

MATRICES AND DETERMINANTS

Idea of matrices:

The idea of matrices was given by Arthur Cayley, an English mathematician of nineteenth century who first developed, "Theory of Matrices" in 1858.

Q1. Define the following terms.

(i) Matrix

"A rectangular array or a formation of a collection of real numbers, say 0, 1, 2, 3, 4 and 7, such as: $\begin{bmatrix} 1 & 3 & 4 \\ 7 & 2 & 0 \end{bmatrix}$ and then enclosed by brackets '[]' is said to form a matrix $\begin{bmatrix} 1 & 3 & 4 \\ 7 & 2 & 0 \end{bmatrix}$. Similarly $\begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix}$ is another matrix.

The matrices are denoted conventionally by the capital letters A, B, C, ..., M, N etc. of the English alphabet.

(ii) Order of a Matrix

The number of rows and columns in a matrix specifies its order. If a matrix M has m rows and n columns then M is said to be of order, m -by- n . For example,

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} \text{ is order 2-by-3,}$$

(iii) Equal Matrices

Let A and B be two matrices. Then A is said to be equal to B, and is denoted by $A = B$, if and only if;

- (i) The order of A = The order of B
- (ii) Their corresponding entries are equal.

Examples

$$(i) \quad A = \begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2+1 \\ -4 & 4-2 \end{bmatrix}$$

are equal matrices.

We see that:

- (a) The order of matrix A = The order of matrix B
- (b) Their corresponding elements are equal.

Thus $A = B$

$$(ii) \quad L = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \quad \text{and} \quad M = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} \text{ are}$$

not equal matrices.

We see that: order of L = order of M but entries in the second row and second column are not same, so $L \neq M$.

$$(iii) \quad P = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \quad \text{and} \quad Q = \begin{bmatrix} 2 & 3 & 4 \\ -1 & 2 & 0 \end{bmatrix}$$

are not equal matrices.

We see that order of P $\neq$ order of Q, so $P \neq Q$.

Exercise 1.1

1. Find the order of the following matrices.

$$A = \begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}, \text{ order of A is 2-by-2}$$

$$B = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}, \text{ order of B is 2-by-2}$$

$$C = \begin{bmatrix} 2 & 4 \end{bmatrix} \quad \text{order of C is 1-by-2}$$

$$D = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}, \quad \text{order of D is 3-by-1}$$

$$E = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}, \text{ order of } E \text{ is 3-by-2}$$

$$F = [2] \text{ order of } F \text{ is 1-by-1}$$

$$G = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}, \text{ order of } G \text{ is 3-by-3}$$

$$H = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix} \text{ order of } H \text{ is 2-by-3}$$

2. Which of the following matrices are equal?

$$A = [3],$$

$$B = [3 \quad 5], C = [5 \quad 2]$$

$$D = [5 \quad 3], E = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix},$$

$$F = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$$G = \begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix},$$

$$H = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}, I = [3 \quad 3+2]$$

$$J = \begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$$

Ans. Equal matrices are

$$A = C \quad B = I$$

$$E = H = J \quad F = G$$

3. Find the values of a, b, c and d which satisfy the matrix equation.

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

$$\text{Ans. } a+c=0 \dots\dots\dots(i)$$

$$a+2b=-7 \dots\dots\dots(ii)$$

$$c-1=3 \dots\dots\dots(iii)$$

$$4d-6=2d \dots\dots\dots(iv)$$

From (iii)

$$c=3+1$$

$$\boxed{c=4}$$

From (iv)

$$4d-2d=6$$

$$2d=6$$

$$d=\frac{6}{2}$$

$$\boxed{d=3}$$

Put value of $c=4$ in (i)

$$a+4=0$$

$$\boxed{a=-4}$$

Put value of $a=-4$ in (ii)

$$-4+2b=-7$$

$$2b=-7+4$$

$$2b=-3$$

$$\boxed{b=-\frac{3}{2}}$$

Types of Matrices

(e) Row Matrix.

A matrix is called a row matrix if it has only one row.

e.g.; the matrix $M = [2 \quad -1 \quad 7]$ is a row matrix of order 1-by-3 and $M = [1 \quad -1]$ is a row matrix of order 1-by-2.

(ii) Column Matrix.

A matrix is called a column matrix if it has only one column e.g., $M = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

and $N = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ are column matrices of order 2-by-1 and 3-by-1 respectively.

(e) **Rectangular Matrix.**

A matrix is called rectangular if, the number of rows of M is not equal to the number of columns of M .

e.g. $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$;

$B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$; $C = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and

$D = \begin{bmatrix} 7 \\ 8 \\ 0 \end{bmatrix}$ are all rectangular matrices. The

order of A is 3-by-2, the order of B is 2-by-3, the order of C is 1-by-3 and order of D is 3-by-1, which indicates that in each matrix the number of rows $\neq$ the number of columns.

(e) **Square Matrix.**

A matrix is called a square matrix if its number of rows is equal to its number of columns.

e.g., $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix}$ and

$C = \begin{bmatrix} 3 \end{bmatrix}$ are square matrices of orders 2-by-2, 3-by-3 and 1-by-1 respectively.

(v) **Null or Zero Matrix.**

A matrix M is called a null or zero matrix if each of its entries is 0.

e.g., $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

and $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ are null matrices of

orders

2-by-2, 1-by-2, 2-by-1, 2-by-3 and 3-by-3 respectively. Null matrix is represented by O .

(vi) **Transpose of a Matrix.**

A matrix obtained by interchanging the rows into columns or columns into rows of a matrix is called transpose of that matrix. If A is a matrix then its transpose is denoted by A^t .

e.g., (i) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ -1 & 4 & -2 \end{bmatrix}$

, then $A^t = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 4 \\ 3 & 0 & -2 \end{bmatrix}$

Note: If a matrix A is of order 2-by-3 then order of its transpose A^t is 3-by-2

(vii) **Negative of a Matrix.**

Let A be a matrix. Then its negative, $-A$, is obtained by changing the signs of all the entries of A , i.e.,

If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, then $-A = \begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix}$.

(viii) **Symmetric Matrix.**

A square matrix is symmetric if it is equal to its transpose i.e., matrix A is symmetric if $A^t = A$.

e.g. (i) If $M = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 4 & 0 \end{bmatrix}$

is a square matrix, then

$$M^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 4 & 0 \end{bmatrix} = M. \text{ Thus } M \text{ is a}$$

symmetric matrix.

(ii) If $A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 2 \\ 3 & 1 & 3 \end{bmatrix},$

then $A^t = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 1 \\ 3 & 2 & 3 \end{bmatrix} \neq A$

Hence A is not a symmetric matrix.

(x) Skew-Symmetric Matrix.

A square matrix A is said to be skew-symmetric if $A^t = -A$.

e.g., If $A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix},$ then

$$A^t = \begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -3 \\ -(-2) & 0 & -1 \\ -(-3) & -(-1) & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -A$$

Since $A^t = -A$, therefore A is a skew-symmetric matrix.

(x) Diagonal Matrix.

A square matrix A is called a diagonal matrix if atleast any one of the entries of its diagonal is not zero and non-diagonal entries must all be zero.

e.g., $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

and $C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ are all diagonal matrices of order 3-by-3.

$M = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ and $N = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

are diagonal matrices of order 2-by-2.

(xi) Scalar Matrix.

A diagonal matrix is called a scalar matrix, if all the diagonal entries are same

and non-zero. For example $\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$

where k is a constant $\neq 0, 1$.

Also $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ and

$C = [5]$ are scalar matrices of order 3-by-3, 2-by-2 and 1-by-1 respectively.

(xii) Identity Matrix.

A diagonal matrix is called identity (unit) matrix if all diagonal entries are 1 and it is denoted by I .

e.g., $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a 3-by-3

identity matrix.

$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a 2-by-2 identity matrix.

$C = [1]$ is a 1-by-1 identity matrix.

