

EXERCISE 5.2

Q.1 If $X = \{1, 3, 5, 7, \dots, 19\}$

$Y = \{0, 2, 4, 6, 8, \dots, 20\}$

$Z = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$, then find the following:

(i) $X \cup (Y \cup Z)$ (ii) $(X \cup Y) \cup Z$

(iii) $X \cap (Y \cap Z)$ (iv) $(X \cap Y) \cap Z$

(v) $X \cup Y \cap Z$ (vi) $(X \cup Y) \cap (X \cup Z)$

(vii) $X \cap (Y \cup Z)$ (viii) $(X \cap Y) \cup (X \cap Z)$

Solution:

(i) $X \cup (Y \cup Z)$

$$= X \cup (\{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$$

$$= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\} \cup \{0, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 23\}$$

$$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23\}$$

(ii) $(X \cup Y) \cup Z$

$$= (\{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 4, 6, 8, \dots, 20\}) \cup Z$$

$$= \{0, 1, 2, 3, 4, 5, 6, 7, \dots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{0, 1, 2, 3, 4, 5, 6, 7, \dots, 20, 23\}$$

(iii) $X \cap (Y \cap Z)$

$$= X \cap (\{0, 2, 4, 6, 8, \dots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$$

$$= \{1, 3, 5, 7, \dots, 19\} \cap \{2\}$$

$$= \phi$$

(iv) $(X \cap Y) \cap Z$

$$= (\{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 4, 6, 8, \dots, 20\}) \cap Z$$

$$= \{ \} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \phi$$

(v) $X \cup (Y \cap Z)$

$$= X \cup (\{0, 2, 4, 6, 8, \dots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$$

$$= \{1, 3, 5, 7, \dots, 19\} \cup \{2\}$$

$$= \{1, 2, 3, 5, 7, \dots, 19\}$$

(vi) $(X \cup Y) \cap (X \cup Z)$

$$X \cup Y = \{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 4, 6, 8, \dots, 20\}$$

$$= \{0, 1, 2, 3, 4, 5, \dots, 20\}$$

$$X \cup Z = \{1, 3, 5, 7, \dots, 19\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 23\}$$

$$(X \cup Y) \cap (X \cup Z)$$

$$= \{0, 1, 2, 3, 4, \dots, 20\} \cap \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 23\}$$

$$= \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$$

(vii) $X \cap (Y \cup Z)$

$$X \cap (Y \cup Z)$$

$$= X \cap (\{0, 2, 4, 6, 8, \dots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$$

$$= \{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 23\}$$

$$= \{3, 5, 7, 11, 13, 17, 19\}$$

(viii) $(X \cap Y) \cup (X \cap Z)$

$$X \cap Y = \{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 4, 6, 8, \dots, 20\}$$

$$= \{ \}$$

$$X \cap Z = \{1, 3, 5, 7, \dots, 19\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{3, 5, 7, 11, 13, 17, 19\}$$

$$(X \cap Y) \cup (X \cap Z) = \{ \} \cup \{3, 5, 7, 11, 13, 17, 19\}$$

$$= \{3, 5, 7, 11, 13, 17, 19\}$$

Q. 2. If $A = \{1, 2, 3, 4, 5, 6\}$

$B = \{2, 4, 6, 8\}$ $C = \{1, 4, 8\}$ Prove the following identities:

(i) $A \cap B = B \cap A$

(ii) $A \cup B = B \cup A$

(iii) $A \cap (B \cap C) = (A \cap B) \cap (A \cap C)$

(iv) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

(i) $A \cap B = B \cap A$

$$\text{L.H.S} = A \cap B$$

$$= \{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}$$

$$= \{2, 4, 6\}$$

$$\text{R.H.S} = B \cap A$$

$$= \{2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6\}$$

$$= \{2, 4, 6\}$$

$$\text{L.H.S} = \text{R.H.S}, \text{ so}$$

$$A \cap B = B \cap A$$

(ii) $A \cup B = B \cup A$

$$\text{L.H.S} = A \cup B$$

$$= \{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$\text{R.H.S} = B \cup A$$

$$= \{2, 4, 6, 8\} \cup \{1, 2, 3, 4, 5, 6\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$\text{L.H.S} = \text{R.H.S},$$

$$\text{So, } A \cup B = B \cup A$$

(iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$\text{L.H.S} = A \cap (B \cup C)$$

$$= A \cap (\{2, 4, 6, 8\} \cup \{1, 4, 8\})$$

$$= \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 4, 6, 8\}$$

$$= \{1, 2, 4, 6\}$$

$$\text{R.H.S} = (A \cap B) \cup (A \cap C)$$

$$A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}$$

$$= \{2, 4, 6\}$$

$$A \cap C = \{1, 2, 3, 4, 5, 6\} \cap \{1, 4, 8\}$$

$$= \{1, 4\}$$

$$(A \cap B) \cup (A \cap C) = \{2, 4, 6\} \cup \{1, 4\}$$

$$= \{1, 2, 4, 6\}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{So, } A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(iv) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$$\text{L.H.S} = A \cup (B \cap C)$$

$$= A \cup (\{2, 4, 6, 8\} \cap \{1, 4, 8\})$$

$$= \{1, 2, 3, 4, 5, 6\} \cup \{4, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$\text{R.H.S} = (A \cup B) \cap (A \cup C)$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$A \cup C = \{1, 2, 3, 4, 5, 6\} \cup \{1, 4, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$(A \cup B) \cap (A \cup C)$$

$$= \{1, 2, 3, 4, 5, 6, 8\} \cap \{1, 2, 3, 4, 5, 6, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$\text{L.H.S} = \text{R.H.S},$$

$$\text{So, } A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Q.3 If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$$A = \{1, 3, 5, 7, 9\}, B = \{2, 3, 5, 7\} \text{ then}$$

verify the De Morgan's laws i.e.,

$$(A \cup B)' = A' \cap B' \text{ and } (A \cap B)' = A' \cup B'$$

Solution:

(i) $(A \cup B)' = A' \cap B'$

$$\text{L.H.S} = (A \cup B)'$$

$$A \cup B = \{1, 3, 5, 7, 9\} \cup \{2, 3, 5, 7\}$$

$$= \{1, 2, 3, 5, 7, 9\}$$

$$(A \cup B)' = U - (A \cup B)$$

$$= \{1, 2, 3, 4, \dots, 10\} - \{1, 2, 3, 5, 7, 9\}$$

$$= \{4, 6, 8, 10\} \dots \dots \dots (i)$$

$$\text{R.H.S} = A' \cap B'$$

$$A' = U - A$$

$$= \{1, 2, 3, 4, \dots, 10\} - \{1, 3, 5, 7, 9\}$$

$$= \{2, 4, 6, 8, 10\}$$

$$B' = U - B$$

$$= \{1, 2, 3, 4, 5, \dots, 10\} - \{2, 3, 5, 7\}$$

$$= \{1, 4, 6, 8, 9, 10\}$$

$$A' \cap B' = \{2, 4, 6, 8, 10\} \cap \{1, 4, 6, 8, 9, 10\}$$

$$= \{4, 6, 8, 10\} \dots \dots \dots (ii)$$

From (i) and (ii)

$$L.H.S = R.H.S$$

$$(A \cup B)' = A' \cap B'$$

$$(ii) (A \cap B)' = A' \cup B'$$

$$L.H.S = (A \cap B)'$$

$$A \cap B = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\}$$

$$= \{3, 5, 7\}$$

$$(A \cap B)' = U - (A \cap B)$$

$$= \{1, 2, 3, 4, \dots, 10\} - \{3, 5, 7\}$$

$$= \{1, 2, 4, 6, 8, 9, 10\} \dots \dots \dots (i)$$

$$R.H.S = A' \cup B'$$

$$A' = U - A$$

$$= \{1, 2, 3, 4, 5, \dots, 10\} - \{1, 3, 5, 7, 9\}$$

$$= \{2, 4, 6, 8, 10\}$$

$$B' = U - B$$

$$= \{1, 2, 3, 4, 5, \dots, 10\} - \{2, 3, 5, 7\}$$

$$= \{1, 4, 6, 8, 9, 10\}$$

$$A' \cup B' = \{2, 4, 6, 8, 10\} \cup \{1, 4, 6, 8, 9, 10\}$$

$$= \{1, 2, 4, 6, 8, 9, 10\} \dots \dots \dots (ii)$$

From (i) and (ii)

$$L.H.S = R.H.S$$

$$\text{So } (A \cap B)' = A' \cup B'$$

Q.4 If $U = \{1, 2, 3, \dots, 20\}$

$$X = \{1, 3, 7, 9, 15, 18, 20\}$$

$$Y = \{1, 3, 5, \dots, 17\} \text{ then show that,}$$

$$(i) X - Y = X \cap Y' \quad (ii) Y - X = Y \cap X'$$

Solution:

$$(i) X - Y = X \cap Y'$$

$$L.H.S = X - Y$$

$$= \{1, 3, 7, 9, 15, 18, 20\} -$$

$$\{1, 3, 5, 7, 9, 11, 13, 15, 17\}$$

$$= \{18, 20\} \dots \dots \dots (i)$$

$$R.H.S = X \cap Y'$$

$$Y' = U - Y$$

$$= \{1, 2, 3, \dots, 20\} - \{1, 3, 5, \dots, 17\}$$

$$= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 19, 20\}$$

$$X \cap Y' = \{1, 3, 7, 9, 15, 18, 20\} \cap$$

$$\{2, 4, 6, 8, 10, 12, 14, 16, 18, 19, 20\}$$

$$= \{18, 20\} \dots \dots \dots (ii)$$

From (i) and (ii)

$$L.H.S = R.H.S,$$

$$\text{So, } X - Y = X \cap Y'$$

$$(ii) Y - X = Y \cap X'$$

$$L.H.S = Y - X$$

$$= \{1, 3, 5, \dots, 17\} - \{1, 3, 7, 9, 15, 18, 20\}$$

$$= \{5, 11, 13, 17\} \dots \dots \dots (i)$$

$$R.H.S = Y \cap X'$$

$$X' = U - X$$

$$= \{1, 2, 3, \dots, 20\} - \{1, 3, 7, 9, 15, 18, 20\}$$

$$= \{2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16, 17, 19\}$$

$$Y \cap X' = \{1, 3, 5, 7, 9, 11, 13, 15, 17\} \cap$$

$$\{2, 4, 5, 6, 8, 10, 11, 12, 13, 14,$$

$$16, 17, 19\}$$

$$= \{5, 11, 13, 17\} \dots \dots \dots (ii)$$

From (i) and (ii)

$$L.H.S = R.H.S$$

$$\text{So, } Y - X = Y \cap X'$$

Verification of Fundamental Properties of Sets

(a) Commutative Property of Union i.e.,

$$A \cup B = B \cup A$$

For example $A = \{1, 3, 5, 7\}$ and

$$B = \{2, 3, 5, 7\}$$

$$\text{Then } A \cup B = \{1, 3, 5, 7\} \cup \{2, 3, 5, 7\}$$

$$= \{1, 2, 3, 5, 7\}$$

$$\text{and } B \cup A = \{2, 3, 5, 7\} \cup \{1, 3, 5, 7\}$$

$$= \{1, 2, 3, 5, 7\}$$

Hence, verified that $A \cup B = B \cup A$

(b) Commutative property of intersection

$$\text{i.e., } A \cap B = B \cap A$$

For example $A = \{1, 3, 5, 7\}$ and

$$B = \{2, 3, 5, 7\}$$

$$\text{Then } A \cap B = \{1, 3, 5, 7\} \cap \{2, 3, 5, 7\}$$

$$= \{3, 5, 7\}$$

$$\text{and } B \cap A = \{2, 3, 5, 7\} \cap \{1, 3, 5, 7\}$$

$$= \{3, 5, 7\}$$

Hence, verified that $A \cap B = B \cap A$

(c) Associative Property of Union

$$\text{i.e., } (A \cup B) \cup C = A \cup (B \cup C).$$

Suppose $A = \{1, 2, 4, 8\}$, $B = \{2, 4, 6\}$

and $C = \{3, 4, 5, 6\}$ then

$$\text{L.H.S} = (A \cup B) \cup C$$

$$= (\{1, 2, 4, 8\} \cup \{2, 4, 6\}) \cup \{3, 4, 5, 6\}$$

$$= \{1, 2, 4, 6, 8\} \cup \{3, 4, 5, 6\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$\text{and R.H.S} = A \cup (B \cup C)$$

$$= \{1, 2, 4, 8\} \cup (\{2, 4, 6\} \cup \{3, 4, 5, 6\})$$

$$= \{1, 2, 4, 8\} \cup \{2, 3, 4, 5, 6\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$\text{L.H.S} = \text{R.H.S}$$

Hence, union of sets is associative

(d) Associative Property of intersection

$$\text{i.e., } (A \cap B) \cap C = A \cap (B \cap C)$$

Suppose $A = \{1, 2, 4, 8\}$

$$B = \{2, 4, 6\}$$

and $C = \{3, 4, 5, 6\}$

Then L.H.S = $(A \cap B) \cap C$

$$= (\{1, 2, 4, 8\} \cap \{2, 4, 6\}) \cap \{3, 4, 5, 6\}$$

$$= \{2, 4\} \cap \{3, 4, 5, 6\}$$

$$= \{4\}$$

and R.H.S = $A \cap (B \cap C)$

$$= \{1, 2, 4, 8\} \cap (\{2, 4, 6\} \cap \{3, 4, 5, 6\})$$

$$= \{1, 2, 4, 8\} \cap \{4, 6\}$$

$$= \{4\}$$

$$\text{L.H.S} = \text{R.H.S}$$

Hence, intersection of sets is associative law.

(e) Distributive Property of Union over Intersection

$$\text{i.e., } A \cup (B \cap C) = (A \cup B) \cap (A \cup C):$$

Suppose $A = \{1, 2, 4, 8\}$

$$B = \{2, 4, 6\} \text{ and}$$

$$C = \{3, 4, 5, 6\}$$

Let L.H.S = $A \cup (B \cap C)$

$$= \{1, 2, 4, 8\} \cup (\{2, 4, 6\} \cap \{3, 4, 5, 6\})$$

$$= \{1, 2, 4, 8\} \cup \{4, 6\}$$

$$= \{1, 2, 4, 6, 8\}$$

R.H.S = $(A \cup B) \cap (A \cup C)$

$$(A \cup B) = (\{1, 2, 4, 8\} \cup \{2, 4, 6\})$$

$$= \{1, 2, 4, 6, 8\}$$

$$(A \cup C) = (\{1, 2, 4, 8\} \cup \{3, 4, 5, 6\})$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$(A \cup B) \cap (A \cup C)$$

$$= \{1, 2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6, 8\}$$

$$= \{1, 2, 4, 6, 8\}$$

$$\text{L.H.S} = \text{R.H.S}$$

(f) **Distributive Property of Intersection over Union**

$$\text{i.e., } A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Suppose

$$A = \{1, 2, 3, 4, 5, \dots, 20\}$$

$$B = \{5, 10, 15, 20, 25, 30\}$$

$$C = \{3, 9, 15, 21, 27, 33\}$$

$$\text{L.H.S} = A \cap (B \cup C)$$

$$= A \cap (\{5, 10, 15, 20, 25, 30\} \cup \{3, 9, 15, 21, 27, 33\})$$

$$= \{1, 2, 3, 4, 5, \dots, 20\} \cap \{3, 5, 9, 10, 15, 20, 21, 25, 27, 30, 33\}$$

$$\text{L.H.S} = \{3, 5, 9, 10, 15, 20\}$$

$$\text{R.H.S} = (A \cap B) \cup (A \cap C)$$

$$(A \cap B) = \{1, 2, 3, 4, \dots, 20\} \cap \{5, 10, 15, 20, 25, 30\} \\ = \{5, 10, 15, 20\}$$

$$(A \cap C) = \{1, 2, 3, 4, 5, \dots, 20\} \cap \{3, 9, 15, 21, 27, 33\} \\ = \{3, 9, 15\}$$

$$(A \cap B) \cup (A \cap C) \\ = \{5, 10, 15, 20\} \cup \{3, 9, 15\} \\ = \{3, 5, 9, 10, 15, 20\}$$

$$\text{L.H.S} = \text{R.H.S}$$

(g) **De Morgan's laws**

If set A and B are the subsets of universal set U then De Morgan's laws are expressed as.

$$(i) (A \cup B)' = A' \cap B'$$

$$(ii) (A \cap B)' = A' \cup B'$$

Proof:

$$(i) (A \cup B)' = A' \cap B'$$

Suppose

$$U = \{1, 2, 3, 4, \dots, 10\}$$

$$A = \{2, 4, 6, 8, 10\} \Rightarrow A' = \{1, 3, 5, 7, 9\}$$

$$B = \{1, 2, 3, 4, 5, 6\} \Rightarrow B' = \{7, 8, 9, 10\}$$

$$\text{L.H.S} = (A \cup B)'$$

$$(A \cup B) = \{2, 4, 6, 8, 10\} \cup \{1, 2, 3, 4, 5, 6\} \\ = \{1, 2, 3, 4, 5, 6, 8, 10\}$$

$$\text{L.H.S} = (A \cup B)' \\ = U - (A \cup B) \\ = \{1, 2, 3, 4, \dots, 10\} - \{1, 2, 3, 4, 5, 6, 8, 10\}$$

$$\text{L.H.S} = \{7, 9\} \dots \dots \dots (i)$$

$$\text{R.H.S} = A' \cap B' \\ = \{1, 3, 5, 7, 9\} \cap \{7, 8, 9, 10\} \\ = \{7, 9\} \dots \dots \dots (ii)$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$(ii) (A \cap B)' = A' \cup B'$$

Suppose

$$U = \{1, 2, 3, 4, \dots, 10\}$$

$$A = \{2, 4, 6, 8, 10\} \Rightarrow A' = \{1, 3, 5, 7, 9\}$$

$$B = \{1, 2, 3, 4, 5, 6\} \Rightarrow B' = \{7, 8, 9, 10\}$$

$$\text{Let L.H.S} = (A \cap B)'$$

$$A \cap B = \{2, 4, 6, 8, 10\} \cap \{1, 2, 3, 4, 5, 6\} \\ = \{2, 4, 6\}$$

$$\text{L.H.S} = (A \cap B)' \\ = U - (A \cap B) \\ = \{1, 2, 3, 4, \dots, 10\} - \{2, 4, 6\} \\ = \{1, 3, 5, 7, 8, 9, 10\} \dots \dots \dots (i)$$

$$\text{R.H.S} = A' \cup B' \\ = \{1, 3, 5, 7, 9\} \cup \{7, 8, 9, 10\} \\ = \{1, 3, 5, 7, 8, 9, 10\} \dots \dots \dots (ii)$$

From (i) and (ii)

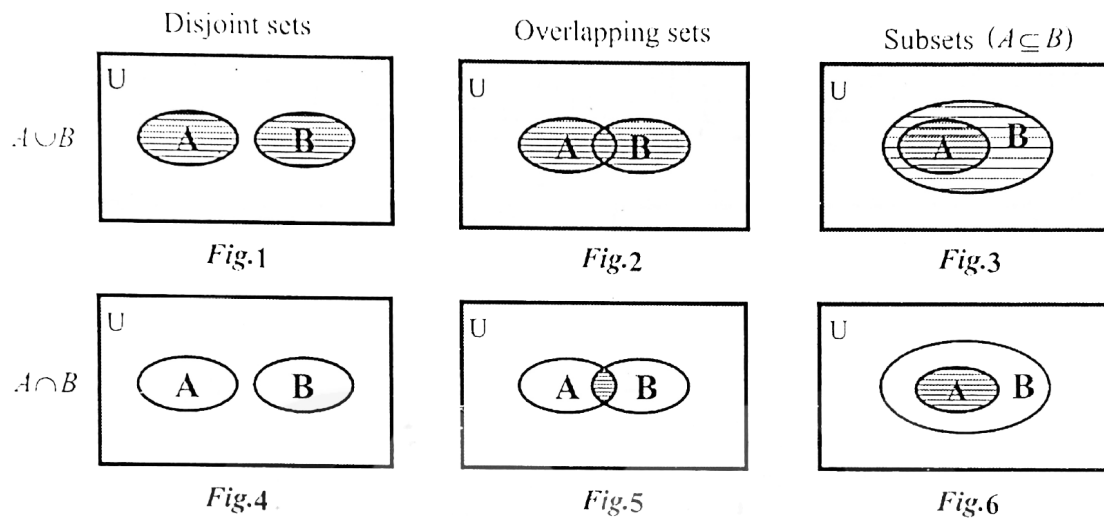
$$\text{L.H.S} = \text{R.H.S}$$

Venn Diagram

British mathematician John Venn (1834–1923) introduced rectangle for a universal set U and its subsets A and B as closed figures inside this rectangle.

Use Venn diagrams to represent:

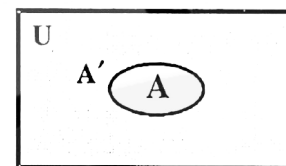
(a) Union and intersection of sets



Regions shown by horizontal line segments in figures 1 to 6 shows $A \cup B$ and $A \cap B$

(b) Complement of a set

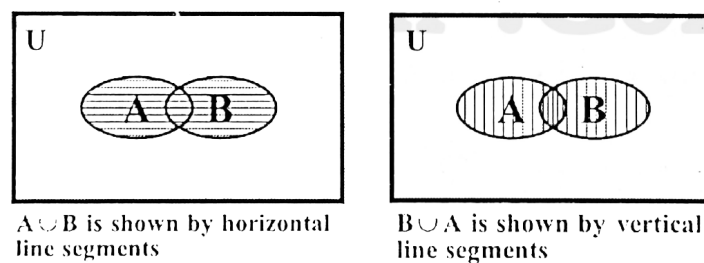
$U - A = A'$ is shown by Shaded area.



Use Venn diagram to verify:

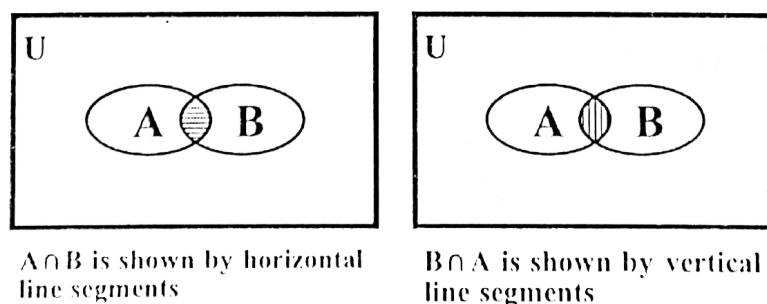
Commutative law for union and intersection of sets.

- Commutative Law for Union:



The region shown in both cases are equal. Thus $A \cup B = B \cup A$.

- Commutative Law for Intersection:



The regions shown in both cases are equal. Thus $A \cap B = B \cap A$

(c) De Morgan's laws

$$(i) (A \cup B)' = A' \cap B'$$

$$(ii) (A \cap B)' = A' \cup B'$$

$$(i) (A \cup B)' = A' \cap B'$$

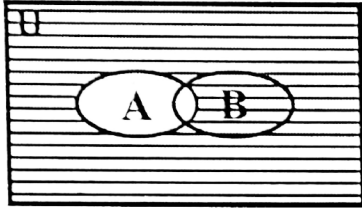


Fig. 1: A' is shown by horizontal line segments

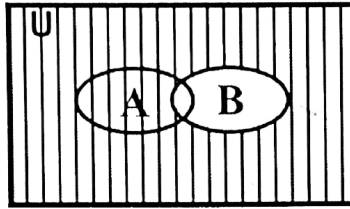


Fig. 2: B' is shown by vertical line segments

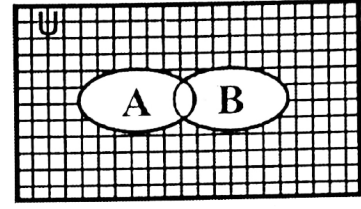


Fig. 3: $A' \cap B'$ is shown by squares

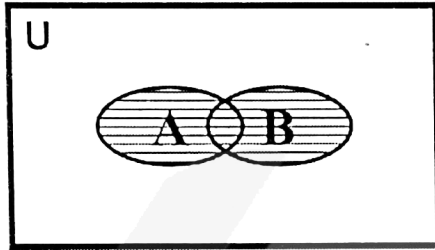


Fig. 4: $A \cup B$ is shown by horizontal line segments

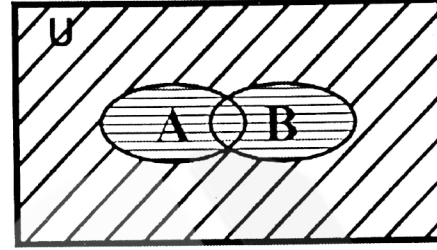


Fig. 5: $(A \cup B)'$ is shown by slanting line segments

Regions shown in figure 3 and 5 are equal thus $(A \cup B)' = A' \cap B'$.

$$(ii) (A \cap B)' = A' \cup B'$$

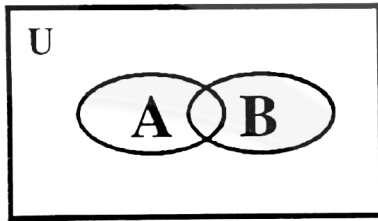


Fig. 6: $U - (A \cap B) (=) A \cap B'$ is shown by shading

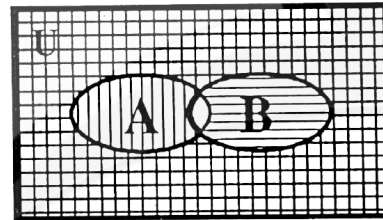


Fig. 7: $A' \cup B'$ is shown by squares, horizontal and vertical line segments.

Regions shown in fig. 6 and fig. 7 are equal.

$$\text{Thus } (A \cap B)' = A' \cup B'$$

(d) Associative law of Union and Intersection:

$$(i) \text{ Associative law of Union: } (A \cup B) \cup C = A \cup (B \cup C)$$

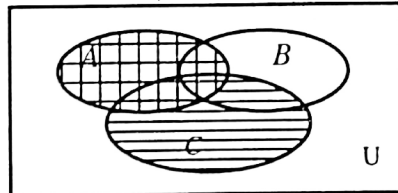


Fig-1

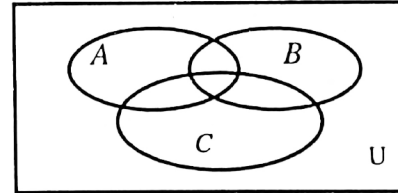


Fig-2

$(A \cup B) \cup C$ is shown in the above Fig-1. $A \cup (B \cup C)$ is shown in the above Fig-2.

Regions shown in Fig. 1 and Fig. 2 by different ways are equal.

$$\text{Thus } (A \cup B) \cup C = A \cup (B \cup C)$$

(ii) Associative law of Intersection: $(A \cap B) \cap C = A \cap (B \cap C)$

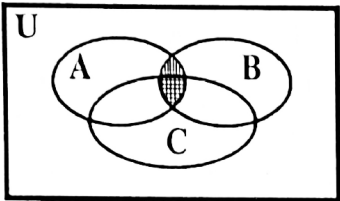


Fig. 3

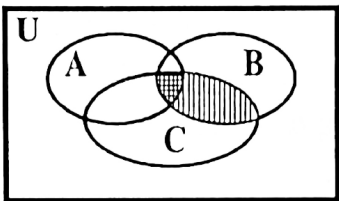


Fig. 4

$(A \cap B) \cap C$ is shown in figure 3 by double crossing line segments.

$A \cap (B \cap C)$ is shown in figure 4 by double crossing line segments.

Regions shown in Fig 3 and Fig. 4 are equal.

Thus $(A \cap B) \cap C = A \cap (B \cap C)$

(e) Distributive law:

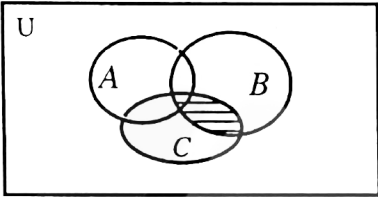


Fig. 1

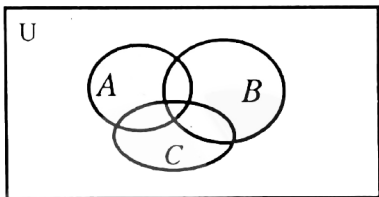


Fig. 2

Fig. 1: $A \cup (B \cap C)$ is shown by horizontal line segments in the above figure 1.

Fig. 2: $A \cup B$ is shown by horizontal line segments in the fig. 2.

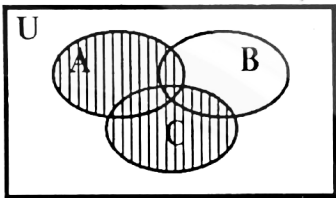


Fig. 3

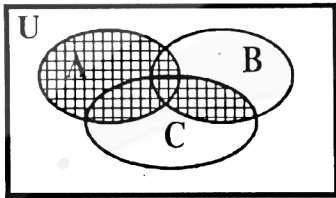


Fig. 4

Fig. 3: $A \cup C$ is shown by vertical line segments in fig.3

Fig. 4: $(A \cup B) \cap (A \cup C)$ is shown by double crossing line segments in fig. 4.

Regions shown in fig 1 and Fig.4 are equal. Thus $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Distributive Law of Intersection over Union:

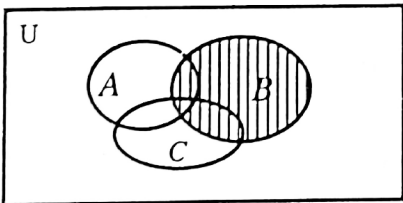


Fig.5

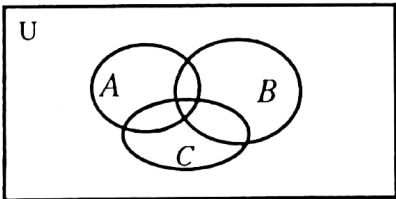


Fig.6

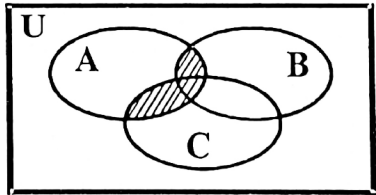


Fig. 7

Fig. 5: $B \cup C$ is shown by vertical line segments in Fig 5.

Fig. 6: $A \cap (B \cup C)$ is shown in Fig.6 by vertical line segments.

Fig. 7: $(A \cap B) \cup (A \cap C)$ is shown in fig. 7 by slanting line segments.

Regions displayed in Fig. 6 and Fig. 7 are equal.

Thus $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$