

EXERCISE 4.4

Q.1 $\frac{x^3}{(x^2+4)^2}$

Solution: $\frac{x^3}{(x^2+4)^2}$

Let $\frac{x^3}{(x^2+4)^2} = \frac{Ax+B}{x^2+4} + \frac{Cx+D}{(x^2+4)^2} \dots\dots\dots (i)$

Multiplying both sides by $(x^2+4)^2$, we get

$$x^3 = (Ax+B)(x^2+4) + (Cx+D)$$

$$x^3 = Ax(x^2+4) + B(x^2+4) + (Cx+D) \dots\dots\dots(ii)$$

Equating the coefficients of x^3 , x^2 , x and constants, we get

Coefficients of x^3 : $A = 1$

Coefficients of x^2 : $B = 0$

Coefficients of x : $4A + C = 0$

$$4(1) + C = 0$$

$$\Rightarrow C = -4$$

Constants: $4B + D = 0$

$$4(0) + D = 0$$

$$\Rightarrow D = 0$$

Putting the value of A,B,C and D in equation(i) we get required partial fractions.

$$\frac{x^3}{(x^2+4)^2} = \frac{x}{x^2+4} - \frac{4x}{(x^2+4)^2}$$

Q.2 $\frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}$

Solution: $\frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}$

Let $\frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2} \dots(i)$

Multiplying both sides by $(x+1)(x^2+1)^2$ we get

$$x^4+3x^2+x+1 = A(x^2+1)^2 + (Bx+C)(x+1)(x^2+1)$$

$$+(Dx+E)(x+1).....(ii)$$

$$x^4 + 3x^2 + x + 1 = A(x^4 + 2x^2 + 1) + Bx(x^3 + x^2 + x + 1)$$

$$+ C(x^3 + x^2 + x + 1) + Dx(x+1) + E(x+1)$$

$$x^4 + 3x^2 + x + 1 = A(x^4 + 2x^2 + 1) + B(x^4 + x^3 + x^2 + x)$$

$$+ C(x^3 + x^2 + x + 1) + D(x^2 + x) + E(x+1) .. (iii)$$

Putting $x+1=0$ i.e $x=-1$ in eq.(ii), we get

$$(-1)^4 + 3(-1)^2 + (-1) + 1 = A [(-1)^2 + 1]^2$$

$$1 + 3(1) - 1 + 1 = A(1+1)^2$$

$$4 = 4A$$

$$\Rightarrow \boxed{A = 1}$$

Now equating the coefficients of x^4 , x^3 , x^2 , x and constants, we get from equation (iii)

$$\text{Coefficients of } x^4: A + B = 1$$

$$1 + B = 1$$

$$B = 1 - 1$$

$$\Rightarrow \boxed{B = 0}$$

$$\text{Coefficients of } x^3: B + C = 0$$

$$0 + C = 0$$

$$\Rightarrow \boxed{C = 0}$$

$$\text{Coefficients of } x^2: 2A + B + C + D = 3$$

$$2(1) + 0 + 0 + D = 3$$

$$D = 3 - 2$$

$$\boxed{D = 1}$$

$$\text{Coefficients of } x: B + C + D + E = 1$$

$$0 + 0 + 1 + E = 1$$

$$E = 1 - 1$$

$$\Rightarrow \boxed{E = 0}$$

Putting the value of A, B, C and D in equation(i) we get required partial fractions.

$$\frac{x^4 + 3x^2 + x + 1}{(x+1)(x^2+1)^2} = \frac{1}{x+1} + \frac{x}{(x^2+1)^2}$$

$$\text{Q.3 } \frac{x^2}{(x+1)(x^2+1)^2}$$

$$\text{Solution: } \frac{x^2}{(x+1)(x^2+1)^2}$$

$$\text{Let } \frac{x^2}{(x+1)(x^2+1)^2} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2} .. (i)$$

Multiply both sides by $(x+1)(x^2+1)^2$ we get

$$x^2 = A(x^2+1)^2 + (Bx+C)(x+1)(x^2+1)$$

$$+ (Dx+E)(x+1).....(ii)$$

$$x^2 = A(x^4 + 2x^2 + 1) + Bx(x^3 + x^2 + x + 1)$$

$$+ C(x^3 + x^2 + x + 1) + Dx(x+1) + E(x+1)$$

$$x^2 = A(x^4 + 2x^2 + 1) + B(x^4 + x^3 + x^2 + x)$$

$$+ C(x^3 + x^2 + x + 1) + D(x^2 + x) + E(x+1)....(iii)$$

Putting $x+1=0$ i.e $x=-1$ in equation (ii) we get

$$(-1)^2 = A [(-1)^2 + 1]^2$$

$$1 = A(1+1)^2$$

$$1 = 4A \Rightarrow \boxed{A = \frac{1}{4}}$$

Now equating the coefficients of x^4 , x^3 , x^2 , x and constants we get from equation (iii)

$$\text{Coefficients of } x^4: A + B = 0$$

$$\frac{1}{4} + B = 0 \Rightarrow \boxed{B = -\frac{1}{4}}$$

$$\text{Coefficients of } x^3: B + C = 0$$

$$-\frac{1}{4} + C = 0 \Rightarrow \boxed{C = \frac{1}{4}}$$

$$\text{Coefficients of } x^2: 2A + B + C + D = 1$$

$$2\left(\frac{1}{4}\right) - \frac{1}{4} + \frac{1}{4} + D = 1$$

$$\frac{1}{2} + D = 1$$

$$D = 1 - \frac{1}{2}$$

$$\boxed{D = \frac{1}{2}}$$

Coefficients of x : $B + C + D + E = 0$

$$-\frac{1}{4} + \frac{1}{4} + \frac{1}{2} + E = 0$$

$$\frac{1}{2} + E = 0$$

$$\boxed{E = -\frac{1}{2}}$$

Putting the value of A , B , C , D and E in equation(i) we get required partial fractions.

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{1}{4(x+1)} - \frac{x-1}{4(x^2+1)} + \frac{x-1}{2(x^2+1)^2}$$

Q.4 $\frac{x^2}{(x-1)(x^2+1)^2}$

Solution: $\frac{x^2}{(x-1)(x^2+1)^2}$

Let $\frac{x^2}{(x-1)(x^2+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2} \dots (i)$

Multiplying both sides by $(x-1)(x^2+1)^2$, we get

$$x^2 = A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) + (Dx+E)(x-1) \dots (ii)$$

$$x^2 = A(x^4+2x^2+1) + Bx(x-1)(x^2+1) + C(x-1)(x^2+1) + Dx(x-1) + E(x-1)$$

$$x^2 = A(x^4+2x^2+1) + B(x^4-x^3+x^2-x) + C(x^3-x^2+x-1) + D(x^2-x) + E(x-1) \dots (iii)$$

Putting $x-1=0$ i.e $x=1$ in equation (ii) we get

$$(1)^2 = A[(1)^2+1]^2$$

$$1 = A(1+1)^2$$

$$1 = 4A \Rightarrow \boxed{A = \frac{1}{4}}$$

Now equating the coefficients of x^4 , x^3 , x^2 and x in equation (iii) we get

Coefficients of x^4 : $A + B = 0$

$$\frac{1}{4} + B = 0$$

$$\Rightarrow \boxed{B = -\frac{1}{4}}$$

Coefficients of x^3 : $-B + C = 0$

$$-\left(-\frac{1}{4}\right) + C = 0$$

$$\Rightarrow C = -\frac{1}{4}$$

Coefficients of x^2 : $2A + B - C + D = 1$

$$2\left(\frac{1}{4}\right) - \frac{1}{4} - \left(-\frac{1}{4}\right) + D = 1$$

$$\frac{1}{2} - \frac{1}{4} + \frac{1}{4} + D = 1$$

$$D = 1 - \frac{1}{2}$$

$$D = \frac{2-1}{2}$$

$$\boxed{D = \frac{1}{2}}$$

Coefficients of x : $-B + C - D + E = 0$

$$-\left(-\frac{1}{4}\right) - \frac{1}{4} - \frac{1}{2} + E = 0$$

$$\frac{1}{4} - \frac{1}{4} - \frac{1}{2} + E = 0$$

$$\frac{-1}{2} + E = 0$$

$$\boxed{E = \frac{1}{2}}$$

Putting the value of A , B , C , D and E in equation(i) we get required partial fractions.

$$\frac{x^2}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} - \frac{x+1}{4(x^2+1)} + \frac{x+1}{2(x^2+1)^2}$$

Q.5 $\frac{x^4}{(x^2+2)^2}$

Solution: $\frac{x^4}{(x^2+2)^2}$

$\frac{x^4}{(x^2+2)^2} = \frac{x^4}{x^4+4x^2+4}$ is an improper fraction. First we resolve it into proper fraction.

$$x^4 + 4x^2 + 4 \sqrt{\frac{x^4}{\pm x^4 \pm 4x^2 \pm 4}} \frac{1}{-4x^2 - 4}$$

$$\frac{x^4}{(x^2+2)^2} = 1 + \frac{-4x^2-4}{(x^2+2)^2}$$

Let $\frac{-4x^2-4}{(x^2+2)^2} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2} \dots\dots(i)$

Multiplying both sides by $(x^2+2)^2$ we get
 $-4x^2-4 = (Ax+B)(x^2+2) + (Cx+D)$
 $-4x^2-4 = A(x^3+2x) + B(x^2+2) + Cx + D \dots\dots(ii)$

Equating the coefficients of x^3, x^2, x and constants in equation (ii) we get

Coefficients of x^3 : $A = 0$

Coefficients of x^2 : $B = -4$

Coefficients of x : $2A + C = 0$

$$2(0) + C = 0$$

$$\Rightarrow \boxed{C = 0}$$

Constants: $2B + D = -4$

$$2(-4) + D = -4$$

$$-8 + D = -4$$

$$D = 8 - 4$$

$$\boxed{D = 4}$$

Putting the value of A, B, C and D in equation(i) we get required partial fractions.

$$\frac{x^4}{(x^2+2)^2} = 1 + \frac{-4}{x^2+2} + \frac{4}{(x^2+2)^2}$$

$$\frac{x^4}{(x^2+2)^2} = 1 - \frac{4}{x^2+2} + \frac{4}{(x^2+2)^2}$$

Q.6 $\frac{x^5}{(x^2+1)^2}$

Solution: $\frac{x^5}{(x^2+1)^2}$

$\frac{x^5}{(x^2+1)^2} = \frac{x^5}{x^4+2x^2+1}$ is an improper fraction.

First we resolve it into proper fraction.

$$x^4 + 2x^2 + 1 \sqrt{\frac{x^5}{\cancel{x^5} \pm 2x^3 \pm x}} \frac{-2x^3 - x}{-2x^3 - x}$$

$$\frac{x^5}{(x^2+1)^2} = x + \frac{-2x^3-x}{(x^2+1)^2}$$

Let $\frac{-2x^3-x}{(x^2+1)^2} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} \dots\dots(i)$

Multiplying both sides by $(x^2+1)^2$ we get

$$-2x^3-x = (Ax+B)(x^2+1) + (Cx+D)$$

$$-2x^3-x = A(x^3+x) + B(x^2+1) + Cx + D$$

Equating the coefficients of x^3, x^2, x and constants we get

Coefficients of x^3 : $A = -2$

Coefficients of x^2 : $B = 0$

Coefficients of x : $A + C = -1$

$$-2 + C = -1$$

$$C = -1 + 2$$

$$\Rightarrow \boxed{C = 1}$$

Constants: $B + D = 0$

$$0 + D = 0$$

$$\Rightarrow D = 0$$

Hence the required partial fractions are

$$\frac{x^5}{(x^2+1)^2} = x + \frac{-2x}{x^2+1} + \frac{x}{(x^2+1)^2}$$

$$\Rightarrow \frac{x^5}{(x^2+1)^2} = x - \frac{2x}{x^2+1} + \frac{x}{(x^2+1)^2}$$