

Exercise 1.5

1. Find the determinant of the following matrices.

Ans. (i) $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

$$\begin{aligned} |A| &= \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix} \\ &= -1(0) - 2(1) \\ &= 0 - 2 = -2 \end{aligned}$$

(ii) $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

$$\begin{aligned} |B| &= 1(-2) - 2(3) \\ &= -2 - 6 \\ &= -8 \end{aligned}$$

(iii) $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

$$\begin{aligned} |C| &= 3(2) - 3(2) \\ &= 6 - 6 = 0 \end{aligned}$$

(iv) $D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

$$\begin{aligned} |D| &= 3(4) - 1(2) \\ &= 12 - 2 = 10 \end{aligned}$$

2. Find which of the following matrices are singular or non-singular?

Ans. (i) $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

$$\begin{aligned} |A| &= \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix} \\ &= 3(4) - 2(6) \\ &= 12 - 12 \\ &= 0 \quad \text{singular} \end{aligned}$$

(ii) $B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

$$\begin{aligned} |B| &= \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} \\ &= 4(2) - 3(1) = 8 - 3 = 5 \quad \text{non-singular} \end{aligned}$$

(iii) $C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$

$$\begin{aligned} |C| &= \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix} \\ &= 7(5) - 3(-9) \\ &= 35 + 27 \\ &= 62 \neq 0 \quad \text{non-singular} \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad D &= \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix} \\
 |D| &= \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix} \\
 &= 5(4) - (-2)(-10) \\
 &= 20 - 20 \\
 &= 0 \text{ singular}
 \end{aligned}$$

3. Find the multiplicative inverse (if it exists) of each.

$$\begin{aligned}
 \text{Ans. (i)} \quad A &= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \\
 |A| &= \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix} \\
 &= -1(0) - 2(3) \\
 &= -6 \\
 \text{Adj } A &= \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix} \\
 A^{-1} &= \frac{1}{|A|} \text{adj } A \\
 &= \frac{1}{-6} \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad B &= \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \\
 |B| &= \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix} \\
 &= 1(-5) - (-3)(2) \\
 &= -5 + 6 \\
 &= 1 \neq 0
 \end{aligned}$$

$$\text{Adj } B = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$$

$$\begin{aligned}
 B^{-1} &= \frac{1}{|B|} \text{adj } B \\
 &= \frac{1}{1} \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}
 \end{aligned}$$

$$\text{(iii)} \quad C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$$

$$\begin{aligned}
 |C| &= \begin{vmatrix} -2 & 6 \\ 3 & -9 \end{vmatrix} \\
 &= -2(-9) - 3(6) \\
 &= 18 - 18 = 0
 \end{aligned}$$

C^{-1} does not exist.

$$\text{(iv)} \quad D = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}$$

$$\begin{aligned}
 |D| &= \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix} \\
 &= \frac{1}{2}(2) - 1\left(\frac{3}{4}\right) \\
 &= 1 - \frac{3}{4} \\
 &= \frac{4-3}{4} = \frac{1}{4} \neq 0
 \end{aligned}$$

$$\text{Adj } D = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$D^{-1} = \frac{1}{|D|} \text{adj } D$$

$$= \frac{1}{\frac{1}{4}} \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 4 \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$$

4. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$, then

(i) $A(\text{Adj } A) = (\text{Adj } A)A = (\det A)I$

(ii) $BB^{-1} = I = B^{-1}B$

Ans. (i) $A(\text{Adj } A) = (\text{Adj } A)A = (\det A)I$

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$$

$$\begin{aligned} A(\text{Adj } A) &= \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1(6) + 2(-4) & 1(-2) + 2(1) \\ 4(6) + 6(-4) & 4(-2) + 6(1) \end{bmatrix} \\ &= \begin{bmatrix} 6-8 & -2+2 \\ 24-24 & -8+6 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{Now } (\text{Adj } A)A &= \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 6(1) + (-2)(4) & 6(2) + (-2)(6) \\ -4(1) + 1(4) & -4(2) + 1(6) \end{bmatrix} \\ &= \begin{bmatrix} 6-8 & 12-12 \\ -4+4 & -8+6 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \end{aligned}$$

Also $(\det A)I$

$$\begin{aligned} \det A &= \begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix} \\ &= 1(6) - 2(4) = 6 - 8 = -2 \end{aligned}$$

$$(\det A)I = -2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

Hence: $A(\text{Adj } A) = (\text{Adj } A)A = (\det A)I$

(ii) $B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$

$$\begin{aligned} |B| &= \begin{vmatrix} 3 & -1 \\ 2 & -2 \end{vmatrix} = 3(2) - 2(-1) \\ &= -6 + 2 = -4 \neq 0 \end{aligned}$$

$$\text{Adj } B = \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$\begin{aligned} B^{-1} &= \frac{1}{|B|} \text{Adj } B \\ &= \frac{1}{-4} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 2 & -1 \\ 2 & -3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} BB^{-1} &= \frac{1}{4} \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 2 & -3 \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} 3(2) + (-1)(2) & 3(-1) + (-1)(-3) \\ 2(2) + (-2)(2) & 2(-1) + (-2)(-3) \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} 6-2 & -3+3 \\ 4-4 & -2+6 \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Similarly:

$$\begin{aligned} B^{-1}B &= \frac{1}{4} \begin{bmatrix} 2 & -1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} 2(3) + (-1)(2) & 2(-1) + (-1)(-2) \\ 2(3) + (-3)(2) & 2(-1) + (-3)(-2) \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Hence: $BB^{-1} = I = B^{-1}B$

5. Determine whether the given matrices are multiplicative inverses of each other.

Ans. (i) $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

$$\begin{aligned} & \begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 3(7)+5(-4) & 3(-5)+5(3) \\ 4(7)+7(-4) & 4(-5)+7(3) \end{bmatrix} \\ &= \begin{bmatrix} 21-20 & -15+15 \\ 28-28 & -20+21 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

∴ Given matrices are multiplicative inverse of each other.

$$\begin{aligned} \text{(ii)} \quad & \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \text{ and } \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \\ & \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 1(-3)+2(2) & 1(2)+2(-1) \\ 2(-3)+3(2) & 2(2)+3(-1) \end{bmatrix} \\ &= \begin{bmatrix} -3+4 & 2-2 \\ -6+6 & 4-3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

6. If $A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$,

$D = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$, then verify that

(i) $(AB)^{-1} = B^{-1} A^{-1}$

(ii) $(DA)^{-1} = A^{-1} D^{-1}$

Ans. (i) $(AB)^{-1} = B^{-1} A^{-1}$

L.H.S = $(AB)^{-1}$

$$\begin{aligned} AB &= \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 4(-4)+0(1) & 4(-2)+0(-1) \\ -1(-4)+2(1) & -1(-2)+2(-1) \end{bmatrix} \\ &= \begin{bmatrix} -16 & -8 \\ 6 & 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} |AB| &= \begin{vmatrix} -16 & -8 \\ 6 & 0 \end{vmatrix} \\ &= -16(0) - 6(-8) \\ &= 0 + 48 = 48 \neq 0 \end{aligned}$$

$$\text{Adj}(AB) = \begin{vmatrix} 0 & 8 \\ -6 & -16 \end{vmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|} \text{Adj}(AB)$$

$$= \frac{1}{48} \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix} = \begin{bmatrix} 0 & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix} = \begin{bmatrix} 0 & 1/6 \\ -1/8 & -1/3 \end{bmatrix}$$

R.H.S = $B^{-1} A^{-1}$

$$B = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

$$|B| = -4(-1) - (1)(-2) = 4 + 2 = 6$$

$$B^{-1} = \frac{1}{|B|} \text{Adj} B = \frac{1}{6} \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix} = 4(2) - (-1)(0) = 8$$

$$A^{-1} = \frac{1}{|A|} \text{Adj} A = \frac{1}{8} \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$B^{-1} A^{-1} = \frac{1}{6} \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \frac{1}{8} \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \frac{1}{48} \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \frac{1}{48} \begin{bmatrix} -1(2)+2(1) & -1(0)+2(4) \\ -1(2)+-4(1) & -1(0)+-4(4) \end{bmatrix}$$

$$= \frac{1}{48} \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix} = \begin{bmatrix} 0 & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1/6 \\ -1/8 & -1/3 \end{bmatrix}$$

L.H.S = R.H.S

Hence: $(AB)^{-1} = B^{-1} A^{-1}$

$$(ii) \quad (DA)^{-1} = A^{-1} D^{-1}$$

$$\text{L.H.S} = (DA)^{-1}$$

$$DA = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3(4)+1(-1) & -2(0)+1(2) \\ -2(4)+2(-1) & -2(0)+2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 12-1 & 0+2 \\ -8-2 & 0+4 \end{bmatrix} = \begin{bmatrix} 11 & 2 \\ -10 & 4 \end{bmatrix}$$

$$|DA| = \begin{vmatrix} 11 & 2 \\ -10 & 4 \end{vmatrix}$$

$$= 11(4) - (-10)(2)$$

$$= 44 + 20$$

$$= 64$$

$$\text{Adj}(DA) = \begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$$

$$(DA)^{-1} = \frac{1}{DA} \text{Adj}(DA)$$

$$= \frac{1}{64} \begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix} = \begin{bmatrix} \frac{4}{64} & \frac{-2}{64} \\ \frac{10}{64} & \frac{11}{64} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & \frac{-1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix}$$

$$\text{R.H.S} = A^{-1} D^{-1}$$

$$A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$$

$$= 4(2) - (-1)(0)$$

$$= 8 \neq 0$$

$$A^{-1} = \frac{1}{|A|} \text{Adj}A$$

$$= \frac{1}{8} \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$$

$$|D| = 3(2) - (-2)(1)$$

$$= 6 + 2 = 8$$

$$D^{-1} = \frac{1}{|D|} \text{Adj}D$$

$$= \frac{1}{8} \begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$$

$$A^{-1} D^{-1} = \frac{1}{8} \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix} \cdot \frac{1}{8} \begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$$

$$A^{-1} D^{-1} = \frac{1}{64} \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= \frac{1}{64} \begin{bmatrix} 2(2)+0(2) & 2(-1)+0(3) \\ 1(2)+4(2) & 1(-1)+4(3) \end{bmatrix}$$

$$= \frac{1}{64} \begin{bmatrix} 4+0 & -2+0 \\ 2+8 & -1+12 \end{bmatrix}$$

$$= \frac{1}{64} \begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix} = \begin{bmatrix} \frac{4}{64} & \frac{-2}{64} \\ \frac{10}{64} & \frac{11}{64} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & \frac{-1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Hence: } (DA)^{-1} = A^{-1} D^{-1}$$

Solution of Simultaneous Linear Equations

System of two linear equations in two variables in general form is given as

$$ax + by = m$$

$$cx + dy = n$$

Where a, b, c, d, m and n are real numbers.

This system is also called simultaneous linear equations.

We discuss here the following methods of solution.

(i) **Matrix inversion method.**

(ii) **Cramer's rule**

(i) **Matrix Inversion Method**

Consider the system of linear questions

$$ax + by = m$$

$$cx + dy = n$$

$$\text{Then } \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix}$$

$$\text{or } AX = B$$

$$\text{Where } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} m \\ n \end{bmatrix}$$

$$\text{or } X = A^{-1} B$$

$$|A| = ad - bc$$

$$\text{or } X = \frac{\text{Adj } A}{|A|} \times B$$

$$\therefore A^{-1} = \frac{\text{Adj } A}{|A|} \text{ and } |A| \neq 0$$

$$\text{or } \begin{bmatrix} x \\ y \end{bmatrix} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \begin{bmatrix} m \\ n \end{bmatrix}}{ad - bc}$$

$$= \begin{bmatrix} \frac{dm - bn}{ad - bc} \\ \frac{-cm + an}{ad - bc} \end{bmatrix}$$

$$\Rightarrow x = \frac{dm - bn}{ad - bc} \text{ and } y = \frac{an - cm}{ad - bc}$$

(ii) **Cramer's Rule.**

Consider the following system of linear equations.

$$ax + by = m$$

$$cx + dy = n$$

We know that

$$AX = B, \text{ where } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\text{and } B = \begin{bmatrix} m \\ n \end{bmatrix}$$

$$\text{or } X = A^{-1} B$$

$$\text{or } X = \frac{\text{Adj } A}{|A|} \times B$$

$$\text{or } \begin{bmatrix} x \\ y \end{bmatrix} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \begin{bmatrix} m \\ n \end{bmatrix}}{|A|}$$

$$= \frac{\begin{bmatrix} dm - bn \\ -cm + an \end{bmatrix}}{|A|}$$

$$= \begin{bmatrix} \frac{dm - bn}{|A|} \\ \frac{-cm + an}{|A|} \end{bmatrix}$$

$$\text{or } x = \frac{dm - bn}{|A|} = \frac{|A_x|}{|A|}$$

$$\text{and } y = \frac{an - cm}{|A|} = \frac{|A_y|}{|A|}$$

$$\text{where } |A_x| = \begin{vmatrix} m & b \\ n & d \end{vmatrix} \text{ and}$$

$$|A_y| = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$$

Example 1

Solve the following system by using matrix inversion method.

$$4x - 2y = 8$$

$$3x + y = -4$$

Solution

Step 1
$$\begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$$

Step 2

The coefficient matrix $M = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$ is

non-singular, since

$$\det M = 4 \times 1 - 3(-2) = 4 + 6 = 10 \neq 0. \text{ So}$$

M^{-1} is possible.

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} 8 \\ -4 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 8 \\ -4 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 8-8 \\ -24-16 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 0 \\ -40 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

$$\Rightarrow x = 0 \text{ and } y = -4$$

Example 2

Solve the following system of linear equations by using Cramer's rule.

$$3x - 2y = 1$$

$$-2x + 3y = 2$$

Solution

$$3x - 2y = 1$$

$$-2x + 3y = 2$$

We have

$$A = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix},$$

$$A_x = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix},$$

$$A_y = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 \\ -2 & 3 \end{vmatrix} = 9 - 4 = 5 \neq 0 \text{ (non-singular)}$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix}}{5} = \frac{3+4}{5} = \frac{7}{5}$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 3 & 1 \\ -2 & 2 \end{vmatrix}}{5} = \frac{6+2}{5} = \frac{8}{5}$$

$$S.S = \left\{ \left(\frac{7}{5}, \frac{8}{5} \right) \right\}$$

Example 3

The length of a rectangle is 6 cm less than three times its width. The perimeter of the rectangle is 140 cm. Find the dimensions of the rectangle.

(by using matrix inversion method)

Solution

If width of the rectangle is x cm, then length of the rectangle y cm. According to first condition

$$y = 3x - 6,$$

According to 2nd condition

$$\text{The perimeter} = 2x + 2y = 140$$

$$\Rightarrow x + y = 70 \quad \dots\dots\dots (i)$$

$$\text{and } 3x - y = 6 \quad \dots\dots\dots (ii)$$

In the matrix form

$$\begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 70 \\ 6 \end{bmatrix}$$

$$\det \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix} = \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix}$$

$$= 1 \times (-1) - 3 \times 1 = -1 - 3 = -4 \neq 0$$

We know that:

$$X = A^{-1} B \text{ and } A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$\begin{aligned}\text{Hence } \begin{bmatrix} x \\ y \end{bmatrix} &= \frac{1}{-4} \begin{bmatrix} -1 & -1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 70 \\ 6 \end{bmatrix} \\ &= \frac{-1}{4} \begin{bmatrix} -70 - 6 \\ -210 + 6 \end{bmatrix} = \begin{bmatrix} \frac{76}{4} \\ \frac{204}{4} \end{bmatrix} = \begin{bmatrix} 19 \\ 51 \end{bmatrix}\end{aligned}$$

Thus, by the equality of matrices, width of the rectangle $x = 19$ cm and the length $y = 51$ cm.