$$

$$(vi) \quad a^2+b^2 : \frac{a^3}{a+b} = c^2+d^2 : \frac{c^3}{c+d}$$

$$(vii) \quad \frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$$

Solution:

$$(i) \quad \frac{4a-9b}{4a+9b} = \frac{4c-9d}{4c+9d}$$

$$\text{Let } a : b = c : d = k$$

$$\frac{a}{b} = \frac{c}{d} = k$$

$$\frac{a}{b} = k, \text{ and } \frac{c}{d} = k$$

$$\boxed{a = bk} \text{ and } \boxed{c = dk}$$

$$L.H.S = \frac{4a-9b}{4a+9b}$$

$$= \frac{4bk-9b}{4bk+9b}$$

$$= \frac{b(4k-9)}{b(4k+9)}$$

$$L.H.S = \frac{4k-9}{4k+9} \quad \dots\dots\dots (i)$$

Now , taking

$$R.H.S = \frac{4c-9d}{4c+9d}$$

$$= \frac{4dk-9d}{4dk+9d}$$

$$= \frac{d(4k-9)}{d(4k+9)}$$

$$R.H.S = \frac{4k-9}{4k+9} \quad \dots\dots\dots (ii)$$

From equation (i) and (ii)

$$L.H.S = R.H.S$$

$$\text{So } \frac{4a-9b}{4a+9b} = \frac{4c-9d}{4c+9d}$$

Hence proved

$$(ii) \frac{6a-5b}{6a+5b} = \frac{6c-5d}{6c+5d}$$

$$\text{Let } a : b = c : d = k$$

$$\frac{a}{b} = \frac{c}{d} = k$$

$$\Rightarrow \frac{a}{b} = k : \frac{c}{d} = k$$

$$\boxed{a = bk}; \boxed{c = dk}$$

$$\text{Let } L.H.S = \frac{6a-5b}{6a+5b}$$

$$= \frac{6bk-5b}{6bk+5b}$$

$$= \frac{b(6k-5)}{b(6k+5)}$$

$$= \frac{6k-5}{6k+5}$$

$$L.H.S = \frac{6k-5}{6k+5} \dots\dots\dots (i)$$

$$\text{Now } R.H.S = \frac{6c-5d}{6c+5d}$$

$$= \frac{6dk-5d}{6dk+5d}$$

$$= \frac{d(6k-5)}{d(6k+5)}$$

$$= \frac{6k-5}{6k+5}$$

$$R.H.S = \frac{6k-5}{6k+5} \dots\dots\dots (ii)$$

From (i) and (ii)

$$L.H.S = R.H.S$$

$$\text{So, } \frac{6a-5b}{6a+5b} = \frac{6c-5d}{6c+5d}$$

Hence proved

$$(iii) \frac{a}{b} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$$

$$\text{Let } a : b = c : d = k$$

$$\frac{a}{b} = \frac{c}{d} = k$$

$$\frac{a}{b} = k : \frac{c}{d} = k$$

$$\boxed{a = bk}; \boxed{c = dk}$$

$$\text{Let } L.H.S = \frac{a}{b} = \frac{\cancel{b}k}{\cancel{b}}$$

$$L.H.S = k \dots\dots\dots (i)$$

$$\text{Now } R.H.S = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$$

$$= \sqrt{\frac{b^2k^2+d^2k^2}{b^2+d^2}}$$

$$= \sqrt{\frac{k^2(\cancel{b^2}+\cancel{d^2})}{(b^2+d^2)}}$$

$$= \sqrt{k^2}$$

$$R.H.S = k \dots\dots\dots (ii)$$

From (i) and (ii)

$$L.H.S = R.H.S$$

Hence proved

$$\text{So } \frac{a}{b} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$$

$$(iv) a^6+c^6:b^6+d^6 = a^3c^3 : b^3d^3$$

$$\text{Let } a : b = c : d = k$$

$$\frac{a}{b} = \frac{c}{d} = k$$

$$\frac{a}{b} = k; \frac{c}{d} = k$$

$$\text{Let } L.H.S = a^6+c^6 : b^6+d^6$$

$$= \frac{a^6+c^6}{b^6+d^6} = \frac{(bk)^6+(dk)^6}{b^6+d^6}$$

$$= \frac{b^6k^6+d^6k^6}{b^6+d^6} = \frac{k^6(\cancel{b^6}+\cancel{d^6})}{(\cancel{b^6}+\cancel{d^6})}$$

$$L.H.S = k^6 \dots\dots\dots (i)$$

$$\text{Now, } R.H.S = a^3c^3 : b^3d^3$$

$$= \frac{a^3c^3}{b^3d^3} = \frac{(bk)^3.(dk)^3}{b^3d^3}$$

$$= \frac{b^3k^3.d^3k^3}{b^3d^3} = \frac{\cancel{b^3}\cancel{d^3}.k^{3+3}}{\cancel{b^3}\cancel{d^3}}$$

$$R.H.S = k^6 \dots\dots\dots (ii)$$

From (i) and (ii)

$$L.H.S = R.H.S$$

$$\text{So } a^6+c^6 : b^6+d^6 = a^3c^3 : b^3d^3$$

Hence proved

$$(v) \quad p(a+b) + qb : p(c+d) + qd = a : c$$

$$\text{Let } a : b = c : d = k$$

$$\frac{a}{b} = \frac{c}{d} = k$$

$$\frac{a}{b} = k ; \frac{c}{d} = k$$

$$\boxed{a = bk} ; \boxed{c = dk}$$

$$\text{Let } L.H.S = p(a+b) + qb : p(c+d) + qd$$

$$= \frac{p(a+b) + qb}{p(c+d) + qd}$$

$$= \frac{p(bk+b) + qb}{p(dk+d) + qd}$$

$$= \frac{pb(k+1) + qb}{pd(k+1) + qd}$$

$$= \frac{b[p(k+1) + q]}{d[p(k+1) + q]}$$

$$L.H.S = \frac{b}{d} \dots\dots\dots (i)$$

$$R.H.S = a : c = \frac{a}{c}$$

$$R.H.S = \frac{bk}{dk}$$

$$R.H.S = \frac{b}{d} \dots\dots\dots (ii)$$

From (i) and (ii)

$$L.H.S = R.H.S$$

$$p(a+b) + qb : p(c+d) + qd = a : b$$

Hence proved

$$(vi) \quad a^2 + b^2 : \frac{a^3}{a+b} = c^2 + d^2 : \frac{c^3}{c+d}$$

$$\text{Let } a : b = c : d = k$$

$$\frac{a}{b} = \frac{c}{d} = k$$

$$\frac{a}{b} = k ; \frac{c}{d} = k$$

$$\boxed{a = bk} ; \boxed{c = dk}$$

$$\text{Let } L.H.S = a^2 + b^2 : \frac{a^3}{a+b}$$

$$= [(bk)^2 + b^2] \div \frac{(bk)^3}{bk+b}$$

$$= (b^2k^2 + b^2) \times \frac{(bk+b)}{b^3k^3}$$

$$= b^2(k^2+1) \times \frac{b(k+1)}{b^3k^3}$$

$$= \frac{\cancel{b^3}(k^2+1)(k+1)}{\cancel{b^3}k^3}$$

$$L.H.S = \frac{(k^2+1)(k+1)}{k^3} \dots\dots\dots (i)$$

$$\text{Now } R.H.S = c^2 + d^2 : \frac{c^3}{c+d}$$

$$= [(dk)^2 + d^2] \div \frac{(dk)^3}{(dk+d)}$$

$$= (d^2k^2 + d^2) \times \frac{(dk+d)}{d^3k^3}$$

$$= d^2(k^2+1) \times \frac{d(k+1)}{d^3k^3}$$

$$= \frac{\cancel{d^3}(k^2+1)(k+1)}{\cancel{d^3}k^3}$$

$$R.H.S = \frac{(k^2+1)(k+1)}{k^3} \dots\dots\dots (ii)$$

From (i) and (ii)

$$L.H.S = R.H.S$$

$$\text{So } a^2 + b^2 : \frac{a^3}{a+b} = c^2 + d^2 : \frac{c^3}{c+d}$$

Hence proved

$$(vii) \quad \frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$$

$$\text{Let } a : b = c : d = k$$

$$\frac{a}{b} = \frac{c}{d} = k$$

$$\frac{a}{b} = k, \frac{c}{d} = k$$

$$\boxed{a = bk}, \boxed{c = dk}$$

Let $L.H.S = \frac{a}{a-b} : \frac{a+b}{b}$

$$= \frac{bk}{bk-b} \div \frac{bk+b}{b}$$

$$= \frac{\cancel{b}k}{\cancel{b}(k-1)} \times \frac{\cancel{b}}{\cancel{b}(k+1)}$$

$$= \frac{k}{(k)^2 - (1)^2}$$

L.H.S = $\frac{k}{k^2-1}$ (i)

Now $R.H.S = \frac{c}{c-d} : \frac{c+d}{d}$

$$= \frac{dk}{dk-d} \div \frac{dk+d}{d}$$

$$= \frac{\cancel{d}k}{\cancel{d}(k-1)} \times \frac{\cancel{d}}{\cancel{d}(k+1)}$$

$$= \frac{k}{(k)^2 - (1)^2}$$

R.H.S = $\frac{k}{k^2-1}$ (ii)

From (i) and (ii)
 L.H.S = R.H.S

So $\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$

Hence proved

Q.2 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ (a,b,c,d,e,f ≠ 0) , then show that

(i) $\frac{a}{b} = \sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}}$

(ii) $\frac{ac + ce + ea}{bd + df + fb} = \left[\frac{ace}{bdf} \right]^{2/3}$

(iii) $\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

Solution:

(i) $\frac{a}{b} = \sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$$\frac{a}{b} = k, \quad \frac{c}{d} = k, \quad \frac{e}{f} = k$$

$a = bk, \quad c = dk, \quad e = fk$

L.H.S = $\frac{a}{b} = \frac{\cancel{b}k}{\cancel{b}} = k$

L.H.S = k (i)

Now $R.H.S = \sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}}$

$$= \sqrt{\frac{b^2k^2 + d^2k^2 + f^2k^2}{b^2 + d^2 + f^2}}$$

$$= \sqrt{\frac{k^2(b^2 + d^2 + f^2)}{(b^2 + d^2 + f^2)}}$$

$$= \sqrt{k^2}$$

R.H.S = k (ii)

From (i) and (ii)
 R.H.S = R.H.S

i.e $\frac{a}{b} = \sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}}$

(ii) $\frac{ac + ce + ea}{bd + df + fb} = \left(\frac{ace}{bdf} \right)^{2/3}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$$\frac{a}{b} = k, \quad \frac{c}{d} = k, \quad \frac{e}{f} = k$$

$a = bk; \quad c = dk, \quad e = fk$

Let $L.H.S = \frac{ac + ce + ea}{bd + df + fb}$

$$= \frac{bk(dk) + dk(fk) + fk(bk)}{bd + df + fb}$$

$$= \frac{k^2bd + k^2df + k^2fb}{bd + df + fb}$$

$$= \frac{k^2(bd + df + fb)}{(bd + df + fb)}$$

L.H.S = k^2 (i)

$$\begin{aligned}\text{Now, } R.H.S &= \left(\frac{ace}{bdf} \right)^3 \\ &= \left(\frac{bk \cdot dk \cdot fk}{bdf} \right)^3 = \left(\frac{k^3 \cdot bdf}{bdf} \right)^3 \\ &= (k^3)^3 = k^{3 \times 3}\end{aligned}$$

$$R.H.S = k^2 \dots\dots\dots (ii)$$

From (i) and (ii)

$$L.H.S = R.H.S$$

$$\text{So, } \frac{ac + ce + ea}{bd + df + fb} = \left(\frac{ace}{bdf} \right)^3$$

Hence proved.

$$(iii) \quad \frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$$

$$\text{Let } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$$

$$\frac{a}{b} = k, \quad \frac{c}{d} = k, \quad \frac{e}{f} = k$$

$$\boxed{a = bk, \quad c = dk, \quad e = fk}$$

$$\text{Let } L.H.S = \frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb}$$

$$= \frac{bk \cdot dk}{bd} + \frac{dk \cdot fk}{df} + \frac{fk \cdot bk}{fb}$$

$$= \frac{k^2 \cancel{b} \cancel{d}}{\cancel{b} \cancel{d}} + \frac{k^2 \cancel{d} \cancel{f}}{\cancel{d} \cancel{f}} + \frac{k^2 \cancel{f} \cancel{b}}{\cancel{f} \cancel{b}}$$

$$= k^2 + k^2 + k^2$$

$$L.H.S = 3k^2 \dots\dots\dots (i) \text{ Now}$$

$$R.H.S = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$$

$$= \frac{\cancel{b}^2 k^2}{\cancel{b}^2} + \frac{\cancel{d}^2 k^2}{\cancel{d}^2} + \frac{\cancel{f}^2 k^2}{\cancel{f}^2}$$

$$= k^2 + k^2 + k^2$$

$$R.H.S = 3k^2 \dots\dots\dots (ii)$$

From (i) and (ii)

$$L.H.S = R.H.S$$

$$\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$$

Real life problems based on variation

Example 1: The strength “s” of a rectangular beam varies directly as the breadth b and the square of the depth d. If a beam 9cm wide and 12 cm deep with support 1200 lb. What weight a beam of 12cm wide and 9 cm deep will support?

Solution:

By the joint variation, we have

$$\boxed{s \propto bd^2}$$

$$s = 1200 \text{ lb}$$

$$b = 9 \text{ cm}$$

$$d = 12 \text{ cm}$$

$$s = ?$$

$$b = 12 \text{ cm}$$

$$d = 9 \text{ cm}$$

$$s \propto bd^2$$

$$\text{i.e. } s = kbd^2 \dots\dots\dots (i)$$

To find k,

put $s = 1200$, $b = 9$ and $d = 12$ in equation (i)

$$1200 = k(9)(12)^2$$

$$1200 = k(9)(144)$$

$$k = \frac{1200}{9 \times 144}$$

$$k = \frac{1200}{1296} = \frac{25}{27} \quad (\text{dividing by 48})$$

$$\Rightarrow k = \frac{25}{27}$$

Put $k = \frac{25}{27}$ in equation (i)

$$s = \frac{25}{27} bd^2 \dots\dots\dots (ii)$$

To find s,

Now put $b = 12$ and $d = 9$ in equation (ii)

$$s = \frac{25}{27} (12)(9)^2$$

$$s = \frac{25(12)(9)(9)}{27}$$

$$s = \frac{24300}{27}$$

$$\boxed{s = 900 \text{ lb}}$$

Example 2: The current in a wire varies directly as the electromotive force E and inversely as the resistance R . If $I = 32$ amperes, when $E = 128$ volts and $R = 8$ ohms. Find I , when $E = 150$ volts and $R = 18$ ohms.

Solution:

In joint variation, we have

$I \propto \frac{E}{R}$	$I = 32 \text{ amp}$	$I = ?$
	$E = 128 \text{ volts}$	$E = 150 \text{ volts}$
	$R = 8 \text{ ohms}$	$R = 18 \text{ ohms}$

$$I \propto \frac{E}{R}, \quad \text{i.e.} \quad I = \frac{kE}{R} \dots\dots\dots(i)$$

To find k ,

Put $I = 32$, $E = 128$ and $R = 8$ in equation (i)

$$32 = \frac{k(128)}{8} \Rightarrow \frac{32 \times 8}{128} = k$$

$$\frac{256}{128} = k \Rightarrow k = 2$$

Put $k = 2$ in equation (i)

$$I = \frac{2E}{R} \dots\dots\dots(ii)$$

To find I ,

Now put $E = 150$ and $R = 18$ in equation (ii)

$$I = \frac{2E}{R}$$

$$I = \frac{2(150)}{18} = \frac{\cancel{300}^{50}}{\cancel{18}_3} \quad (\text{dividing by "6"})$$

$I = \frac{50}{3} \text{ amp}$
