EXERCISE 5.3

Q.1 If $U = \{1, 2, 3, 4, \ldots, 10\}$ $A = \{1, 3, 5, 7, 9\}$, $B = \{1, 4, 7, 10\}$ then verify the following questions:

Solution:

(i) $A - B = A \cap B'$

L.H.S $= A - B = \{1,3,5,7,9\} - \{1,4,7,10\}$
$= \{3, 5, 9\}$ ..... (i)

R.H.S $= A \cap B'$

$B' = U - B = \{1, 2, 3, \ldots, 10\} - \{1, 4, 7, 10\}$
$= \{2, 3, 5, 6, 8, 9\}$

$A \cap B' = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 6, 8, 9\}$
$= \{3, 5, 9\}$ ..... (ii)

From (i) and (ii)

L.H.S = R.H.S

$A - B = A \cap B'$

(ii) $B - A = B \cap A'$

L.H.S $= B - A = \{1,4,7,10\} - \{1, 3, 5, 7, 9\}$
$= \{4, 10\}$ ..... (i)

R.H.S $= B \cap A'$

$A' = U - A = \{1,2,3, \ldots, 10\} - \{1,3,5,7,9\}$
$= \{2, 4, 6, 8, 10\}$

$B \cap A' = \{1, 4, 7, 10\} \cap \{2, 4, 6, 8, 10\}$
$= \{4, 10\}$ ..... (ii)

From (i) and (ii)

L.H.S = R.H.S

$B - A = B \cap A'$

(iii) $(A \cup B)' = A' \cap B'$

L.H.S $= (A \cup B)'$

$A \cup B = \{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\}$
$= \{1, 3, 4, 5, 7, 9, 10\}$

$(A \cup B)' = U - (A \cup B)$
$= \{1,2,3,\ldots,10\} - \{1,3,4,5,7,9,10\}$
$= \{2, 6, 8\}$ ..... (i)

R.H.S $= A' \cap B'$

$A' = U - A = \{1,2,3,\ldots, 10\} - \{1,3,5,7,9\}$
$= \{2, 4, 6, 8, 10\}$

$B' = U - B = \{1,2,3,\ldots, 10\} - \{1,4,7,10\}$
$= \{2, 3, 5, 6, 8, 9\}$

$A' \cap B' = \{2, 4, 6, 8, 10\} \cap \{2, 3, 5, 6, 8, 9\}$
$= \{2, 6, 8\}$ ..... (ii)

From (i) and (ii)

L.H.S = R.H.S

(iv) $(A \cap B)' = A' \cup B'$

L.H.S $= (A \cap B)'$

$A \cap B = \{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\}$
$= \{1, 7\}$

$(A \cap B)' = U - (A \cap B)$
$= \{1, 2, 3, \ldots, 10\} - \{1, 7\}$
$= \{2, 3, 4, 5, 6, 8, 9, 10\}$ ..... (i)

R.H.S $= A' \cup B'$

$A' = U - A = \{1,2,3,\ldots,10\} - \{1,3,5,7,9\}$
$= \{2, 4, 6, 8, 10\}$

$B' = U - B = \{1,2,3,\ldots, 10\} - \{1, 4,7,10\}$
$= \{2, 3, 5, 6, 8, 9\}$

$A' \cup B' = \{2, 4, 6, 8, 10\} \cup \{2, 3, 5, 6, 8, 9\}$
$= \{2, 3, 4, 5, 6, 8, 9, 10\}$..... (ii)

From (i) and (ii)

L.H.S = R.H.S

(v) $(A - B)' = A' \cup B$

L.H.S $= (A - B)'$

$A - B = \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$
$= \{3, 5, 9\}$

$(A - B)' = U - (A - B)$
$= \{1, 2, 3, \ldots, 10\} - \{3, 5, 9\}$
$= \{1, 2, 4, 6, 7, 8, 10\}$..... (i)

R.H.S $= A' \cup B$

$A' = U - A = \{1,2,3, \ldots,10\} - \{1,3,5,7,9\}$
$= \{2, 4, 6, 8, 10\}$

$A' \cup B = \{2, 4, 6, 8, 10\} \cup \{1, 4, 7, 10\}$
$= \{1, 2, 4, 6, 7, 8, 10\}$..... (ii)

From (i) and (ii)

L.H.S = R.H.S

(vi) $(B - A)' = B' \cup A$

L.H.S $= (B - A)'$

$B - A = \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\}$
$= \{4, 10\}$

$(B - A)' = U - (B - A)$
$= \{1, 2, 3, \ldots, 10\} - \{4, 10\}$
$= \{1, 2, 3, 5, 6, 7, 8, 9\}$..... (i)

R.H.S $= B' \cup A$

$B' = U - B = \{1,2,3,4,\ldots,10\} - \{1, 4, 7, 10\}$
$= \{2, 3, 5, 6, 8, 9\}$

$B' \cup A = \{2,3,5,6,8,9\} \cup \{1, 3, 5, 7, 9\}$
$= \{1, 2, 3, 5, 6, 7, 8, 9\}$..... (ii)

From (i) and (ii)

L.H.S = R.H.S

Q.2 If A = {1, 3, 5, 7, 9} B = {1, 4, 7, 10} C = {1, 5, 8, 10} Then verily the following:

Solution:

(i) (A∪B) ∪C = A∪(B∪C)

L.H.S = (A∪B)∪C

= ({1,3,5,7,9}∪{1,4,7,10}) ∪ {1,5,8,10}

= {1, 3, 4, 5, 7, 9, 10} ∪ {1, 5, 8, 10}

= {1, 3, 4, 5, 7, 8, 9, 10} ..... (i)

R.H.S = A∪(B∪C)

= {1,3,5,7,9}∪({1,4,7,10} ∪ {1,5,8,10})

= {1,3,5,7,9} ∪ {1,4,5,7,8,10}

= {1, 3, 4, 5, 7, 8, 9, 10} ..... (ii)

From (i) and (ii)

L.H.S = R.H.S

(ii) (A∩B)∩C = A∩(B∩C)

L.H.S = (A∩B)∩C

= ({1,3,5,7,9}∩{1,4,7,10}) ∩ {1,5,8,10}

= {1, 7} ∩ {1, 5, 8, 10}

= {1} ..... (i)

R.H.S = A∩(B∩C)

= {1,3,5,7,9}∩({1,4,7,10} ∩ {1,5,8,10})

= {1, 3, 5, 7, 9} ∩ {1, 10}

= {1} ..... (ii)

From (i) and (ii)

L.H.S = R.H.S

(iii) A∪(B∩C) = (A∪B)∩(A∪C)

L.H.S = A∪(B∩C)

= {1,3,5,7,9}∪({1,4,7,10} ∩ {1,5,8,10})

= {1, 3, 5, 7, 9} ∪ {1, 10}

= {1, 3, 5, 7, 9, 10} ..... (i)

R.H.S = (A∪B)∩(A∪C)

A∪B = {1, 3, 5, 7, 9} ∪ {1, 4, 7, 10}

= {1, 3, 4, 5, 7, 9, 10}

A∪C = {1, 3, 5, 7, 9} ∪ {1, 5, 8, 10}

= {1, 3, 5, 7, 8, 9, 10}

Now (A∪B)∩(A∪C)

= {1,3,4,5,7,9,10} ∩ {1,3,5,7,8,9,10}

= {1, 3, 5, 7, 9, 10} ..... (ii)

From (i) and (ii)

L.H.S = R.H.S

(iv) A∩(B∪C) = (A∩B)∪(A∩C)

L.H.S = A∩(B∪C)

= {1,3,5,7,9}∩({1,4,7,10}∪{1, 5, 8, 10})

= {1, 3, 5, 7, 9} ∩ {1, 4, 5, 7, 8, 10}

= {1, 5, 7} ..... (i)

R.H.S = (A∩B)∪(A∩C)

A∩B = {1,3,5,7,9} ∩ {1, 4, 7, 10}

= {1, 7}

A∩C = {1,3, 5, 7, 9} ∩ {1, 5, 8, 10}

= {1, 5}

Now (A∩B)∪(A∩C) = {1,7}∪{1,5}

= {1,5,7}..... (ii)

From (i) and (ii)

L.H.S = R.H.S

Q.3 If U = N, then verify De-Morgan's laws by using:

A = ϕ, B = P

Solution:

A = { }

B = {2, 3, 5, 7, ...}

U = {1, 2, 3, 4, 5, 6, 7....}

(i) (A∪B)′ = A′∩B′

L.H.S = (A∪B)′

A∪B = { } ∪ {2, 3, 5, 7, ...}

= {2, 3, 5, 7, ...}

(A∪B)′ = U – (A∪B)

= {1, 2, 3, 4,5,6,7 ....} – {2, 3, 5, 7, ...

= {1, 4, 6, ...} ..... (i)

R.H.S = A′∩B′

A′ = U – A = {1, 2, 3, 4, 5, 6, 7.....} – { }

= {1, 2, 3, 4, 5, 6, 7....}

B′ = U – B = {1,2,3,4,5,6,7....}–{2,3,5,7,....}

= {1, 4, 6, ...}

A′∩B′ = {1,2,3,4,5,6,7 ....} ∩ {1, 4, 6, ...}

= {1, 4, 6, ...} ..... (ii)

From (i) and (ii)

L.H.S = R.H.S

(ii) $(A \cap B)' = A' \cup B'$

L.H.S $= (A \cap B)'$

$(A \cap B) = \{\ \} \cap \{2, 3, 5, 7, \ldots\}$

$= \{\ \}$

$(A \cap B)' = U - (A \cap B)$

$= \{1, 2, 3, 4, 5, 6, 7 \ldots.\} - \{\ \}$

$= \{1, 2, 3, 4, 5, 6, 7\ldots\}$..... (i)

R.H.S $= A' \cup B'$

$A' = U - A = \{1, 2, 3, 4, \ldots\} - \{\ \}$

$= \{1, 2, 3, 4, \ldots\}$

$B' = U - B = \{1,2,3,4,5,6,7\ldots.\} - \{2,3,5,7,\ldots\}$

$= \{1, 4, 6, \ldots\}$

$A' \cup B' = \{1,2,3,4,5,6,7 \ldots\} \cup \{1, 4, 6, \ldots\}$

$= \{1, 2, 3, 4, \ldots\}$..... (ii)

From (i) and (ii)

L.H.S = R.H.S

Q.4 If $\cup = \{1, 2, 3, 4, \ldots.10\}$, $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 3, 4, 5, 8\}$ then prove the following questions by Venn Diagram.

Solution: $A \cap B = \{1, 3, 5, 7, 9\} \cap \{2, 3, 4, 5, 8\} = \{3, 5\}$

So given sets A and B are overlapping sets

(i) **$A - B = A \cap B'$**

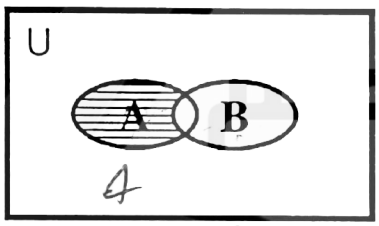

Fig: 1 (A-B)

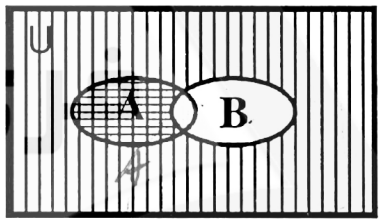

Fig: 2 $(A \cap B')$

- A – B is shown by horizontal line segments in fig. 1.
- B′ is shown by vertical line segments and squares in fig. 2.
- $A \cap B'$ is shown by squares in fig. 2.

Regions shown in fig. 1 and fig. 2 are equal, thus $A - B = A \cap B'$

(ii) **$B - A = B \cap A'$**

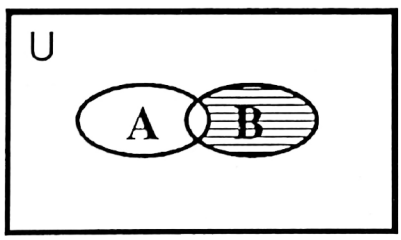

Fig. 1 (B - A)

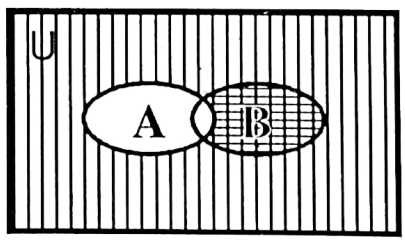

Fig: 2 $(B \cap A')$

- B – A is shown by horizontal line segments in fig. 1.
- A′ is shown by vertical line segments and squares in fig. 2.
- $B \cap A'$ is shown by squares in fig. 2.

Regions shown in fig. 1 and fig. 2 are equal, thus $B - A = B \cap A'$.

(iii) $(A \cup B)' = A' \cap B'$ **(De-Morgan's Law)**

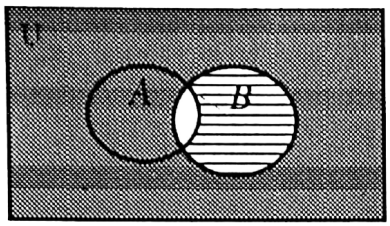

Fig.1

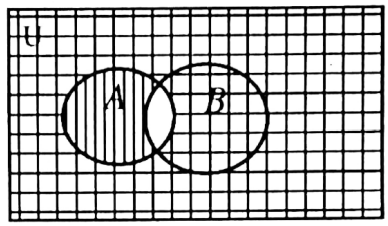

Fig.2

- $A \cup B$ is shown by horizontal line segments in Fig. 1.
- $(A \cup B)'$ is shown by shaded area in Fig. 1.
- A' is shown by horizontal line segments and squares in Fig. 2.
- B' is shown by vertical line segments and squares in Fig. 2.
- $A' \cap B'$ is shown by squares in Fig. 2.

  Shaded area shown in Fig. 1 and square area shown in Fig. 2 are equal.

  thus $(A \cup B)' = A' \cap B'$

(iv) $(A \cap B)' = A' \cup B'$

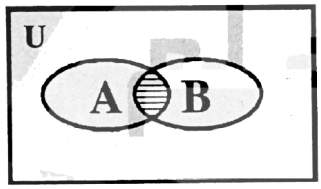

Fig. 1

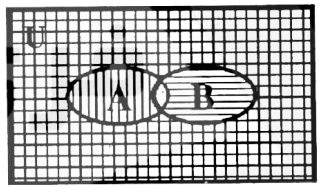

Fig. 2

- $A \cap B$ is shown by horizontal line segments in Fig. 1.
- $(A \cap B)'$ is shown by shaded area in Fig. 1.
- A' is shown by horizontal line segments and squares in Fig. 2.
- B' is shown by vertical line segments and squares in Fig. 2.
- $A' \cup B'$ is shown by squares, horizontal and vertical line segments in Fig. 2.

  Shaded area shown in Fig. 1 and area of squares, vertical and horizontal line segments shown in Fig. 2 are equal. thus $(A \cap B)' = A' \cup B'$

(v) $(A - B)' = A' \cup B$

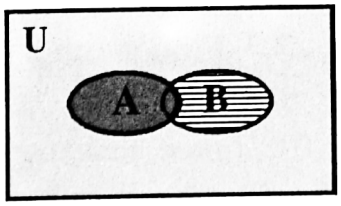

Fig. 1

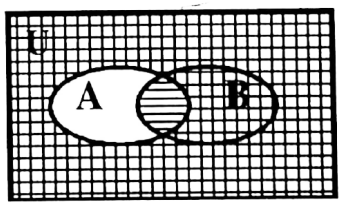

Fig. 2

- $A - B$ is shown by horizontal line segments in Fig. 1.
- $(A - B)'$ is shown by shaded area in Fig.1.
- A' is shown by squares Fig. 2.
- $A' \cup B$ is shown by squares and horizontal line segments in Fig. 2.

  Shaded area in Fig. 1 and area of squares & horizontal line segments in Fig. 2 are equal.

  thus $(A - B)' = A' \cup B$

(vi) $(B-A)' = B' \cup A$

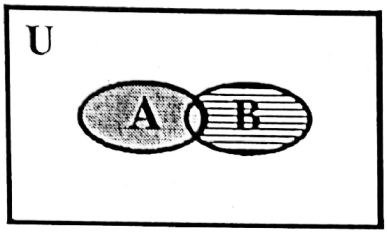

Fig. 1

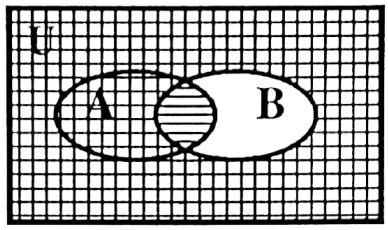

Fig. 2

- B–A is shown by horizontal line segments in Fig. 1.
- $(B-A)'$ is shown by shaded area in Fig.1.
- B' is shown by squares in Fig. 2.
- $B' \cup A$ is shown by squares and horizontal line segments in Fig. 2.
  Shaded area in Fig. 1 and area of squares & vertical line segments in Fig. 2 are equal.
  thus $(B-A)' = B' \cup A$

Ordered Pairs and Cartesian Product:

(a) Ordered pairs

Any two numbers x and y, written in the form (x, y) is called an ordered pair. In an ordered pair (x, y) the order of numbers is important, that is, x is the first co-ordinate and y is the second co-ordinate. For example, (3, 2) is different from (2, 3).

It is obvious that $(x, y) \neq (y, x)$ unless $x = y$.

Note that $(x, y) = (s, t)$, iff $x=s$ and $y = t$

(b) Cartesian product:

Cartesian product of two non-empty sets A and B denoted by $A \times B$ consists of all ordered pairs (x, y) such that $x \in A$ and $y \in B$.

$$A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$$

Example: If A = {1, 2, 3} and B = {2, 5}, then find A × B and B × A

Solution:

$A \times B = \{1,2,3\} \times \{2,5\}$

$A \times B = \{(1,2),(1,5),(2,2),(2,5),(3,2)(3,5)\}$

Since set A has 3 elements and set B has 2 elements.

Hence product set $A \times B$ has $3 \times 2 = 6$ ordered pairs. We note that

$B \times A = \{(2,1),(2,2),(2,3),(5,1),(5,2),(5,3)\}$

Evidently $A \times B \neq B \times A$