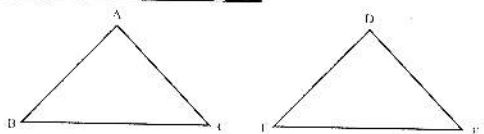


CONGRUENT TRIANGLES

Congruent Triangle



Let there be two triangles ABC and DEF. Out of the total six (1 - 1) correspondences that can be established between $\triangle ABC$ and $\triangle DEF$. One of the choices is explained below.

In the correspondence $\triangle ABC \leftrightarrow \triangle DEF$ it means.

$\angle A \leftrightarrow \angle D$ ($\angle A$ corresponds to $\angle D$)

$\angle B \leftrightarrow \angle E$ ($\angle B$ corresponds to $\angle E$)

$\angle C \leftrightarrow \angle F$ ($\angle C$ corresponds to $\angle F$)

$\overline{AB} \leftrightarrow \overline{DE}$ ($\overline{AB}$ corresponds to $\overline{DE}$)

$\overline{BC} \leftrightarrow \overline{EF}$ ($\overline{BC}$ corresponds to $\overline{EF}$)

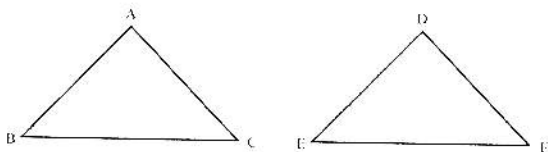
$\overline{CA} \leftrightarrow \overline{FD}$ ($\overline{CA}$ corresponds to $\overline{FD}$)

Congruency of Triangles

Two triangles are said to be congruent written symbolically as, $\cong$, if there exists a correspondence between them such that all the corresponding sides and angles are congruent i.e.

$$\text{If } \begin{cases} \overline{AB} \cong \overline{DE} \\ \overline{BC} \cong \overline{EF} \\ \overline{CA} \cong \overline{FD} \end{cases} \quad \text{and} \quad \begin{cases} \angle A \cong \angle D \\ \angle B \cong \angle E \\ \angle C \cong \angle F \end{cases}$$

Then $\triangle ABC \cong \triangle DEF$



Note

(i) These triangles are congruent w.r.t. the above mentioned choice of the (1 - 1) correspondence.

(ii) $\triangle ABC \cong \triangle ABC$

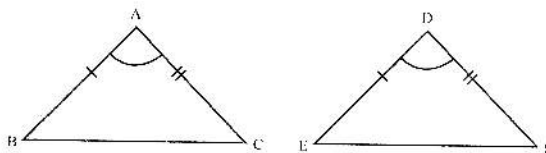
(iii) $\triangle ABC \cong \triangle DEF \Leftrightarrow \triangle DEF \cong \triangle ABC$

(iv) If $\triangle ABC \cong \triangle DEF$ and $\triangle ABC \cong \triangle PQR$, then $\triangle DEF \cong \triangle PQR$

In any correspondence of two triangles, if two sides and their included angle of one triangle are congruent to the corresponding two sides and their included angle of the other, then the triangles are congruent.

In $\triangle ABC \leftrightarrow \triangle DEF$, shown in the following figure.

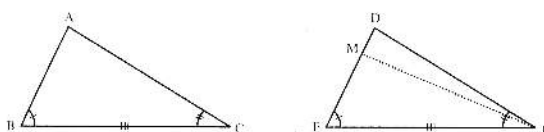
$$\text{If } \begin{cases} \overline{AB} \cong \overline{DE} \\ \angle A \cong \angle D \\ \overline{AC} \cong \overline{DF} \end{cases}$$



Then $\triangle ABC \cong \triangle DEF$ (S.A.S. Postulate)

Theorem

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding, side and angles of the other, then the triangles are congruent. (A.S.A $\cong$ A.S.A)



Given

In $\triangle ABC \leftrightarrow \triangle DEF$

$$\angle B \cong \angle E$$

$$\overline{BC} \cong \overline{EF}$$

Proof

Statements	Reasons
In $\triangle ABC \leftrightarrow \triangle MEF$	Construction
$\overline{AB} \cong \overline{ME}$(i)	Given
$\overline{BC} \cong \overline{EF}$(ii)	Given
$\angle B \cong \angle E$(iii)	S.A.S. postulate
$\therefore \triangle ABC \cong \triangle MEF$	(Corresponding angles of congruent triangles)
So, $\angle C \cong \angle MFE$	Given
But $\angle C \cong \angle DFE$	Both congruent to $\angle C$
$\therefore \angle DFE \cong \angle MFE$	
This is possible only if D and M are the same points, and $\overline{ME} \cong \overline{DE}$	
So, $\overline{AB} \cong \overline{DE}$(iv)	$\overline{AB} \cong \overline{ME}$ (construction) and
Thus from (i), (iii) and (iv), we have	$\overline{ME} \cong \overline{DE}$ (proved)
$\triangle ABC \cong \triangle DEF$	S.A.S. postulate

Example

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the correspondence side and angles of the other, then the triangles are congruent. (S.A.A $\cong$ S.A.A.)

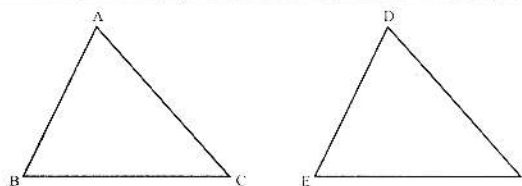
$$\angle C \cong \angle F$$

To prove

$$\triangle ABC \leftrightarrow \triangle DEF$$

Construction

Suppose $\overline{AB} \not\cong \overline{DE}$, take a point M on $\overline{DE}$ such that $\overline{AB} \cong \overline{ME}$. Join M to F



Given

In $\triangle ABC \leftrightarrow \triangle DEF$

$$\overline{BC} \cong \overline{EF}, \angle A \cong \angle D, \angle B \cong \angle E$$

To Prove

$$\triangle ABC \cong \triangle DEF$$

Proof

Statements	Reasons
In $\triangle ABC \leftrightarrow \triangle DEF$ $\angle B \cong \angle E$ $\overline{BC} \cong \overline{EF}$ $\angle C \cong \angle F$ $\therefore \triangle ABC \cong \triangle DEF$	Given Given $\angle A \cong \angle D, \angle B \cong \angle E, (\text{Given})$ A.S.A. $\cong$ A.S.A

Example

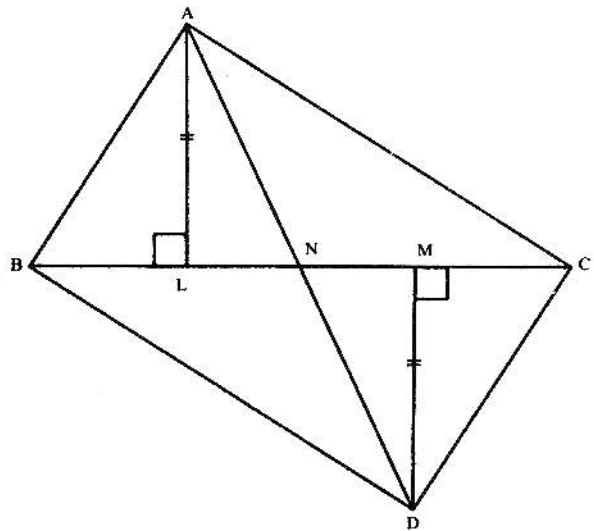
If $\triangle ABC$ and $\triangle DCB$ are on the opposite sides of common base $\overline{BC}$ such that $\overline{AL} \perp \overline{BC}$, $\overline{DM} \perp \overline{BC}$, $\overline{AL} \cong \overline{DM}$, then $\overline{BC}$ bisects $\overline{AD}$.

Given

$\triangle ABC$ and $\triangle DCB$ are on the opposite sides of $\overline{BC}$ such that $\overline{AL} \perp \overline{BC}$, $\overline{DM} \perp \overline{BC}$, $\overline{AL} \cong \overline{DM}$ and $\overline{AD}$ is cut by $\overline{BC}$ at N .

To Prove

$\overline{AN} \cong \overline{DN}$



Statements	Reasons
In $\triangle ALN \leftrightarrow \triangle DMN$ $\overline{AL} \cong \overline{DM}$ $\angle ALN \cong \angle DMN$ $\angle ANL \cong \angle DNM$ $\triangle ALN \cong \triangle DMN$ Hence $\overline{AN} \cong \overline{DN}$	Given Each angle is right angle Vertical angles S.A.A. $\cong$ S.A.A Corresponding sides of $\cong$ Δ s.

Exercise 10.1

1. In the given figure.

$$\overline{AB} \cong \overline{CB}, \angle 1 \cong \angle 2.$$

Prove that

$$\triangle ABD \cong \triangle CBE$$

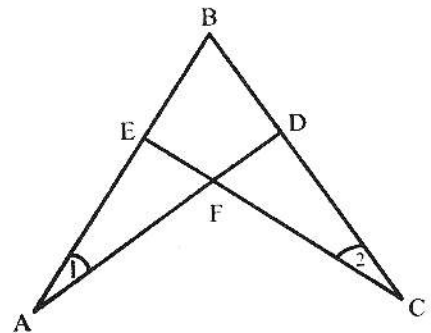
Given

$$\overline{AB} \cong \overline{CB}$$

$$\angle 1 = \angle 2$$

To Prove

$$\triangle ABD \cong \triangle CBE$$



Statements	Reasons
<p>In $\triangle ABD \leftrightarrow \triangle CBE$</p> <p>$\overline{AB} \cong \overline{CB}$</p> <p>$\angle 1 \cong \angle 2$</p> <p>$\angle ABD \cong \angle CBE$</p> <p>$\therefore \triangle ABD \cong \triangle CBE$</p>	<p>Given</p> <p>Given</p> <p>Common angle</p> <p>A.S.A $\cong$ A.S.A</p>

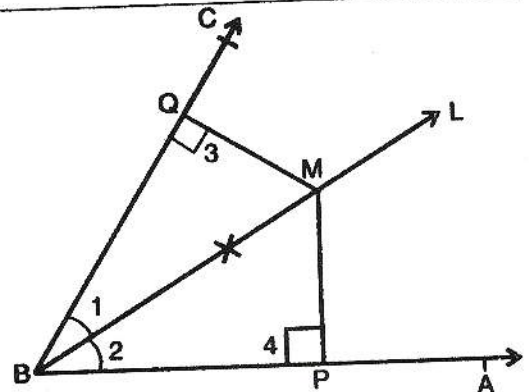
- (2) From a point on the bisector of an angle, perpendiculars are drawn to the arms of the angle. Prove that these perpendiculars are equal in measure.

Given

$\angle ABC$, $\overline{BL}$ the bisector of $\angle ABC$, M any point on $\overline{BL}$, $\overline{MP}$ perpendicular on $\overline{AB}$, $\overline{MQ} \perp \overline{BC}$.

To Prove

$$\overline{MP} \cong \overline{MQ}$$



Statements	Reasons
<p>In $\triangle BMP \leftrightarrow \triangle BMQ$</p> <p>$\angle 1 \cong \angle 2$</p> <p>$\angle 3 \cong \angle 4$</p> <p>$\overline{BM} \cong \overline{BM}$</p> <p>$\triangle BMP \cong \triangle BMQ$</p> <p>$\overline{PM} \cong \overline{QM}$</p>	<p>$\overline{BL}$ bisects $\angle PBQ$</p> <p>Each = 90°</p> <p>Common</p> <p>A.S.A $\cong$ A.S.A</p> <p>Corresponding sides of the congruent triangles.</p>

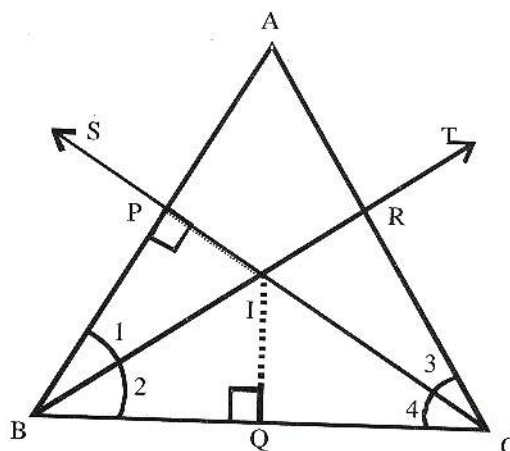
(3) In a triangle ABC, the bisectors of $\angle B$ and $\angle C$ meet in a point I. Prove that I is equidistant from the three sides of $\triangle ABC$.

Given

In $\triangle ABC$, $\overline{BT}$, $\overline{CS}$ are the bisectors of the angles B and C respectively.

To Prove

I is equidistant from the three sides of $\triangle ABC$ i.e. $\overline{IP} \cong \overline{IQ} \cong \overline{IR}$



Construction

$\overline{IR} \perp \overline{AC}$, $\overline{IQ} \perp \overline{BC}$, $\overline{IP} \perp \overline{AB}$

Statements	Reasons
In $\triangle IPB \leftrightarrow \triangle IQB$	
$\angle 1 \cong \angle 2$	Given
$\angle P \cong \angle Q$	Each = 90°
$\overline{IB} \cong \overline{IB}$	Common
$\triangle IPB \cong \triangle IQB$	A.S.A $\cong$ A.S.A
$\overline{IP} \cong \overline{IQ}$(i)	Corresponding sides of congruent triangles
Similarly $\triangle IRC \cong \triangle IQC$	
$\overline{IQ} \cong \overline{IR}$(ii)	Corresponding sides of congruent triangles
$\overline{IP} \cong \overline{IQ} \cong \overline{IR}$	By (i) and (ii)

Theorem

If two angles of a triangle are congruent, then the sides opposite to them are also congruent.

Given

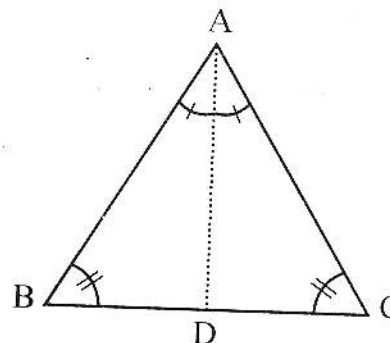
In $\triangle ABC$, $\angle B \cong \angle C$

To Prove

$\overline{AB} \cong \overline{AC}$

Construction

Draw the bisector of $\angle A$, meeting $\overline{BC}$ at the point D.



Proof

Statements	Reasons
In $\triangle ABD \leftrightarrow \triangle ACD$	
$\overline{AD} \cong \overline{AD}$	Common
$\angle B \cong \angle C$	Given
$\angle BAD \cong \angle CAD$	Construction
$\therefore \triangle ABD \cong \triangle ACD$	S.A.A. $\cong$ S.A.A.
Hence $\overline{AB} \cong \overline{AC}$	Corresponding sides of congruent triangles

Example

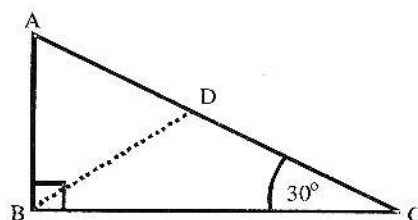
If one angle of a right triangle is of 30° , the hypotenuse is twice as long as the side opposite to the angle.

Given

In $\triangle ABC$, $m\angle B = 90^\circ$ and $m\angle C = 30^\circ$

To Prove

$$m\overline{AC} = 2m\overline{AB}$$

**Construction**

At B, construct $\angle CBD$ of 30° . Let $\overline{BD}$ cut $\overline{AC}$ at the point D.

Proof

Statements	Reasons
In $\triangle ABD$, $m\angle A = 60^\circ$	$m\angle ABC = 90^\circ$, $m\angle C = 30^\circ$
$m\angle ABD = m\angle ABC - m\angle CBD = 60^\circ$	$m\angle ABC = 90^\circ$, $m\angle CBD = 30^\circ$
$\therefore m\angle ADB = 60^\circ$	Sum of measures of $\angle$ s of a $\triangle$ is 180°
$\therefore \triangle ABD$ is equilateral	Each of its angles is equal to 60°
$\therefore \overline{AB} \cong \overline{BD} \cong \overline{AD}$	Sides of equilateral $\triangle$
In $\triangle BCD$, $\overline{BD} \cong \overline{CD}$	$\angle C = \angle CBD$ (each of 30°).
Thus	$\overline{AD} \cong \overline{AB}$ and $\overline{CD} \cong \overline{BD} \cong \overline{AB}$
$\left. \begin{aligned} m\overline{AC} &= m\overline{AD} + m\overline{CD} \\ &= m\overline{AB} + m\overline{AB} \\ &= 2(m\overline{AB}) \end{aligned} \right\}$	

Example

If the bisector of an angle of a triangle bisects the side opposite to it, the triangle is isosceles.

Given

In $\triangle ABC$, $\overline{AD}$ bisects $\angle A$ and $\overline{BD} \cong \overline{CD}$

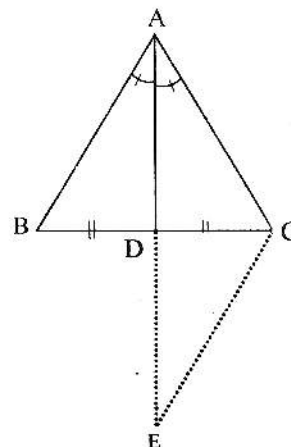
To Prove

$$\overline{AB} \cong \overline{AC}$$

Construction

Produce $\overline{AD}$ to E, and take $\overline{ED} \cong \overline{AD}$.

Join C to E

Proof

Statements	Reasons
<p>In $\triangle ABD \leftrightarrow \triangle EDC$</p> <p>$\overline{AD} \cong \overline{ED}$</p> <p>$\angle ADB \cong \angle EDC$</p> <p>$\overline{BD} \cong \overline{CD}$</p> <p>$\therefore \triangle ADB \cong \triangle EDC$</p> <p>$\therefore \overline{AB} \cong \overline{EC} \dots\dots\dots(1)$</p> <p>and $\angle BAD \cong \angle E$</p> <p>But $\angle BAD \cong \angle CAD$</p> <p>$\therefore \angle E \cong \angle CAD$</p> <p>In $\triangle ACE$, $\overline{AC} \cong \overline{EC} \dots\dots\dots(2)$</p> <p>Hence $\overline{AB} \cong \overline{AC}$</p>	<p>Construction</p> <p>Vertical angles</p> <p>Given</p> <p>S.A.S. Postulate</p> <p>Corresponding sides of $\cong \Delta s$</p> <p>Corresponding angles of $\cong \Delta s$</p> <p>Given</p> <p>Each $\cong \angle BAD$</p> <p>$\angle E \cong \angle CAD$ (proved)</p> <p>From (1) and (2)</p>