

Exercise 1.2

1. From the following matrices, identify unit matrices, row matrices, column matrices and null matrices.

Ans. $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, Null matrix

$B = [2 \quad 3 \quad 4]$, Row matrix

$C = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$, Column matrix

$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, Unit matrix

$E = [0]$, Null matrix

$F = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$, Column matrix

2. From the following matrices, identify

- (a) Square matrices
- (b) Rectangular matrices
- (c) Row matrices
- (d) Column matrices
- (e) Identity matrices
- (f) Null matrices

Ans. (a) **Square Matrices:**

(iii) $\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(viii) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Ans. (b) **Rectangular Matrices:**

(i) $\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$

(ii) $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

(v) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

Ans. (c) **Row Matrices:**

(vi) $[3 \quad 10 \quad -1]$

Ans. (d) **Column Matrices:**

(ii) $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

(vii) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Ans. (e) **Identity Matrices:**

(iv) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Ans. (f) **Null matrices:**

(ix) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

3. From the following matrices, identify diagonal, scalar and unit (identity) matrices.

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix},$$

$$B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix},$$

$$E = \begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$$

Ans. Scalar matrices:

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, E = \begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$$

Unit Matrices:

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Diagonal Matrices:

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$$

4. Find negative of matrices A, B, C, D and E when:

$$A = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix}, D = \begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix},$$

$$E = \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$$

Negative of matrices

$$\text{Ans. } -A = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix},$$

$$\text{Ans. } -B = \begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix}$$

$$\text{Ans. } -C = \begin{bmatrix} -2 & -6 \\ -3 & -2 \end{bmatrix}$$

$$\text{Ans. } -D = \begin{bmatrix} 3 & -2 \\ 4 & -5 \end{bmatrix},$$

$$\text{Ans. } -E = \begin{bmatrix} -1 & 5 \\ -2 & -3 \end{bmatrix}$$

5. Find the transpose of each of the following matrices:

Ans. (i)

$$A = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \Rightarrow A^t = [0 \quad 1 \quad -2]$$

$$B = \begin{bmatrix} 5 & 1 & -6 \end{bmatrix} \Rightarrow B^t = \begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix} \Rightarrow C^t = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} \Rightarrow D^t = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$$

$$E = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix} \Rightarrow E^t = \begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow F^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

6. Verify that if

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}, \text{ then}$$

$$(i) (A^t)^t = A$$

$$(ii) (B^t)^t = B$$

$$\text{Ans. (i) } (A^t)^t = A$$

$$\text{L.H.S} = (A^t)^t$$

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$(A^t)^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$(A^t)^t = A = \text{R.H.S.}$$

$$\text{Hence L.H.S} = \text{R.H.S.}$$

$$\text{Ans. (ii) } (B^t)^t = B$$

$$\text{L.H.S} = (B^t)^t$$

$$B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$(B^t)^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$(B^t)^t = B \\ = \text{R.H.S}$$

$$\text{Hence L.H.S} = \text{R.H.S.}$$

Addition and Subtraction of Matrices

Define Addition of Matrices.

Let A and B be any two matrices with real number entries. The matrices A and B are conformable for addition, if they have the same order.

$$\text{e.g., } A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix} \text{ are}$$

conformable for addition.

Addition of A and B, written $A+B$ is obtained by adding the entries of the matrix A to the corresponding entries of the matrix B.

$$\text{e.g., } A+B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 6 \end{bmatrix} + \begin{bmatrix} -2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix} \\ = \begin{bmatrix} 2+(-2) & 3+3 & 0+4 \\ 1+1 & 0+2 & 6+3 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 4 \\ 2 & 2 & 9 \end{bmatrix}$$

Define Subtraction of Matrices.

If A and B are two matrices of same order then subtraction of matrix B from matrix A is obtained by subtracting the entries of matrix B from the corresponding entries of matrix A and it is denoted by $A - B$.

$$\text{e.g., } A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2 & 2 \\ -1 & 4 & 3 \end{bmatrix} \text{ are}$$

conformable for subtraction.

$$\text{i.e., } A-B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 2 \\ -1 & 4 & 3 \end{bmatrix} \\ = \begin{bmatrix} 2-0 & 3-2 & 4-2 \\ 1-(-1) & 5-4 & 0-3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 1 & -3 \end{bmatrix}$$

Multiplication of a Matrix by a Real Number

Let A be any matrix and the real number k be a scalar. Then the scalar

multiplication of matrix A with k is obtained by multiplying each entry of matrix A with k. It is denoted by kA .

Let $A = \begin{bmatrix} 1 & -1 & 4 \\ 2 & -1 & 0 \\ -1 & 3 & 2 \end{bmatrix}$ be a matrix of

order 3-by-3 and $k=-2$ be a real number.

Then

$$kA = (-2)A$$

$$\begin{aligned} &= (-2) \begin{bmatrix} 1 & -1 & 4 \\ 2 & -1 & 0 \\ -1 & 3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} (-2)(1) & (-2)(-1) & (-2)(4) \\ (-2)(2) & (-2)(-1) & (-2)(0) \\ (-2)(-1) & (-2)(3) & (-2)(2) \end{bmatrix} \\ &= \begin{bmatrix} -2 & 2 & -8 \\ -4 & 2 & 0 \\ 2 & -6 & -4 \end{bmatrix} \end{aligned}$$

Commutative and Associative Laws of Matrices

(a) Commutative Law under Addition

If A and B are two matrices of the same order, then $A + B = B + A$ is called commutative law under addition.

Let $A = \begin{bmatrix} 2 & 3 & 0 \\ 5 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix}$,

$$B = \begin{bmatrix} 3 & -2 & 5 \\ -1 & 4 & 1 \\ 4 & 2 & -4 \end{bmatrix}$$

Then

$$\begin{aligned} A + B &= \begin{bmatrix} 2 & 3 & 0 \\ 5 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 5 \\ -1 & 4 & 1 \\ 4 & 2 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 2+3 & 3-2 & 0+5 \\ 5-1 & 6+4 & 1+1 \\ 2+4 & 1+2 & 3-4 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 5 \\ 4 & 10 & 2 \\ 6 & 3 & -1 \end{bmatrix} \end{aligned}$$

Similarly

$$\begin{aligned} B + A &= \begin{bmatrix} 3 & -2 & 5 \\ -1 & 4 & 1 \\ 4 & 2 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 0 \\ 5 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 1 & 5 \\ 4 & 10 & 2 \\ 6 & 3 & -1 \end{bmatrix} \end{aligned}$$

Thus the commutative law of addition of matrices is verified.

$$A + B = B + A$$

(b) Associative Law under Addition

If A, B and C are three matrices of same order, such that $(A+B)+C=A+(B+C)$ is called associative law under addition.

Let $A = \begin{bmatrix} 2 & 3 & 0 \\ 5 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix}$,

$$B = \begin{bmatrix} 3 & -2 & 5 \\ -1 & 4 & 1 \\ 4 & 2 & -4 \end{bmatrix}$$

and

$$C = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 0 & 4 \\ 1 & 2 & 0 \end{bmatrix}$$

Then

$$(A+B)+C = \left(\begin{bmatrix} 2 & 3 & 0 \\ 5 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 5 \\ -1 & 4 & 1 \\ 4 & 2 & -4 \end{bmatrix} \right)$$

$$+ \begin{bmatrix} 1 & 2 & 3 \\ -2 & 0 & 4 \\ 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3 & 3-2 & 0+5 \\ 5-1 & 6+4 & 1+1 \\ 2+4 & 1+2 & 3-4 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ -2 & 0 & 4 \\ 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 1 & 5 \\ 4 & 10 & 2 \\ 6 & 3 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ -2 & 0 & 4 \\ 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 3 & 8 \\ 2 & 10 & 6 \\ 7 & 5 & -1 \end{bmatrix}$$

$$A+(B+C) = \begin{bmatrix} 2 & 3 & 0 \\ 5 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$+ \left(\begin{bmatrix} 3 & -2 & 5 \\ -1 & 4 & 1 \\ 4 & 2 & -4 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ -2 & 0 & 4 \\ 1 & 2 & 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 & 3 & 0 \\ 5 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 3+1 & -2+2 & 5+3 \\ -1-2 & 4+0 & 1+4 \\ 4+1 & 2+2 & -4+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 0 \\ 5 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 8 \\ -3 & 4 & 5 \\ 5 & 4 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 3 & 8 \\ 2 & 10 & 6 \\ 7 & 5 & -1 \end{bmatrix}$$

Thus the associative law of addition is verified:

$$(A+B)+C = A+(B+C)$$

Additive Identity of a Matrix

If A and B are two matrices of same order such that $A + B = A = B + A$ then matrix B is called additive identity of matrix A.

For any matrix A and zero matrix O of same order, O is called additive identity of A as

$$A + O = A = O + A$$

$$\text{e.g., let } A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \text{ and } O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

then

$$A + O = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} = A$$

$$O + A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} = A$$

Additive Inverse of a Matrix

If A and B are two matrices of same order such that $A + B = O = B + A$ then A and B are called additive inverse of each other.

Additive inverse of any matrix A is obtained by changing the signs of all the non zero entries of A.

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -2 \\ 3 & 1 & 0 \end{bmatrix}$$

then

$$B = (-A) = - \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -2 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 1 & 2 \\ -3 & -1 & 0 \end{bmatrix}$$

is additive inverse of A. It can be verified as:

$$A + B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -2 \\ 3 & 1 & 0 \end{bmatrix} + \begin{bmatrix} -1 & -2 & -1 \\ 0 & 1 & 2 \\ -3 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1)+(-1) & (2)+(-2) & (1)+(-1) \\ 0+0 & (-1)+(1) & (-2)+(2) \\ (3)+(-3) & (1)+(-1) & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

$$B + A = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 1 & 2 \\ -3 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -2 \\ 3 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)+(1) & (-2)+(2) & (-1)+(1) \\ 0+0 & (1)+(-1) & (2)+(-2) \\ (-3)+(3) & (-1)+(1) & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

Since $A + B = O = B + A$

Therefore B is additive inverse of A.