

Exercise 1.4

1. Which of the following product of matrices is conformable for multiplication?

Ans. (i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Number of Columns = Number of Rows

$\therefore$ product is possible.

(ii) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

Number of columns = Number of Rows.

$\therefore$ product is possible.

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

Number of columns $\neq$ Number of Rows.

$\therefore$ product is not possible.

(iv) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

Number of columns = Number of Rows.

$\therefore$ product is possible.

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Number of Columns = Number of Rows.

$\therefore$ Product is possible.

2. If $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$, find (i)

AB (ii) BA (if possible).

(i) $AB = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$$= \begin{bmatrix} 3(6) + 0(5) \\ -1(6) + 2(5) \end{bmatrix}$$

$$= \begin{bmatrix} 18+0 \\ -6+10 \end{bmatrix} = \begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

(ii) $BA = \begin{bmatrix} 6 \\ 5 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$

$\therefore$ Product is not possible.

Because number of columns $\neq$ number of rows.

3. Find the following products.

$$\begin{aligned} \text{Ans. (i)} \quad & [1 \ 2] \begin{bmatrix} 4 \\ 0 \end{bmatrix} \\ & = [1(4) + 2(0)] \\ & = [4 + 0] \\ & = [4] \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & [1 \ 2] \begin{bmatrix} 5 \\ -4 \end{bmatrix} \\ & = [1(5) + 2(-4)] \\ & = [5 - 8] \\ & = [-3] \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad & [-3 \ 0] \begin{bmatrix} 4 \\ 0 \end{bmatrix} \\ & = [-3(4) + 0(0)] = [-12] \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad & [6 \ 0] \begin{bmatrix} 4 \\ 0 \end{bmatrix} \\ & = [6(4) + (0)(0)] = [24] \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad & \begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix} \\ & = \begin{bmatrix} 1(4) + 2(0) & 1(5) + 2(-4) \\ -3(4) + 0(0) & -3(5) + 0(-4) \\ 6(4) + (-1)(0) & 6(5) + (-1)(-4) \end{bmatrix} \\ & = \begin{bmatrix} 4 + 0 & 5 - 8 \\ -12 + 0 & -15 + 0 \\ 24 + 0 & 30 + 4 \end{bmatrix} \\ & = \begin{bmatrix} 4 & -3 \\ -12 & -15 \\ 24 & 34 \end{bmatrix} \end{aligned}$$

4. Multiply the following matrix

$$\text{(a)} \quad \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$$

$$\text{(b)} \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$$

$$\text{(c)} \quad \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\text{(d)} \quad \begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 2 & -\frac{5}{4} \\ 4 & 4 \end{bmatrix}$$

$$\text{(e)} \quad \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} \text{Ans. (a)} \quad & \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix} \\ & = \begin{bmatrix} 2(2) + 3(3) & 2(-1) + 3(0) \\ 1(2) + 1(3) & 1(-1) + 1(0) \\ 0(2) + (-2)(3) & 0(-1) + (-2)(0) \end{bmatrix} \\ & = \begin{bmatrix} 13 & -2 \\ 5 & -1 \\ -6 & 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix} \\ & = \begin{bmatrix} 1(1) + 2(3) + 3(-1) & 1(2) + 2(4) + 3(1) \\ 4(1) + 5(3) + 6(-1) & 4(2) + 5(4) + 6(1) \end{bmatrix} \\ & = \begin{bmatrix} 1 + 6 - 3 & 2 + 8 + 3 \\ 4 + 15 - 6 & 8 + 20 + 6 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 4 & 13 \\ 13 & 34 \end{bmatrix}$$

$$(c) \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+2(4) & 1(2)+2(5) & 1(3)+2(6) \\ 3(1)+4(4) & 3(2)+4(5) & 3(3)+4(6) \\ -1(1)+1(4) & -1(2)+1(5) & -1(3)+1(6) \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 3 & 3 & 3 \end{bmatrix}$$

$$(d) \begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8(2)+5(-4) & 8\left(-\frac{5}{2}\right)+5(4) \\ 6(2)+4(-4) & 6\left(-\frac{5}{2}\right)+4(4) \end{bmatrix}$$

$$= \begin{bmatrix} 16-20 & -20+20 \\ 12-16 & -15+16 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ -4 & 1 \end{bmatrix}$$

$$(e) \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1(0)+2(0) & -1(0)+2(0) \\ 1(0)+3(0) & 1(0)+3(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

5. Let $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$ and

$C = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$. Verify whether

(i) $AB = BA$.

(ii) $A(BC) = (AB)C$

(iii) $A(B+C) = AB+AC$

(iv) $A(B-C) = AB-AC$

Ans. (i) $AB = BA$.

To check whether $AB = BA$ Or not

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -1(1)+3(-3) & -1(2)+3(-5) \\ 2(1)+0(-3) & 2(2)+0(-5) \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & -2-15 \\ 2-0 & 4+0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-1)+2(2) & 1(3)+2(0) \\ -3(-1)+(-5)(2) & -3(3)+(-5)(0) \end{bmatrix}$$

$$= \begin{bmatrix} -1+4 & 3+0 \\ 3-10 & -9+0 \end{bmatrix},$$

$$= \begin{bmatrix} 3 & 3 \\ -7 & -9 \end{bmatrix}$$

So $AB \neq BA$

(ii) $A(BC) = (AB)C$

L.H.S = $A(BC)$

$$BC = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1(2)+2(1) & 1(1)+2(3) \\ -3(2)+(-5)(1) & -3(1)+(-5)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 1+6 \\ -6-5 & -3-15 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 7 \\ -11 & -18 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 4 & 7 \\ -11 & -18 \end{bmatrix}$$

$$= \begin{bmatrix} -1(4)+3(-11) & -1(7)+3(-18) \\ 2(4)+0(-11) & 2(7)+0(-18) \end{bmatrix}$$

$$= \begin{bmatrix} -4-33 & -7-54 \\ 8+0 & 14+0 \end{bmatrix}$$

$$= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}$$

$$\text{R.H.S} = (AB)C$$

$$(AB) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -1(1)+3(-3) & -1(2)+3(-5) \\ 2(1)+0(-3) & 2(2)+0(-5) \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix} = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -10(2)+(-17)(1) & -10(1)+(-17)(3) \\ 2(2)+4(1) & 2(1)+4(3) \end{bmatrix}$$

$$= \begin{bmatrix} -20-17 & -10-51 \\ 4+4 & 2+12 \end{bmatrix}$$

$$= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}$$

$$\text{Hence } A(BC) = (AB)C$$

$$\text{(iii) } A(B+C) = AB+AC$$

$$\text{L.H.S} = A(B+C)$$

$$(B+C) = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 2+1 \\ -3+1 & -5+3 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix} = \begin{bmatrix} -3-6 & -3-6 \\ 6+0 & 6+0 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix}$$

$$\text{L.H.S.}$$

$$AB+AC$$

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -1(1)+3(-3) & -1(2)+3(-5) \\ 2(1)+0(-3) & 2(2)+0(-5) \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix} = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

$$AC = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1(2)+3(1) & -1(1)+3(3) \\ 2(2)+0(1) & 2(1)+0(3) \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$$

$$AB+AC = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -10+1 & -17+8 \\ 2+4 & 4+2 \end{bmatrix} = \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix}$$

6. For the matrices.

$$A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}, C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$$

Verify that (i) $(AB)^t = B^t A^t$ (ii) $(BC)^t = C^t B^t$

Ans. (i) $(AB)^t = B^t A^t$

$$\text{L.H.S} = (AB)^t$$

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -1(1)+3(-3) & -1(2)+3(-5) \\ 2(1)+0(-3) & 2(2)+0(-5) \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

$$(AB)^t = \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix}$$

$$\text{R.H.S} = B^t A^t$$

$$A^t = \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-1)+(-3)(3) & 1(2)+(-3)(0) \\ 2(-1)+(-5)(3) & 2(2)+5(0) \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & 2-0 \\ -2-15 & 4 \end{bmatrix} = \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Hence } (AB)^t = B^t A^t$$

$$(ii) (BC)^t = C^t B^t$$

$$\text{L.H.S} = (BC)^t$$

$$\begin{aligned} BC &= \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix} \\ &= \begin{bmatrix} 1(-2) + 2(3) & 1(6) + 2(-9) \\ -3(-2) + (-5)(3) & -3(6) + (-5)(-9) \end{bmatrix} \\ &= \begin{bmatrix} -2+6 & 6-18 \\ 6-15 & -18+45 \end{bmatrix} \\ &= \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix} \end{aligned}$$

$$(BC)^t = \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix}$$

$$\text{R.H.S} = C^t B^t$$

$$C^t = \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$\begin{aligned} C^t B^t &= \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} \\ &= \begin{bmatrix} -2(1) + 3(2) & -2(-3) + 3(-5) \\ 6(1) + (-9)(2) & 6(-3) + (-9)(-5) \end{bmatrix} \\ &= \begin{bmatrix} -2+6 & 6-15 \\ 6-18 & -18+45 \end{bmatrix} \\ &= \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix} \end{aligned}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Hence } (BC)^t = C^t B^t$$

Determinant of a 2-by-2 Matrix.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2-by-2 square matrix. The determinant of A , denoted by $\det A$ or $|A|$ is defined as $|A| = \det A = \det \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$= \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc = \lambda \in \text{Re.g.},$$

$$\text{Let } B = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}. \text{ Then } |B| = \det B = \begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} = 1 \times 3 - (-2)(1) = 3 + 2 = 5$$

$$\text{If } M = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}, \text{ then}$$

$$\det M = \begin{vmatrix} 2 & 6 \\ 1 & 3 \end{vmatrix} = 2 \times 3 - 1 \times 6 = 0$$

Singular and non-singular matrix.

A square matrix A is called singular if determinant of A is equal to zero.

$$\text{i.e., } |A| = 0.$$

For example, $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ is a singular

matrix, since $\det A = 1 \times 0 - 0 \times 2 = 0$

A square matrix A is called non-singular if the determinant of A is not equal to zero.

$$\text{i.e., } |A| \neq 0$$

For example $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ is non-

singular, since $\det A = 1 \times 2 - 0 \times 1 = 2 \neq 0$.

Note that, each square matrix with real entries is either singular or non-singular.

Adjoint of a Matrix.

Adjoint of a square matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

obtained by interchanging the diagonal entries and changing the sign of other entries. Adjoint of matrix A is denoted as $\text{Adj } A$.

$$\text{i.e., } \text{Adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\text{e.g., if } A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}, \text{ then}$$

$$\text{Adj } A = \begin{bmatrix} 0 & -2 \\ -3 & 1 \end{bmatrix}$$

$$\text{If } B = \begin{bmatrix} 2 & -1 \\ 3 & -4 \end{bmatrix}, \text{ then } \text{Adj } B = \begin{bmatrix} -4 & 1 \\ -3 & 2 \end{bmatrix}$$

Multiplicative inverse of a non-singular matrix.

Let A and B be two non-singular square matrices of same order. Then A and B are said to be multiplicative inverse of each other if

$$AB = BA = I$$

The inverse of A is denoted by A^{-1} , thus $AA^{-1} = A^{-1}A = I$.

Inverse of a matrix is possible only if matrix is non-singular.

Inverse of a Matrix using Adjoint

$$\text{Let } M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ be a square}$$

matrix. To find the inverse of M, i.e., M^{-1} , first we find the determinant as inverse is possible only of a non-singular matrix.

$$|M| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \neq 0$$

$$\text{and } \text{Adj } M = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \text{ then}$$

$$M^{-1} = \frac{\text{Adj } M}{|M|}$$

$$\text{e.g., Let } A = \begin{bmatrix} 2 & 1 \\ -1 & -3 \end{bmatrix}$$

$$\text{Then } |A| = -6 - (-1) = -6 + 1 = -5 \neq 0$$

$$|A| = -6 - (-1) = -6 + 1 = -5 \neq 0.$$

$$\text{Thus } A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} -3 & -1 \\ 1 & 2 \end{bmatrix}}{-5}$$

$$= \frac{-1}{5} \begin{bmatrix} -3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-1}{5} & \frac{-2}{5} \end{bmatrix}$$

$$\text{and } AA^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-1}{5} & \frac{-2}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{6}{5} - \frac{1}{5} & \frac{2}{5} - \frac{2}{5} \\ -\frac{3}{5} + \frac{3}{5} & -\frac{1}{5} + \frac{6}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I = A^{-1}A$$

Verification of $(AB)^{-1} = B^{-1}A^{-1}$

$$\text{Let } A = \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -1 \\ 3 & 2 \end{bmatrix}$$

$$\text{Then } \det A = 3 \times 0 - (-1) \times 1 = 1 \neq 0$$

$$\text{And } \det B = 0 \times 2 - 3(-1) = 3 \neq 0$$

Therefore, A and B are invertible i.e., their inverses exist.

Then, to verify the law of inverse of the product, take

AB

$$= \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 0 + 1 \times 3 & 3 \times (-1) + 1 \times 2 \\ -1 \times 0 + 0 \times 3 & -1 \times (-1) + 0 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \det (AB) = \begin{vmatrix} 3 & -1 \\ 0 & 1 \end{vmatrix} = 3 \neq 0$$

and L.H.S. = $(AB)^{-1}$

$$(AB)^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ 0 & 1 \end{bmatrix}$$

$$\text{R.H.S.} = B^{-1}A^{-1}, \text{ where } B^{-1} = \frac{1}{3} \begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix},$$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix} \frac{1}{1} \begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 2 \times 0 + 1 \times 1 & 2 \times (-1) + 1 \times 3 \\ -3 \times 0 + 0 \times 1 & -3 \times (-1) + 0 \times 3 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 0+1 & -2+3 \\ 0 & 3 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ 0 & 1 \end{bmatrix} = (AB)^{-1}$$

Thus the law $(AB)^{-1} = B^{-1} A^{-1}$ is verified.