

EXERCISE 7.4

In problem 1—6, simply each expression to single trigonometric function:

Q.1. $\frac{\sin^2 x}{\cos^2 x}$

Solution: $\frac{\sin^2 x}{\cos^2 x} = \tan^2 x$

Q.2. $\tan x \sin x \sec x$

Solution: $\tan x \sin x \sec x$

$$= \frac{\sin x}{\cos x} \cdot \sin x \cdot \frac{1}{\cos x}$$

$$= \frac{\sin^2 x}{\cos^2 x}$$

$$= \tan^2 x$$

Q.3. $\frac{\tan x}{\sec x}$

Solution: $\frac{\tan x}{\sec x} = \tan x \div \sec x$

$$= \frac{\sin x}{\cos x} \div \frac{1}{\cos x}$$

$$= \frac{\sin x}{\cancel{\cos x}} \times \cancel{\cos x}$$

$$= \sin x$$

Q.4. $1 - \cos^2 x$

Solution: $1 - \cos^2 x$

$$= \sin^2 x + \cancel{\cos^2 x} - \cancel{\cos^2 x}$$

$$= \sin^2 x$$

Q.5. $\sec^2 x - 1$

Solution: $\sec^2 x - 1$
 $= 1 + \tan^2 x - 1$
 $= \tan^2 x$

Q.6. $\sin^2 x \cdot \cot^2 x$

Solution:
 $\sin^2 x \cdot \cot^2 x$
 $= \cancel{\sin^2 x} \cdot \frac{\cos^2 x}{\cancel{\sin^2 x}}$
 $= \cos^2 x$

In problem 7 — 24, verify the identities

Q.7. $(1 - \sin \theta)(1 + \sin \theta) = \cos^2 \theta$

Solution:

L.H.S = $(1 - \sin \theta)(1 + \sin \theta)$
 $= (1)^2 - (\sin \theta)^2$
 $= 1 - \sin^2 \theta$
 $= \cos^2 \theta$

L.H.S = R.H.S

Q.8. $\frac{\sin \theta + \cos \theta}{\cos \theta} = 1 + \tan \theta$

Solution: Let

L.H.S = $\frac{\sin \theta + \cos \theta}{\cos \theta}$
 $= \frac{\cos \theta + \sin \theta}{\cos \theta}$
 $= \frac{\cos \theta}{\cos \theta} + \frac{\sin \theta}{\cos \theta}$
 $= 1 + \tan \theta \quad \because \frac{\sin \theta}{\cos \theta} = \tan \theta$

L.H.S = R.H.S

Q.9. $(\tan \theta + \cot \theta) \tan \theta = \sec^2 \theta$

Solution: Let

L.H.S = $(\tan \theta + \cot \theta) \tan \theta$
 $= \tan^2 \theta + \cot \theta \cdot \tan \theta$
 $= \tan^2 \theta + \frac{1}{\cancel{\tan \theta}} \cdot \cancel{\tan \theta}$
 $= 1 + \tan^2 \theta$
 $= \sec^2 \theta \quad (1 + \tan^2 \theta = \sec^2 \theta)$

L.H.S = R.H.S

Q.10. $(\cot \theta + \operatorname{cosec} \theta)(\tan \theta - \sin \theta) = \sec \theta - \cos \theta$

Solution: Let

L.H.S = $(\cot \theta + \operatorname{cosec} \theta)(\tan \theta - \sin \theta)$
 $= \frac{1}{\tan \theta} + \frac{1}{\sin \theta} (\tan \theta - \sin \theta)$
 $= \frac{\sin \theta + \tan \theta}{\tan \theta \cdot \sin \theta} (\tan \theta - \sin \theta)$
 $= \frac{(\tan \theta + \sin \theta)(\tan \theta - \sin \theta)}{\tan \theta \cdot \sin \theta}$
 $= \frac{(\tan \theta)^2 - (\sin \theta)^2}{\tan \theta \cdot \sin \theta}$
 $= \frac{\tan^2 \theta - \sin^2 \theta}{\tan \theta \cdot \sin \theta}$
 $= \frac{\tan^2 \theta}{\tan \theta \cdot \sin \theta} - \frac{\sin^2 \theta}{\tan \theta \cdot \sin \theta}$
 $= \frac{\tan \theta}{\sin \theta} - \frac{\sin \theta}{\tan \theta}$
 $= (\tan \theta \div \sin \theta) - (\sin \theta \div \tan \theta)$
 $= \frac{\sin \theta}{\cos \theta} \div \sin \theta - \sin \theta \div \frac{\sin \theta}{\cos \theta}$
 $= \frac{\sin \theta}{\cos \theta} \times \frac{1}{\sin \theta} - \sin \theta \times \frac{\cos \theta}{\sin \theta}$
 $= \sec \theta - \cos \theta$

L.H.S = R.H.S

Q.11. $\frac{\sin \theta + \cos \theta}{\tan^2 \theta - 1} = \frac{\cos^2 \theta}{\sin \theta - \cos \theta}$

Solution: Let

L.H.S = $\frac{\sin \theta + \cos \theta}{\tan^2 \theta - 1}$
 $= (\sin \theta + \cos \theta) \div (\tan^2 \theta - 1)$
 $= (\sin \theta + \cos \theta) \div \frac{\sin^2 \theta}{\cos^2 \theta} - 1$
 $= (\sin \theta + \cos \theta) \div \frac{\sin^2 \theta - \cos^2 \theta}{\cos^2 \theta}$
 $= (\sin \theta + \cos \theta) \times \frac{\cos^2 \theta}{(\sin^2 \theta - \cos^2 \theta)}$
 $= \frac{(\cancel{\sin \theta + \cos \theta}) \times \cos^2 \theta}{(\cancel{\sin \theta + \cos \theta})(\sin \theta - \cos \theta)}$
 $= \frac{\cos^2 \theta}{\sin \theta - \cos \theta}$

L.H.S = R.H.S

$$\text{Q.12. } \frac{\cos^2 \theta}{\sin \theta} + \sin \theta = \operatorname{cosec} \theta$$

Solution: Let

$$\begin{aligned} \text{L.H.S} &= \frac{\cos^2 \theta}{\sin \theta} + \sin \theta \\ &= \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta} \\ &= \frac{1}{\sin \theta} \\ &= \operatorname{cosec} \theta \end{aligned}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Q.13. } \sec \theta - \cos \theta = \tan \theta \cdot \sin \theta$$

Solution: Let

$$\begin{aligned} \text{L.H.S} &= \sec \theta - \cos \theta \\ &= \frac{1}{\cos \theta} - \cos \theta \\ &= \frac{1 - \cos^2 \theta}{\cos \theta} \\ &= \frac{\sin^2 \theta}{\cos \theta} \\ &= \frac{\sin \theta}{\cos \theta} \sin \theta \\ &= \tan \theta \cdot \sin \theta \end{aligned}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Q.14. } \frac{\sin^2 \theta}{\cos \theta} + \cos \theta = \sec \theta$$

Solution: Let

$$\begin{aligned} \text{L.H.S} &= \frac{\sin^2 \theta}{\cos \theta} + \cos \theta \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} \\ &= \frac{1}{\cos \theta} \\ &= \sec \theta \end{aligned}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Q.15. } \tan \theta + \cot \theta = \sec \theta \cdot \operatorname{cosec} \theta$$

Solution: Let

$$\begin{aligned} \text{L.H.S} &= \tan \theta + \cot \theta \\ &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \cdot \sin \theta} \\ &= \frac{1}{\cos \theta \cdot \sin \theta} \quad (\sin^2 \theta + \cos^2 \theta = 1) \\ &= \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} \\ &= \sec \theta \cdot \operatorname{cosec} \theta \end{aligned}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Q.16. } (\tan \theta + \cot \theta)(\cos \theta + \sin \theta) = \sec \theta + \operatorname{cosec} \theta$$

Solution: Let

$$\begin{aligned} \text{L.H.S} &= (\tan \theta + \cot \theta)(\cos \theta + \sin \theta) \\ &= \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) (\cos \theta + \sin \theta) \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \cdot \sin \theta} (\cos \theta + \sin \theta) \\ &= \frac{1}{\cos \theta \cdot \sin \theta} (\cos \theta + \sin \theta) \\ &= \frac{\cos \theta + \sin \theta}{\cos \theta \cdot \sin \theta} \\ &= \frac{\cancel{\cos \theta}}{\cancel{\cos \theta} \cdot \sin \theta} + \frac{\cancel{\sin \theta}}{\cos \theta \cdot \cancel{\sin \theta}} \\ &= \frac{1}{\sin \theta} + \frac{1}{\cos \theta} \\ &= \operatorname{cosec} \theta + \sec \theta \\ &= \sec \theta + \operatorname{cosec} \theta \end{aligned}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Q.17. } \sin \theta (\tan \theta + \cot \theta) = \sec \theta$$

Solution: Let

$$\begin{aligned} \text{L.H.S} &= \sin \theta (\tan \theta + \cot \theta) \\ &= \sin \theta \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) \\ &= \cancel{\sin \theta} \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \cdot \cancel{\sin \theta}} \\ &= \frac{1}{\cos \theta} \\ &= \sec \theta \end{aligned}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Q.18. } \frac{1+\cos}{\sin} + \frac{\sin}{1+\cos} = 2\operatorname{cosec}$$

Solution: Let

$$\begin{aligned} \text{L.H.S} &= \frac{1+\cos \theta}{\sin \theta} + \frac{\sin \theta}{1+\cos \theta} \\ &= \frac{(1+\cos \theta)^2 + (\sin \theta)^2}{(\sin \theta)(1+\cos \theta)} \\ &= \frac{(1)^2 + 2(1)(\cos \theta) + \cos^2 \theta + \sin^2 \theta}{\sin \theta(1+\cos \theta)} \\ &= \frac{1+2\cos \theta+1}{\sin \theta(1+\cos \theta)} \\ &= \frac{2+2\cos \theta}{\sin \theta(1+\cos \theta)} \\ &= \frac{2(1+\cos \theta)}{\sin \theta(1+\cos \theta)} \\ &= \frac{2}{\sin \theta} \end{aligned}$$

$$= 2\operatorname{cosec} \theta$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Q.19. } \frac{1}{1-\cos \theta} + \frac{1}{1+\cos \theta} = 2\operatorname{cosec}^2 \theta$$

Solution:

$$\begin{aligned} \text{L.H.S} &= \frac{1}{1-\cos \theta} + \frac{1}{1+\cos \theta} \\ &= \frac{1+\cancel{\cos \theta} + 1-\cancel{\cos \theta}}{(1-\cos \theta)(1+\cos \theta)} \\ &= \frac{2}{(1)^2 - (\cos^2 \theta)} \\ &= \frac{2}{1-\cos^2 \theta} \\ &= \frac{2}{\sin^2 \theta} \end{aligned}$$

$$= 2\operatorname{cosec}^2 \theta$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Q.20. } \frac{1+\sin}{1-\sin} - \frac{1-\sin}{1+\sin} = 4\tan \sec$$

Solution: Let

$$\begin{aligned} \text{L.H.S} &= \frac{1+\sin \theta}{1-\sin \theta} - \frac{1-\sin \theta}{1+\sin \theta} \\ &= \frac{(1+\sin \theta)^2 - (1-\sin \theta)^2}{(1-\sin \theta)(1+\sin \theta)} \\ &= \frac{(1+\sin^2 \theta + 2\sin \theta) - (1+\sin^2 \theta - 2\sin \theta)}{(1)^2 - (\sin \theta)^2} \\ &= \frac{1+\sin^2 \theta + 2\sin \theta - 1 - \sin^2 \theta + 2\sin \theta}{1-\sin^2 \theta} \\ &= \frac{4\sin \theta}{\cos^2 \theta} \\ &= \frac{4\sin \theta}{\cos \theta \cdot \cos \theta} \\ &= 4 \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} \\ &= 4\tan \theta \cdot \sec \theta \end{aligned}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Q.21. } \sin^3 \theta = \sin \theta - \sin \theta \cdot \cos^2 \theta$$

Solution: Let

$$\begin{aligned} \text{L.H.S} &= \sin^3 \theta \\ &= \sin \theta \cdot \sin^2 \theta \\ &= \sin \theta (1 - \cos^2 \theta) \\ &= \sin \theta - \sin \theta \cdot \cos^2 \theta \end{aligned}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Q.22. } \cos^4 \theta - \sin^4 \theta = \cos^2 \theta - \sin^2 \theta$$

Solution: Let

$$\begin{aligned} \text{L.H.S} &= \cos^4 \theta - \sin^4 \theta \\ &= (\cos^2 \theta)^2 - (\sin^2 \theta)^2 \\ &= (\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta) \\ &= (1)(\cos^2 \theta - \sin^2 \theta) \\ &= \cos^2 \theta - \sin^2 \theta \end{aligned}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$Q.23. \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{\sin\theta}{1-\cos\theta}$$

Solution: Let

$$\begin{aligned} \text{L.H.S} &= \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}} \\ &= \sqrt{\frac{(1+\cos\theta)^2}{(1)^2 - (\cos\theta)^2}} = \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} \\ &= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} = \frac{1+\cos\theta}{\sin\theta} \\ &= \frac{1+\cos\theta}{\sin\theta} \times \frac{1-\cos\theta}{1-\cos\theta} \\ &= \frac{(1)^2 - (\cos\theta)^2}{\sin\theta(1-\cos\theta)} \\ &= \frac{1-\cos^2\theta}{\sin\theta(1-\cos\theta)} \\ &= \frac{\sin^2\theta}{\sin\theta(1-\cos\theta)} \\ &= \frac{\sin\theta}{1-\cos\theta} \end{aligned}$$

* L.H.S = R.H.S

$$Q.24. \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = \frac{\sec\theta+1}{\tan\theta}$$

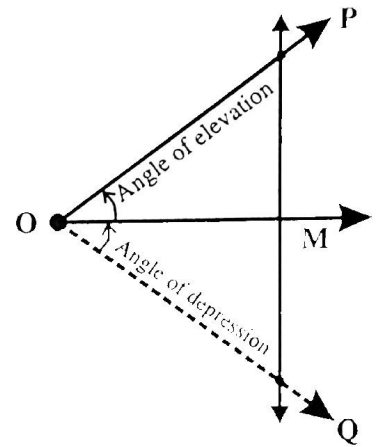
Solution: Let

$$\begin{aligned} \text{L.H.S} &= \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} \\ &= \sqrt{\frac{\sec\theta+1}{\sec\theta-1} \times \frac{\sec\theta+1}{\sec\theta+1}} \\ &= \sqrt{\frac{(\sec\theta+1)^2}{(\sec\theta)^2 - (1)^2}} \\ &= \sqrt{\frac{(\sec\theta+1)^2}{\sec^2\theta - 1}} \\ &= \sqrt{\frac{(\sec\theta+1)^2}{\tan^2\theta}} \\ &= \frac{\sec\theta+1}{\tan\theta} \end{aligned}$$

L.H.S = R.H.S

Angle of Elevation and Angle of Depression

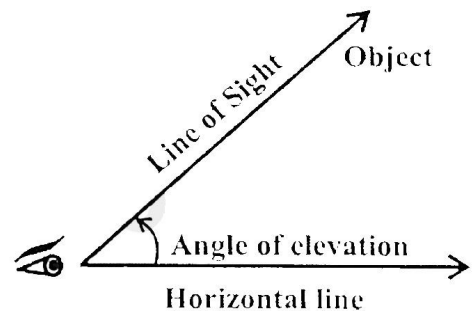
Suppose O, P and Q are three points, P being at a higher level of O and Q being at lower level than O. Let a horizontal line drawn through O meet in M, the vertical line drawn through P and Q.



The angle MOP is called the angle of elevation of point P as seen from O. For looking at Q below the horizontal line we have to lower our eyes and $\angle MOQ$ is called the angle of depression.

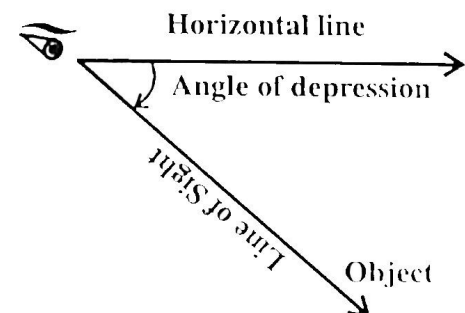
Angle of Elevation:

The angle between the horizontal line through eye and a line from eye to the object, above the horizontal line is called angle of elevation.



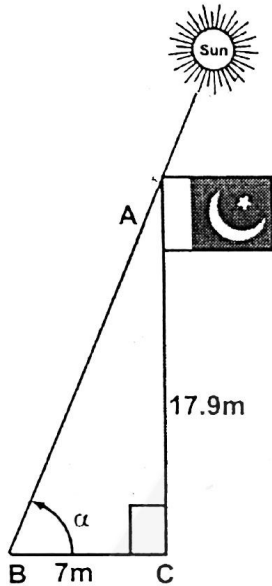
Angle of depression:

The angle between the horizontal line through eye and a line from eye to the object, below the horizontal line is called angle of depression.



Example 1: A flagpole 17.9 meter high casts a 7 meter shadow. Find the angle of elevation of the sun.

Solution:



From the figure, we observe that α is the angle of elevation.

Using the fact that

$$\tan \alpha = \frac{\overline{mAC}}{\overline{mBC}}$$

$$\tan \alpha = \frac{17.9m}{7m}$$

$$\tan \alpha = 2.55714$$

Solving for α gives us

$$\alpha = \tan^{-1}(2.55714)$$

$$\alpha = (68.6666)^\circ$$

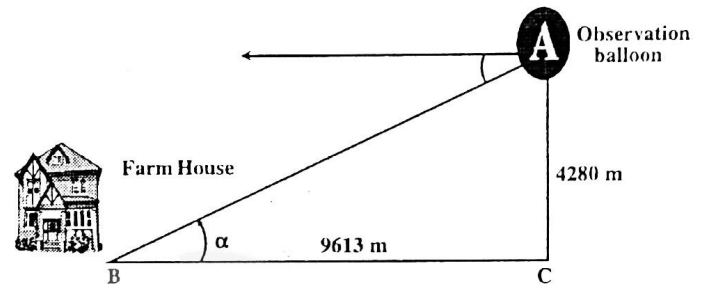
$$68^\circ 40'$$

$$\alpha = 68^\circ 40'$$

So angle of elevation is $68^\circ 40'$.

Example 2: An observation balloon is 4280 meter above the ground and 9613 meter away from a farmhouse. Find the angle of depression of the farmhouse as observed from the observation balloon.

Solution:



For problems of this type the angle of elevation of A from B is considered equal to the angle of depression of B from A, as shown in the diagram.

$$\tan \alpha = \frac{\overline{mAC}}{\overline{mBC}}$$

$$\tan \alpha = \frac{4280m}{9613m}$$

$$\tan \alpha = 0.44523$$

$$\alpha = \tan^{-1}(0.44523)$$

$$\alpha = 24^\circ$$

So, angle of depression is 24° .