

Exercise 1.3

1. Which of the following matrices are conformable for addition?

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}, \quad D = \begin{bmatrix} 2+1 \\ 3 \end{bmatrix},$$

$$E = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix},$$

$$F = \begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix} \text{ Ans. (i)}$$

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}, \text{ and } E = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$$

are conformable for addition.

$$(ii) \quad B = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 2+1 \\ 3 \end{bmatrix}$$

are conformable for addition.

$$(iii) \quad C = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix} \text{ and } F = \begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix}$$

are conformable for addition.

2. Find the additive inverse of following matrices.

$$A = \begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix},$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{bmatrix}$$

Ans.

$$(i) \quad A = \begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$$

Additive inverse of Matrix A is

$$-A = -\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix} \Rightarrow -A = \begin{bmatrix} -2 & -4 \\ 2 & -1 \end{bmatrix}$$

$$(ii) \quad B = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$$

Additive inverse of Matrix B is

$$-B = -\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$$

$$-B = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & -3 \\ -3 & 2 & -1 \end{bmatrix}$$

$$(iii) \quad C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

Additive inverse of Matrix C is

$$-C = -\begin{bmatrix} 4 \\ -2 \end{bmatrix} \Rightarrow -C = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

$$(iv) \quad D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$$

Additive inverse of Matrix D is

$$-D = -\begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix} \Rightarrow -D = \begin{bmatrix} -1 & 0 \\ 3 & 2 \\ -2 & -1 \end{bmatrix}$$

$$(v) \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Additive inverse of Matrix E is

$$-E = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow -E = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$(vi) \quad F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$$

Additive inverse of Matrix F is

$$-F = -\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix} \Rightarrow -F = \begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{2} \end{bmatrix}$$

$$3. \quad \text{If} \quad A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}, \text{ then find,}$$

$$(i) \quad A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad (ii) \quad B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$(iii) \quad C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$$

$$(iv) \quad D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \quad (v) \quad 2A$$

$$(vi) \quad (-1)B \quad (vii) \quad (-2)C$$

$$(viii) \quad 3D \quad (ix) \quad 3C$$

$$\text{Ans.} \quad (i) \quad A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1+1 & 1+2 \\ 2+1 & 1+1 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$$

$$(ii) \quad B + \begin{bmatrix} -2 \\ 3 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1-2 \\ -1+3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$(iii) \quad C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & -1+1 & 2+3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 5 \end{bmatrix}$$

$$(iv) \quad D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 2+1 & 0+3 \\ -1+2 & 0+0 & 2+1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 3 \end{bmatrix}$$

$$(v) \quad 2A = 2 \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 4 & 2 \end{bmatrix}$$

$$(vi) \quad -1(B) = (-1) \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$(vii) \quad (-2)C = (-2) \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 2 & -4 \end{bmatrix}$$

$$(viii) \quad 3D = 3 \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ -3 & 0 & 6 \end{bmatrix}$$

$$(ix) \quad 3C = 3 \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -3 & 6 \end{bmatrix}$$

4. Perform the indicated operations and simplify the following.

$$(i) \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$(ii) \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$(iii) \quad \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} + ([1 \ 0 \ 2] - [2 \ 2 \ 2])$$

$$(iv) \quad \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

$$(v) \quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

$$(vi) \quad \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\text{Ans. (i)} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \\ = \begin{bmatrix} 1+0+1 & 0+2+1 \\ 0+3+1 & 1+0+0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

$$(ii) \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \\ = \begin{bmatrix} 1+0-1 & 0+2-1 \\ 0+3-1 & 1+0-0 \end{bmatrix} \\ = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

$$(iii) \quad \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} + ([1 \ 0 \ 2] - [2 \ 2 \ 2]) \\ = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} + [1-2 \ 0-2 \ 2-2] \\ = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} + [-1 \ -2 \ 0] \\ = \begin{bmatrix} 2-1 & 3-2 & 1+0 \end{bmatrix} \\ = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

$$(iv) \quad \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} \\ = \begin{bmatrix} 1+1 & 2+1 & 3+1 \\ -1+2 & -1+2 & -1+2 \\ 0+3 & 1+3 & 2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 3 & 4 & 5 \end{bmatrix}$$

$$(v) \quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+0 & 3-2 \\ 2-2 & 3-1 & 1-0 \\ 3+0 & 1+2 & 2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$(vi) \quad \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \\ = \begin{bmatrix} 1+2+1 & 2+1+1 \\ 0+1+1 & 1+0+1 \end{bmatrix} \\ = \begin{bmatrix} 4 & 4 \\ 2 & 2 \end{bmatrix}$$

5. For the matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \text{ and}$$

$$C = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \text{ verify the}$$

following rules.

- (i) $A + C = C + A$
- (ii) $A + B = B + A$
- (iii) $B + C = C + B$
- (iv) $A + (B + A) = 2A + B$
- (v) $(C - B) + A = C + (A - B)$
- (vi) $2A + B = A + (A + B)$
- (vii) $(C - B) - A = (C - A) - B$
- (viii) $(A + B) + C = A + (B + C)$
- (ix) $A(B - C) = (A - C) + B$
- (x) $2A + 2B = 2(A + B)$

Ans.

(i) $A + C = C + A$

L.H.S = $A + C$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2+0 & 3+0 \\ 2+0 & 3-2 & 1+3 \\ 1+1 & -1+1 & 0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & 0 & 2 \end{bmatrix}$$

R.H.S = $C + A$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 0+2 & 0+3 \\ 0+2 & -2+3 & 3+1 \\ 1+1 & 1-1 & 0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & 0 & 2 \end{bmatrix}$$

L.H.S = R.H.S

(ii) $A + B = B + A$

L.H.S = $A + B$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

R.H.S = $B + A$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & -1+2 & 1+3 \\ 2+2 & -2+3 & 2+1 \\ 3+1 & 1-1 & 3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

L.H.S. = R.H.S

(iii) $B + C = C + B$

L.H.S = $B + C$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & -1+0 & 1+0 \\ 2+0 & -2-2 & 2+3 \\ 3+1 & 1+1 & 3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix}$$

$$\text{R.H.S} = C + B$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 0-1 & 0+1 \\ 0+2 & -2-2 & 3+2 \\ 1+3 & 1+1 & 3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix}$$

$$\text{L.H.S} = \text{R.H.S.}$$

$$(iv) \quad A + (B + A) = 2A + B$$

$$\text{L.H.S} = A + (B + A)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 2+1 & 3+4 \\ 2+4 & 3+1 & 1+3 \\ 1+4 & -1+0 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix}$$

$$\text{R.H.S} = 2A + B$$

$$= 2 \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 4-1 & 6+1 \\ 4+2 & 6-2 & 2+2 \\ 2+3 & -2+1 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$(v) \quad (C - B) + A = C + (A - B)$$

$$\text{L.H.S.} = (C - B) + A$$

$$C - B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & 0+1 & 0-1 \\ 0-2 & -2+2 & 3-2 \\ 1-3 & 1-1 & 2-3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 & -1 \\ -2 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix}$$

$$(C - B) + A = \begin{bmatrix} -2 & 1 & -1 \\ -2 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2+1 & 1+2 & -1+3 \\ -2+2 & 0+3 & 1+1 \\ -2+1 & 0-1 & -1+0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 & 2 \\ 0 & 3 & 2 \\ -1 & -1 & -1 \end{bmatrix}$$

$$\text{R.H.S.} = C + (A - B)$$

$$A - B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2+1 & 3-1 \\ 2-2 & 3+2 & 1-2 \\ 1-3 & -1-1 & 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 & 2 \\ 0 & 5 & -1 \\ -2 & -2 & -3 \end{bmatrix}$$

$$C + (A - B) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 & 2 \\ 0 & 5 & -1 \\ -2 & -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0 & 0+3 & 0+2 \\ 0+0 & -2+5 & 3-1 \\ 1-2 & 1-2 & 2-3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 & 2 \\ 0 & 3 & 2 \\ -1 & -1 & -1 \end{bmatrix}$$

$$\text{L.H.S.} = \text{R.H.S.}$$

$$(vi) \quad 2A+B=A+(A+B)$$

$$\text{L.H.S.} = 2A+B$$

$$2A+B = 2 \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 4-1 & 6+1 \\ 4+2 & 6-2 & 2+2 \\ 2+3 & -2+1 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix}$$

$$\text{R.H.S.} = A+(A+B)$$

$$A+(A+B) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} +$$

$$\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 3 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 2+1 & 3+4 \\ 2+4 & 3+1 & 1+3 \\ 1+4 & -1+0 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 7 \\ 6 & 4 & 4 \\ 5 & -1 & 3 \end{bmatrix}$$

$$\text{L.H.S.} = \text{R.H.S.}$$

$$(vii) \quad (C-B)-A=(C-A)-B$$

$$\text{L.H.S.} = (C-B)-A$$

$$C-B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & 0+1 & 0-1 \\ 0-2 & -2+2 & 3-2 \\ 1-3 & 1-1 & 2-3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 & -1 \\ -2 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix}$$

$$(C-B)-A = \begin{bmatrix} -2 & 1 & -1 \\ -2 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2-1 & 1-2 & -1-3 \\ -2-2 & 0-3 & 1-1 \\ -2-1 & 0+1 & -1-0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 & -4 \\ -4 & -3 & 0 \\ -3 & 1 & -1 \end{bmatrix}$$

$$\text{R.H.S.} = (C-A)-B$$

$$(C-A) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & 0-2 & 0-3 \\ 0-2 & -2-3 & 3-1 \\ 1-1 & 1+1 & 2-0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 & -3 \\ -2 & -5 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

$$\begin{aligned}
 (C-A)-B &= \begin{bmatrix} -2 & -2 & -3 \\ -2 & -5 & 2 \\ 0 & 2 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} -2-1 & -2+1 & -3-1 \\ -2-2 & -5+2 & 2-2 \\ 0-3 & 2-1 & 2-3 \end{bmatrix} \\
 &= \begin{bmatrix} -3 & -1 & -4 \\ -4 & -3 & 0 \\ -3 & 1 & -1 \end{bmatrix}
 \end{aligned}$$

L.H.S = R.H.S.

(viii) $(A+B) + C = A + (B+C)$

L.H.S = $(A+B) + C$

$$\begin{aligned}
 A+B &= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix} \\
 (A+B)+C &= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 4 \\ 4 & -1 & 6 \\ 5 & 1 & 5 \end{bmatrix}
 \end{aligned}$$

R.H.S = $A + (B+C)$

$$\begin{aligned}
 B+C &= \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1-1 & -1+0 & 1+0 \\ 2+0 & -2-2 & 2+3 \\ 3+1 & 1+1 & 2+3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 A+(B+C) &= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & 5 \\ 4 & 2 & 5 \end{bmatrix} \\
 &= \begin{bmatrix} 1+0 & 2-1 & 3+1 \\ 2+2 & 3-4 & 1+5 \\ 1+4 & -1+5 & 0+5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 4 \\ 4 & -1 & 6 \\ 5 & 1 & 5 \end{bmatrix}
 \end{aligned}$$

R.H.S = R.H.S

(ix) $A + (B-C) = (A-C) + B$

L.H.S = $A + (B-C)$

$$\begin{aligned}
 B-C &= \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1+1 & -1-0 & 1-0 \\ 2-0 & -2+2 & 2-3 \\ 3-1 & 1-1 & 3-2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 1 \\ 2 & 0 & -1 \\ 2 & 0 & 1 \end{bmatrix} \\
 A+(B-C) &= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 1 \\ 2 & 0 & -1 \\ 2 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1+2 & 2-1 & 3+1 \\ 2+2 & 3+0 & 1-1 \\ 1+2 & -1+0 & 0+1 \end{bmatrix} \\
 &= \begin{bmatrix} 3 & 1 & 4 \\ 4 & 3 & 0 \\ 3 & -1 & 1 \end{bmatrix}
 \end{aligned}$$

R.H.S = $(A-C)+B$

$$\begin{aligned}
 A-C &= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1+1 & 2-0 & 3-0 \\ 2-0 & 3+2 & 1-3 \\ 1-1 & -1-1 & 0-2 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 5 & -2 \\ 0 & -2 & -2 \end{bmatrix} \\
 (A-C)+B &= \begin{bmatrix} 2 & 2 & 3 \\ 2 & 5 & -2 \\ 0 & -2 & -2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 2+1 & 2-1 & 3+1 \\ 2+2 & 5-2 & -2+2 \\ 0+3 & -2+1 & -2+3 \end{bmatrix} \\
 &= \begin{bmatrix} 3 & 1 & 4 \\ 4 & 3 & 0 \\ 3 & -1 & 1 \end{bmatrix}
 \end{aligned}$$

L.H.S. = R.H.S.

(x) $2A+2B=2(A+B)$

L.H.S. = $2A+2B$

$$\begin{aligned}
 2A+2B &= 2 \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + 2 \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 & 2 \\ 4 & -4 & 4 \\ 6 & 2 & 6 \end{bmatrix} \\
 &= \begin{bmatrix} 2+2 & 4-2 & 6+2 \\ 4+4 & 6-4 & 2+4 \\ 2+6 & -2+2 & 0+6 \end{bmatrix} \\
 &= \begin{bmatrix} 4 & 2 & 8 \\ 8 & 2 & 6 \\ 8 & 0 & 6 \end{bmatrix}
 \end{aligned}$$

R.H.S = $2(A+B)$

$$\begin{aligned}
 2(A+B) &= 2 \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right) \\
 &= 2 \left(\begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix} \right) \\
 &= 2 \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 4 & 2 & 8 \\ 8 & 2 & 6 \\ 8 & 0 & 6 \end{bmatrix}
 \end{aligned}$$

L.H.S = R.H.S

6. If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$,

find (i) $3A-2B$ (ii) $2A^t - 3B^t$.

Ans. (i)

$$\begin{aligned}
 3A-2B &= 3 \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} - 2 \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix} \\
 &= \begin{bmatrix} 3 & -6 \\ 9 & 12 \end{bmatrix} - \begin{bmatrix} 0 & 14 \\ -6 & 16 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 &= \begin{bmatrix} 3-0 & -6-14 \\ 9+6 & 12-16 \end{bmatrix} \\
 &= \begin{bmatrix} 3 & -20 \\ 15 & -4 \end{bmatrix}
 \end{aligned}$$

(ii) $2A^t - 3B^t$

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$$

$$2A^t = 2 \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$$

$$3B^t = 3 \begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -9 \\ 21 & 24 \end{bmatrix}$$

$$2A^t - 3B^t = \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -9 \\ 21 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & 6+9 \\ -4-21 & 8-24 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 15 \\ -25 & -16 \end{bmatrix}$$

7. If $2 \begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3 \begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix}$

$$= \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}, \text{ then find } a \text{ and } b.$$

Ans. $2 \begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3 \begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$

$$\begin{bmatrix} 4 & 8 \\ -6 & 2a \end{bmatrix} + \begin{bmatrix} 3 & 3b \\ 24 & -12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4+3 & 8+3b \\ -6+24 & 2a-12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 8+3b \\ 18 & 2a-12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$\Rightarrow 8+3b = 10 \dots\dots\dots (i)$$

$$2a - 12 = 1 \dots\dots\dots (ii)$$

From (i)

$$3b = 10 - 8$$

$$3b = 2$$

$$b = \frac{2}{3}$$

From (ii)

$$2a = 1 + 12$$

$$a = \frac{13}{2}$$

8. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$,

then verify that

(i) $(A+B)^t = A^t + B^t$

(ii) $(A-B)^t = A^t - B^t$

(iii) $A + A^t$ is symmetric

(iv) $A - A^t$ is skew symmetric

(v) $B + B^t$ is symmetric

(vi) $B - B^t$ is skew symmetric

Ans. (i) $(A+B)^t = A^t + B^t$

$$\text{L.H.S} = (A+B)^t$$

$$(A+B) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+1 \\ 0+2 & 1+0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$$

$$(A+B)^t = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

$$\text{R.H.S} = A^t + B^t$$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$A^t + B^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 0+2 \\ 2+1 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

$$\text{L.H.S.} = \text{R.H.S.}$$

(ii) $(A-B)^t = A^t - B^t$

$$\text{L.H.S.} = (A-B)^t$$

$$(A-B) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$(A-B) = \begin{bmatrix} 1-1 & 2-1 \\ 0-2 & 1-0 \end{bmatrix}$$

$$(A-B) = \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}$$

$$(A-B)^t = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$\text{R.H.S} = A^t - B^t$$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$A^t - B^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 0-2 \\ 2-1 & 1-0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$\text{L.H.S} = \text{R.H.S}$$

(iii) $A + A^t$ is symmetric

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\begin{aligned} A + A^t &= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+1 & 2+0 \\ 0+2 & 1+1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \end{aligned}$$

$$(A + A^t)^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = A + A^t$$

So, $A + A^t$ is symmetric.

(iv) $A - A^t$ is skew symmetric

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\begin{aligned} A - A^t &= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1-1 & 2-0 \\ 0-2 & 1-1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \end{aligned}$$

$$(A - A^t)^t = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$(A - A^t)^t = -\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$-(A - A^t)^t$ is skew symmetric

(v) $B + B^t$ is symmetric

$$\begin{aligned} B + B^t &= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1+1 & 1+2 \\ 2+1 & 0+0 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix} \end{aligned}$$

$$(B + B^t)^t = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$$

$(B + B^t)$ is symmetric

(vi) $B - B^t$ is skew symmetric

$$\begin{aligned} B - B^t &= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1-1 & 1-2 \\ 2-1 & 0-0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \end{aligned}$$

$$(B - B^t)^t = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$-(B - B^t)$ is skew symmetric

Multiplication of Matrices.

Two matrices A and B are conformable for multiplication, giving product AB if the number of columns of A is equal to the number of rows of B.

e.g., let $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$. Here

number of columns of A is equal to the number of rows of B. So A and B matrices are conformable for multiplication.

Examples

(i) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$,

$$\text{then } AB = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$$

$$= [1 \times 2 + 2 \times 3 \quad 1 \times 0 + 2 \times 1]$$

$$= [2 + 6 \quad 0 + 2] = [8 \quad 2]$$

It is a matrix of order 1-by-2.

(ii)

If $A = \begin{bmatrix} 1 & 3 \\ 2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix}$, then

$$\begin{aligned}
 AB &= \begin{bmatrix} 1 & 3 \\ 2 & -3 \end{bmatrix} \times \begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1 \times (-1) + 3 \times 3 & 1 \times 0 + 3 \times 2 \\ 2 \times (-1) + (-3) \times 3 & 2 \times 0 + (-3) \times 2 \end{bmatrix} \\
 &= \begin{bmatrix} -1+9 & 0+6 \\ -2-9 & 0-6 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ -11 & -6 \end{bmatrix}, \text{ is a}
 \end{aligned}$$

2-by-2 matrix.

Associative Law under Multiplication

If A, B and C are three matrices conformable for multiplication then associative law under multiplication is given as

$$(AB)C = A(BC)$$

e.g., If $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 3 & 1 \end{bmatrix}$ and

$C = \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix}$, then

$$\begin{aligned}
 \text{L.H.S.} &= (AB)C \\
 &= \left(\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 3 & 1 \end{bmatrix} \right) \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 2 \times 0 + 3 \times 3 & 2 \times 1 + 3 \times 1 \\ -1 \times 0 + 0 \times 3 & -1 \times 1 + 0 \times 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 0+9 & 2+3 \\ 0+0 & -1+0 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 9 & 5 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 9 \times 2 + 5 \times (-1) & 9 \times 2 + 5 \times 0 \\ 0 \times 2 + (-1) \times (-1) & 0 \times 2 + (-1) \times 0 \end{bmatrix} \\
 &= \begin{bmatrix} 18-5 & 18+0 \\ 0+1 & 0+0 \end{bmatrix} = \begin{bmatrix} 13 & 18 \\ 1 & 0 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \text{R.H.S.} &= A(BC) = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix} \right) \\
 &= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \times 2 + 1 \times (-1) & 0 \times 2 + 1 \times 0 \\ 3 \times 2 + 1 \times (-1) & 3 \times 2 + 1 \times 0 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 5 & 6 \end{bmatrix} \\
 &= \begin{bmatrix} 2(-1) + 3 \times 5 & 2 \times 0 + 3 \times 6 \\ (-1)(-1) + 0 \times 5 & -1 \times 0 + 0 \times 6 \end{bmatrix} \\
 &= \begin{bmatrix} -2+15 & 0+18 \\ 1+0 & 0+0 \end{bmatrix} \\
 &= \begin{bmatrix} 13 & 18 \\ 1 & 0 \end{bmatrix} = (AB)C
 \end{aligned}$$

The associative law under multiplication of matrices is verified.

Distributive Laws of Multiplication over Addition and Subtraction

(a) Let A, B and C be three matrices. Then distributive laws of multiplication over addition are given below.

- (i) $A(B+C) = AB+AC$
(Left distributive law)
- (ii) $(A+B)C = AC+BC$
(Right distributive law)

Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 3 & 1 \end{bmatrix}$

and $C = \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix}$ then in (i)

$$\begin{aligned}
 \text{L.H.S.} &= A(B+C) \\
 &= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 & 1 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix} \right) \\
 &= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0+2 & 1+2 \\ 3-1 & 1+0 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 &= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 2 \times 2 + 3 \times 2 & 2 \times 3 + 3 \times 1 \\ -1 \times 2 + 0 \times 2 & -1 \times 3 + 0 \times 1 \end{bmatrix} \\
 &= \begin{bmatrix} 4+6 & 6+3 \\ -2+0 & -3+0 \end{bmatrix} = \begin{bmatrix} 10 & 9 \\ -2 & -3 \end{bmatrix}
 \end{aligned}$$

R.H.S. = $AB + AC$

$$\begin{aligned}
 &= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 2 \times 0 + 3 \times 3 & 2 \times 1 + 3 \times 1 \\ -1 \times 0 + 0 \times 3 & -1 \times 1 + 0 \times 1 \end{bmatrix} \\
 &+ \begin{bmatrix} 2 \times 2 + 3 \times (-1) & 2 \times 2 + 3 \times 0 \\ -1 \times 2 + 0 \times (-1) & -1 \times 2 + 0 \times 0 \end{bmatrix} \\
 &= \begin{bmatrix} 9 & 5 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ -2 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 9+1 & 5+4 \\ 0-2 & -1-2 \end{bmatrix} = \begin{bmatrix} 10 & 9 \\ -2 & -3 \end{bmatrix} = \text{L.H.S.}
 \end{aligned}$$

Which shows that

$$A(B+C) = AB+AC;$$

b) Let $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$

and $C = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, then in (i)

L.H.S. = $A(B-C)$

$$\begin{aligned}
 &= \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right) \\
 &= \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1-2 & 1-1 \\ 1-1 & 0-2 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} (2)(-3) + (3)(0) & 2(0) + 3(-2) \\ (0)(-3) + 1 \times 0 & 0 \times 0 + (1)(-2) \end{bmatrix}
 \end{aligned}$$

$$= \begin{bmatrix} -6+0 & 0-6 \\ 0+0 & 0-2 \end{bmatrix} = \begin{bmatrix} -6 & -6 \\ 0 & -2 \end{bmatrix}$$

R.H.S. = $AB - AC$

$$\begin{aligned}
 &= \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 2(-1) + 3(1) & 2(1) + 3(0) \\ 0(-1) + 1(1) & 0(1) + 1(0) \end{bmatrix} \\
 &- \begin{bmatrix} 2 \times 2 + 3 \times 1 & 2 \times 1 + 3 \times 2 \\ 0 \times 2 + 1 \times 1 & 0 \times 1 + 1 \times 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 7 & 8 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1-7 & 2-8 \\ 1-1 & 0-2 \end{bmatrix} \\
 &= \begin{bmatrix} -6 & -6 \\ 0 & -2 \end{bmatrix}
 \end{aligned}$$

Which shows that

$$A(B-C) = AB - AC$$

Commutative Law of Multiplication of Matrices

Consider the matrices $A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ and

$B = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$ then

$$\begin{aligned}
 AB &= \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 0 \times 1 + 1 \times 0 & 0 \times 0 + 1(-2) \\ 2 \times 1 + 3 \times 0 & 2 \times 0 + 3(-2) \end{bmatrix} \\
 &= \begin{bmatrix} 0 & -2 \\ 2 & -6 \end{bmatrix}
 \end{aligned}$$

and

$$\begin{aligned}
 BA &= \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 1 \times 0 + 0 \times 2 & 1 \times 1 + 0 \times 3 \\ 0 \times 0 + (-2) \times 2 & 0 \times 1 + 3(-2) \end{bmatrix}
 \end{aligned}$$

$$= \begin{bmatrix} 0 & 1 \\ -4 & -6 \end{bmatrix}$$

Which shows that. $AB \neq BA$.

Note: Commutative law under multiplication in matrices does not hold in general i.e., if A and B are two matrices then $AB \neq BA$.

Commutative law under multiplication holds in particular case.

e.g., If $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 0 \\ 0 & 4 \end{bmatrix}$

then

$$\begin{aligned} AB &= \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 0 \\ 0 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 2 \times (-3) + 0 \times 0 & 2 \times 0 + 0 \times 4 \\ 0 \times (-3) + 1 \times 0 & 0 \times 0 + 1 \times 4 \end{bmatrix} \\ &= \begin{bmatrix} -6 & 0 \\ 0 & 4 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{and } BA &= \begin{bmatrix} -3 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -3 \times 2 + 0 \times 0 & -3 \times 0 + 0 \times 1 \\ 0 \times 2 + 4 \times 0 & 0 \times 0 + 4 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} -6 & 0 \\ 0 & 4 \end{bmatrix} \end{aligned}$$

Which shows that $AB = BA$.

Multiplicative Identity of a Matrix.

Let A be a matrix. Another matrix B is called the identity matrix of A under multiplication if

$$AB = A = BA$$

If $A = \begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 1 + 2 \times 0 & 1 \times 0 + 2 \times 1 \\ 0 \times 1 + (-3) \times 0 & 0 \times 0 + (-3) \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} BA &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 1 + 0 \times 0 & 1 \times 2 + 0 \times (-3) \\ 0 \times 1 + 1 \times 0 & 0 \times 2 + 1 \times (-3) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix} \end{aligned}$$

Which shows that $AB = A = BA$.

Verification of $(AB)^t = B^t A^t$.

If A and B are two matrices and A^t , B^t are their respective transpose,

then $(AB)^t = B^t A^t$.

e.g., $A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$

$$\text{L.H.S.} = (AB)^t$$

$$\begin{aligned} &= \left(\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} \right)^t \\ &= \begin{bmatrix} 2 \times 1 + 1 \times (-2) & 2 \times 3 + 1 \times 0 \\ 0 \times 1 + (-1) \times (-2) & 0 \times 3 + (-1) \times 0 \end{bmatrix}^t \\ &= \begin{bmatrix} 2-2 & 6+0 \\ 0+2 & 0+0 \end{bmatrix}^t = \begin{bmatrix} 0 & 6 \\ 2 & 0 \end{bmatrix}^t \\ &= \begin{bmatrix} 0 & 2 \\ 6 & 0 \end{bmatrix} \end{aligned}$$

$$\text{R.H.S.} = B^t A^t,$$

$$(A)^t = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}^t = \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$$

$$(B)^t = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}^t = \begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 2 + (-2) \times 1 & 1 \times 0 + (-2)(-1) \\ 3 \times 2 + 0 \times 1 & 3 \times 0 + 0 \times (-1) \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & 0+2 \\ 6+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ 6 & 0 \end{bmatrix} = \text{L.H.S}$$

L.H.S = R.H.S

Thus $(AB)^t = B^t A^t$.