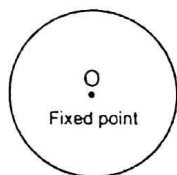


Basic Concepts of the circle

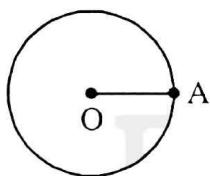
Circle

A **circle** is the locus of a moving point P in a plane which is always equidistant from the fixed point O. This fixed point O not lying on the circle is called the centre of the circle.



Radial Segment

The line segment joining the centre of a circle to any point of the circle is called radial segment. It is determined by the centre and a point on the circle. All the radial segments of a circle are equal in length.



In figure $\overline{OA}$ is a radial segment.

Radius

The distance between the centre and any point of the circle is called its radius. The length of radial segment of a circle is equal to its radius.

Circumference

The length of the boundary traced by a moving point P in a circular path is called circumference of the circle. Circumference is calculated by $C = 2\pi r$. Here r is a radius and π is an irrational number.

What do you know about π ?

π is an irrational number and π is the ratio of the circumference and the diameter of a given circle.

Proof: We know that $2\pi r$ is the circumference of a circle. i.e. $C = 2\pi r$.

$$\text{or} \quad C = (2r)\pi$$

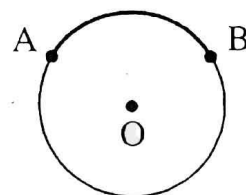
$$\therefore C = D\pi \quad \because 2r = D$$

$$\text{or} \quad \frac{C}{D} = \pi$$

$$\Rightarrow \pi = \frac{C}{D}$$

Arc

Any part or portion of a circle is called its arc. An arc AB, as shown in figure, is denoted by $\widehat{AB}$.

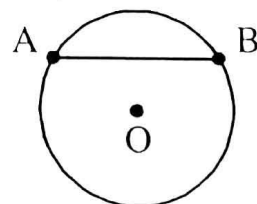


There are two types of arc:

- Major Arc (Arc greater than semi circle)
- Minor Arc (Arc less than semi circle)

Chord

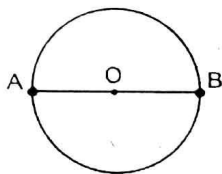
The line segment joining any two points of the circle with each other is called chord of the circle.



In figure $\overline{AB}$ is chord of the circle.

Diameter

The Chord passing through the centre of the circle is called diameter of the circle. Evidently diameter bisects a circle.



In figure $\overline{AB}$ is diameter of the circle.

Segment of the Circle

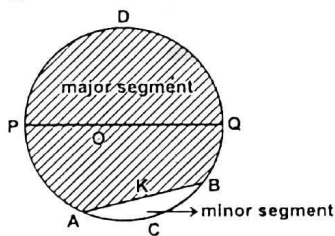
The circular region bounded by an arc and a corresponding chord is called segment of the circle. Evidently any chord divides a circle into two segments. There are two types of segments.

i. Major Segment

The circular region bounded by a major arc and a corresponding chord is called major segment.

ii. Minor Segment

The circular region bounded by a minor arc and a corresponding chord is called minor segment.



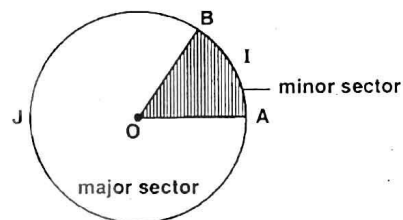
In figure the bigger area shown by slanting line segments is the major segment whereas the smaller area shown by shading is the minor segment.

Sector of a Circle

A sector of a circle is the plane figure founded by two radii and the arc intercepted between them. Any pair of radii divides a circle into two sectors.

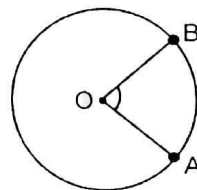
- Major Sector
- Minor Sector

In the figure $OAIB$ is the minor sector, whereas $OAJB$ is the major sector of the circle.



Central Angle

An angle whose vertex is at the centre of the circle and its arms meet at the end points of an arc is called central angle.



In figure $\angle AOB$ is the central angle of a circle.

THEOREM 1

One and only one circle can pass through three non-collinear points.

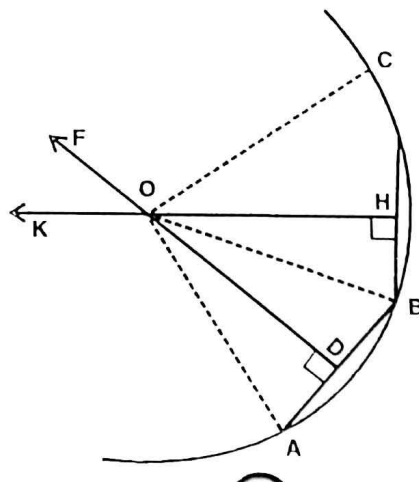
Given: A, B and C are three non collinear points in a plane.

To prove: One and only one circle can pass through three non-collinear points A, B and C.

Construction: Join A with B and B with C.

Draw $\overline{DF} \perp$ bisector to $\overline{AB}$ and $\overline{HK} \perp$ bisector to $\overline{BC}$.

So, $\overline{DF}$ and $\overline{HK}$ are not parallel and they intersect each other at point O. Also join A, B and C with point O.



Proof:

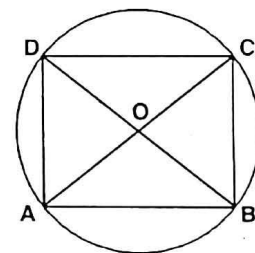
Statements	Reasons
Every point on $\overline{DF}$ is equidistant from A and B.	$\overline{DF}$ is $\perp$ bisector to $\overline{AB}$ (construction)
In particular $m\overline{OA} = m\overline{OB}$(i)	
Similarly every point on $\overline{HK}$ is equidistant from B and C.	$\overline{HK}$ is $\perp$ bisector to $\overline{BC}$ (construction)
In particular $m\overline{OB} = m\overline{OC}$(ii)	
Now O is the only point common to $\overline{DF}$ and $\overline{HK}$ which is equidistant from A, B and C.	
i.e., $m\overline{OA} = m\overline{OB} = m\overline{OC}$	Using (i) and (ii).
However there is no such other point except O.	
Hence a circle with centre O and radius OA will pass through A, B and C.	
Ultimately there is only one circle which passes through three given points A, B and C.	

Example: Show that only one circle can be drawn to pass through the vertices of any rectangle.

Given: ABCD is a rectangle.

To Prove: Only one circle can be drawn through the vertices of the rectangle ABCD.

Construction: Diagonals $\overline{AC}$ and $\overline{BD}$ of the rectangle meet each other at point O.



Statements	Reasons
ABCD is a rectangle.	Given
$\therefore m\overline{AC} = m\overline{BD}$(i)	Diagonals of a rectangle are equal.
$\therefore \overline{AC}$ and $\overline{BD}$ meet each other at O	Construction
$\therefore m\overline{OA} = m\overline{OC}$ and $m\overline{OB} = m\overline{OD}$(ii)	Diagonals of rectangle bisect each other
$\Rightarrow m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD}$ (iii)	Using (i) and (ii)
i.e., point O is equidistant from all vertices of the rectangle ABCD.	
Hence $\overline{OA}$, $\overline{OB}$, $\overline{OC}$ and $\overline{OD}$ are the radii of the circle which is passing through the vertices of the rectangle having centre O.	

THEOREM 2

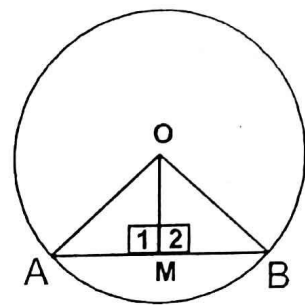
A straight line, drawn from the centre of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord.

Given: M is the midpoint of any chord $\overline{AB}$ of a circle with centre at O.

Where chord $\overline{AB}$ is not the diameter of the circle.

To prove: $\overline{OM} \perp$ the chord $\overline{AB}$.

Construction: Join A and B with centre O. write $\angle 1$ and $\angle 2$ as shown in the figure.



Proof:

Statements	Reasons
In $\triangle OAM \leftrightarrow \triangle OBM$	
$m\overline{OA} = m\overline{OB}$	Radii of the same circle
$m\overline{AM} = m\overline{BM}$	Given
$m\overline{OM} = m\overline{OM}$	Common
$\triangle OAM \cong \triangle OBM$	S.S.S $\cong$ S.S.S
$\Rightarrow m\angle 1 = m\angle 2 \dots \dots \dots (i)$	Corresponding angles of congruent Δ 's
$i.e., m\angle 1 + m\angle 2 = m\angle AMB = 180^\circ \dots \dots \dots (ii)$	Adjacent supplementary angles
$m\angle 1 = m\angle 2 = 90^\circ$	From (i) and (ii)
$i.e. \overline{OM} \perp \overline{AB}$	

THEOREM 3

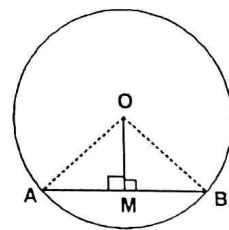
Perpendicular from the centre of a circle on a chord bisects it.

Given: $\overline{AB}$ is the chord of a circle with centre at O so that $\overline{OM} \perp$ chord $\overline{AB}$.

To prove: M is the mid point of chord $\overline{AB}$ i.e. $m\overline{AM} = m\overline{BM}$

Construction: Join A and B with centre O.

Proof:



Statements	Reasons
In $\angle rt \triangle OAM \leftrightarrow \angle rt \triangle OBM$	
$m\angle OMA = m\angle OMB = 90^\circ$	Given
hyp. $\overline{OA} = \text{hyp. } \overline{OB}$	Radii of the same circle
$m\overline{OM} = m\overline{OM}$	Common
$\triangle OAM \cong \triangle OBM$	In $\angle rt \Delta^s$ H.S $\cong$ H.S
Hence, $m\overline{AM} = m\overline{BM}$	Corresponding sides of congruent triangles.
$\Rightarrow \overline{OM}$ bisects the chord $\overline{AB}$.	

Corollary 1: $\perp$ bisector of the chord of a circle passes through the centre of a circle.

Corollary 2: The diameter of a circle passes through the mid points of two parallel chords of a circle.

Example:

Parallel lines passing through the points of intersection of two circles and intercepted by them are equal.

Given:

Two circles have centres O_1 and O_2 . They intersect each other at points E and F.

Line segment $\overline{AB} \parallel$ Line segment $\overline{CD}$

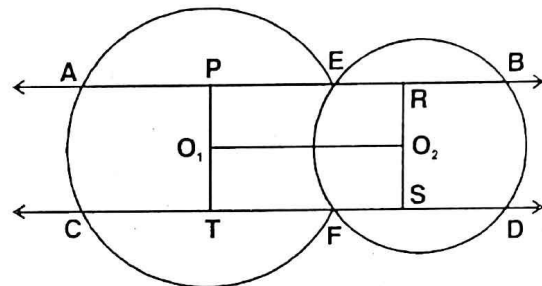
To Prove:

$$m\overline{AB} = m\overline{CD}$$

Construction:

Draw $\overline{PT}$ and $\overline{RS} \perp$ both $\overline{AB}$ and $\overline{CD}$ and join the centres O_1 and O_2 .

Proof:



Statements	Reasons
PRST is rectangle	Construction
$m\overline{PR} = m\overline{TS}$(i)	
Now $m\overline{PR} = m\overline{PE} + m\overline{ER}$	
$= \frac{1}{2} m\overline{AE} + \frac{1}{2} m\overline{EB}$	By theorem 3
$= \frac{1}{2} (m\overline{AE} + m\overline{EB})$	
$m\overline{PR} = \frac{1}{2} m\overline{AB}$(ii)	$m\overline{AE} + m\overline{EB} = m\overline{AB}$
Similarly	
$m\overline{TS} = \frac{1}{2} m\overline{CD}$(iii)	
$\Rightarrow \frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$	Using (i), (ii) and (iii)
$\Rightarrow m\overline{AB} = m\overline{CD}$	

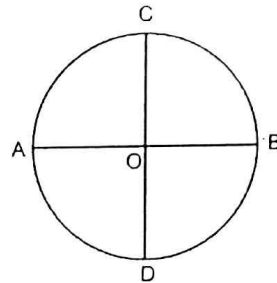
EXERCISE 9.1

Q. 1 Prove that, the diameters of a circle bisect each other.

Given: A circle with centre O. Two diameters $\overline{AB}$ and $\overline{CD}$.

To Prove: Two diameters $\overline{AB}$ and $\overline{CD}$ bisect each other.

Proof:



Statements	Reasons
$\overline{AB}$ and $\overline{CD}$ intersect each other at point O. $\overline{OA} \cong \overline{OB}$ ----- (i) O is the midpoint of $\overline{AB}$ thus $\overline{CD}$ bisects the $\overline{AB}$ at O. Similarly $\overline{OC} \cong \overline{OD}$ ----- (ii) O is the midpoint of $\overline{CD}$ thus $\overline{AB}$ bisects the $\overline{CD}$ at O. Hence, two diameters $\overline{AB}$ and $\overline{CD}$ bisect each other.	$\overline{AB}$ and $\overline{CD}$ are non-parallel. Radii of the same circle from (i) Radii of the same circle from (ii)

Q. 2 Two chords of a circle do not pass through the centre. Prove that they cannot bisect each other.

Given:

A Circle with centre “Q”. Two different chords $\overline{AB}$ and $\overline{CD}$ not passing through the centre, intersect each other at point E.

To Prove:

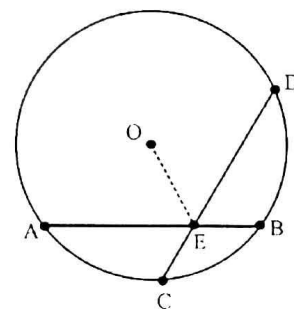
$\overline{AB}$ and $\overline{CD}$ do not bisect each other, i.e.

E is not midpoint of $\overline{AB}$ and $\overline{CD}$.

Construction:

Suppose chords $\overline{AB}$ and $\overline{CD}$ bisect each other at point E i.e. E is the common midpoint of $\overline{AB}$ and $\overline{CD}$. Join O to E.

Proof:



Statements	Reasons
As $\overline{OE}$ is perpendicular from “O” to the midpoint E of $\overline{AB}$ and $\overline{CD}$ so , $m\angle OEA = 90^\circ$ (i) $m\angle OED = 90^\circ$ (ii) $m\angle OEA + m\angle OED = 180^\circ$ (iii) $m\angle AED = 180^\circ$ It is only possible when A and D are on the same line segment. But A and D are not on the same line segment. So, our supposition, $\overline{AB}$ and $\overline{CD}$ bisect each other, is wrong. Thus chords $\overline{AB}$ and $\overline{CD}$ do not bisect each other	A line segment from the centre “O” to the midpoint of a chord is $\perp$ on the chord. (Construction) Adding (i) and (ii) Given

Q. 3 If length of the chord $\overline{AB} = 8$ cm. Its distance from the centre is 3 cm, then find the diameter of such circle.

Solution:

Given:

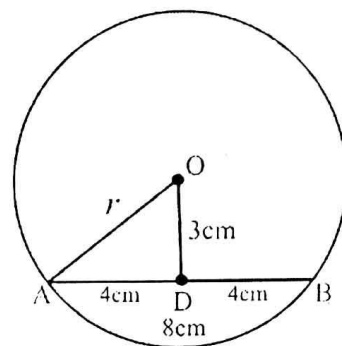
Length of chord = $m\overline{AB} = 8$ cm

Distance from centre = $m\overline{OD} = 3$ cm

To Find: Diameter = $2r$

Calculations:

$m\overline{AB} = 8$ cm (Given)



Perpendicular from the centre to the chord bisects the chord ($\overline{OD} \perp \overline{AB}$)

$$m\overline{AD} = \frac{1}{2} m\overline{AB} = \frac{1}{2} (8\text{cm}) = 4\text{cm}$$

In right angled $\triangle ADO$ by Pythagoras theorem

$$(m\overline{OA})^2 = (m\overline{AD})^2 + (m\overline{OD})^2$$

$$r^2 = (4\text{cm})^2 + (3\text{cm})^2$$

$$r^2 = 16\text{cm}^2 + 9\text{cm}^2$$

$$r^2 = 25\text{cm}^2$$

$$\sqrt{r^2} = \sqrt{25\text{cm}^2}$$

$$r = 5\text{cm}$$

We know that diameter = $2r = 2(5\text{cm}) = 10\text{cm}$

Q.4 Calculate the length of a chord which stands at a distance 5cm from the centre of a circle whose radius is 9cm.

Given:

In a circle with centre O radius = 9cm,

$m\overline{OD} = 5\text{cm}$

To find:

Length of chord $\overline{AB}$

Calculations:

In right angled $\triangle ADO$, by Pythagoras theorem

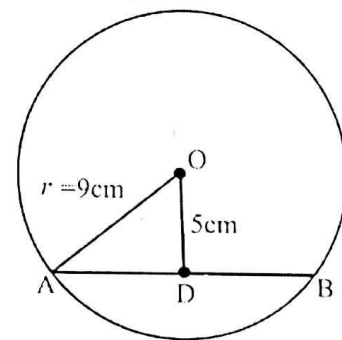
$$(m\overline{OA})^2 = (m\overline{AD})^2 + (m\overline{OD})^2$$

$$(9\text{cm})^2 = (m\overline{AD})^2 + (5\text{cm})^2$$

$$81\text{cm}^2 = (m\overline{AD})^2 + 25\text{cm}^2$$

$$81\text{cm}^2 - 25\text{cm}^2 = (m\overline{AD})^2$$

$$56\text{cm}^2 = (m\overline{AD})^2$$



$$\sqrt{(m\overline{AD})^2} = \sqrt{56cm^2}$$

$$m\overline{AD} = \sqrt{56cm}$$

We know that

$$m\overline{AD} = \frac{1}{2} m\overline{AB} \text{ (Perpendicular from the centre to the cord bisects the chord)}$$

$$\Rightarrow m\overline{AB} = 2m\overline{AD} \\ = 2 \times \sqrt{56cm}$$

$$m\overline{AB} = 14.966cm$$

$$\text{or } m\overline{AB} \approx 14.97cm$$

THEOREM 4

If two chords of a circle are congruent then they will be equidistant from the centre.

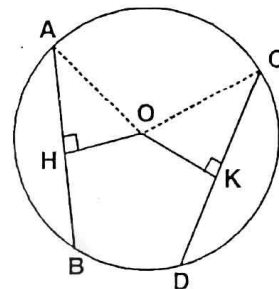
Given: $\overline{AB}$ and $\overline{CD}$ are two equal chords of a circle with centre at O.

So that $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$.

To Prove: $m\overline{OH} = m\overline{OK}$

Construction: Join O with A and O with C.

So that we have $\angle rt \Delta^s$ OAH and OCK.



Proof:

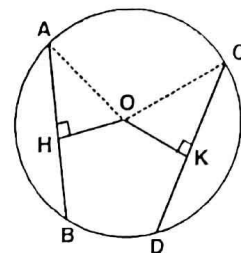
Statements	Reasons
$\overline{OH}$ bisects chord $\overline{AB}$	
i.e., $m\overline{AH} = \frac{1}{2} m\overline{AB}$ (i)	$\overline{OH} \perp \overline{AB}$ By Theorem 3
Similarly $\overline{OK}$ bisects chord $\overline{CD}$	
i.e., $m\overline{CK} = \frac{1}{2} m\overline{CD}$ (ii)	$\overline{OK} \perp \overline{CD}$ By Theorem 3
But $m\overline{AB} = m\overline{CD}$ (iii)	Given
Hence $m\overline{AH} = m\overline{CK}$ (iv)	Using (i), (ii) and (iii) Both are half of equal segment
Now in $\angle rt \Delta^s$ OAH $\leftrightarrow$ OCK	Given $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$
hyp. $\overline{OA} = \text{hyp. } \overline{OC}$	Radii of the same circle
$m\overline{AH} = m\overline{CK}$	Already proved in (iv)
$\Delta OAH \cong \Delta OCK$	H.S $\cong$ H.S
$\Rightarrow m\overline{OH} = m\overline{OK}$	Corresponding sides of congruent triangles.

THEOREM 5

Two chords of a circle which are equidistant from the centre, are congruent.

Given: $\overline{AB}$ and $\overline{CD}$ are two chords of a circle with centre at O.

$\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$, so that $m\overline{OH} = m\overline{OK}$



To Prove: $m\overline{AB} = m\overline{CD}$

Construction: Join A and C with O. So that we can form $\angle rt \Delta^s$ OAH and OCK.

Proof:

Statements	Reasons
In $\angle rt \Delta^s$ OAH $\leftrightarrow$ OCK.	
$\therefore$ hyp. $\overline{OA} =$ hyp. $\overline{OC}$	Radii of the same circle.
$m\overline{OH} = m\overline{OK}$	Given
$\therefore \Delta OAH \cong \Delta OCK$	H.S $\cong$ H.S
So $m\overline{AH} = m\overline{CK}$ (i)	
But $m\overline{AH} = \frac{1}{2} m\overline{AB}$ (ii)	$\overline{OH} \perp$ chord $\overline{AB}$ (Given)
Similarly $m\overline{CK} = \frac{1}{2} m\overline{CD}$	$\overline{OK} \perp$ chord $\overline{CD}$ (Given)
Since $m\overline{AH} = m\overline{CK}$	Already proved in (i)
$\therefore \frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$	Using (ii) & (iii)
or $m\overline{AB} = m\overline{CD}$	

Example: Prove that the largest chord in a circle is the diameter.

Given: $\overline{AB}$ is a chord and $\overline{CD}$ is the diameter of a circle with centre point O.

Prove: If $\overline{AB}$ and $\overline{CD}$ are distinct, then $m\overline{CD} > m\overline{AB}$.

Construction: Join O with A and B to form a ΔOAB .

Proof: Sum of two sides of a triangle is greater than its third side.

$\therefore$ In $\Delta OAB \Rightarrow m\overline{OA} + m\overline{OB} > m\overline{AB}$ (i)

But $\overline{OA}$ and $\overline{OB}$ are the radii of the same circle with centre O.

So that $m\overline{OA} + m\overline{OB} = m\overline{CD}$ (ii)

$m\overline{CD} > m\overline{AB}$ [From (i) and (ii)]

$\Rightarrow$ Diameter $CD >$ chord $\overline{AB}$.

Hence, diameter CD is greater than any other chord drawn in the circle.

