

## Exercise 10.2

Prove that a point, which is equidistant from the end points of a line segment, is on the right bisector of the line segment.

**Given**

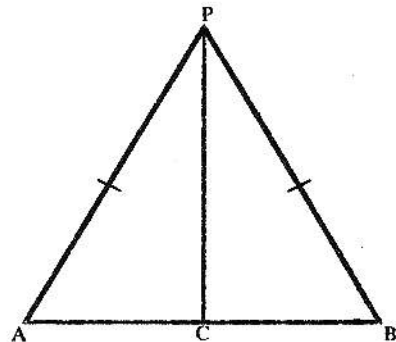
$\overline{AB}$  is a line segment. Point P is such that  $\overline{PA} \cong \overline{PB}$

**To Prove**

Point P is on the right bisector of  $\overline{AB}$ .

**Construction**

Join P to C, the midpoint of  $\overline{AB}$



**Proof**

| Statements                                                                        | Reasons                                     |
|-----------------------------------------------------------------------------------|---------------------------------------------|
| In $\triangle ACP \leftrightarrow \triangle BCP$                                  | Given                                       |
| $\overline{PA} \cong \overline{PB}$                                               | Common                                      |
| $\overline{PC} \cong \overline{PC}$                                               | Construction                                |
| $\overline{AC} \cong \overline{BC}$                                               | S.S.S $\cong$ S.S.S                         |
| $\triangle ACP \cong \triangle BCP$                                               | Corresponding angles of congruent triangles |
| $\angle ACP \cong \angle BCP$ ....(i)                                             | supplementary angles,                       |
| But $m\angle ACP + m\angle BCP = 180^\circ$ ... (ii)                              | From (i) and (ii)                           |
| $m\angle ACP = m\angle BCP = 90^\circ$                                            | $m\angle ACP = 90^\circ$ (proved)           |
| or $\overline{PC} \perp \overline{AB}$ ....(iii)                                  | construction                                |
| Also $\overline{CA} \cong \overline{CB}$ ....(iv)                                 |                                             |
| $\therefore \overline{PC}$ is a right bisector                                    |                                             |
| Of $\overline{AB}$ i.e, the point P is on the right bisector of $\overline{AB}$ . | from (iii) and (vi)                         |

**Theorem**

In a correspondence of two triangles, if three sides of one triangle are congruent to the corresponding three sides of the other, then the two triangles are congruent.

(S.S.S.  $\cong$  S.S.S.)

**Given**

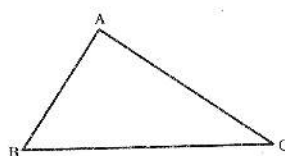
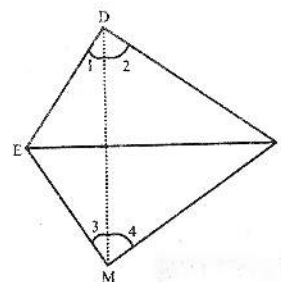
In  $\triangle ABC \leftrightarrow \triangle DEF$   
 $\overline{AB} \cong \overline{DE}$ ,  $\overline{BC} \cong \overline{EF}$  and  $\overline{CA} \cong \overline{FD}$

**To Prove**

$\triangle ABC \cong \triangle DEF$

**Construction**

Suppose that in  $\triangle DEF$  the side  $\overline{EF}$  is not smaller than any of the remaining two sides. On  $\overline{EF}$  construct a  $\triangle MEF$  in which,  $\angle FEM \cong \angle B$  and  $\overline{ME} \cong \overline{AB}$ . Join D and M. As shown in the above figures we label some of the angles as 1, 2, 3 and 4.



# **Proof**

| Statements                                                 | Reasons                                       |
|------------------------------------------------------------|-----------------------------------------------|
| In $\triangle ABC \leftrightarrow \triangle MEF$           |                                               |
| $\overline{BC} \cong \overline{EF}$                        | Given                                         |
| $\angle B \cong \angle FEM$                                | Construction                                  |
| $\overline{AB} \cong \overline{ME}$                        | Construction                                  |
| $\therefore \triangle ABC \cong \triangle MEF$             | S.A.S postulate                               |
| and $\overline{CA} \cong \overline{FM}$ .....(i)           | (Corresponding sides of congruent triangles)  |
| Also $\overline{CA} \cong \overline{FD}$ .....(ii)         | Given                                         |
| $\therefore \overline{FM} \cong \overline{FD}$             | From (i) and (ii)                             |
| In $\triangle FDM$                                         |                                               |
| $\angle 2 \cong \angle 4$ .....(iii)                       | $\overline{FM} \cong \overline{FD}$ (proved)  |
| Similarly $\angle 1 \cong \angle 3$ .....(iv)              |                                               |
| $\therefore m\angle 2 + m\angle 1 = m\angle 4 + m\angle 3$ | { from (iii) and (iv) }                       |
| $\therefore m\angle EDF = m\angle EMF$                     |                                               |
| Now, In $\triangle DEF \leftrightarrow \triangle MEF$      |                                               |
| $\overline{FD} \cong \overline{FM}$                        | Proved                                        |
| And $m\angle EDF \cong m\angle EMF$                        | Proved                                        |
| $\overline{DE} \cong \overline{ME}$                        | Each one $\cong \overline{AB}$                |
| $\therefore \triangle DEF \cong \triangle MEF$             | S.A.S postulate                               |
| Also $\triangle ABC \cong \triangle MEF$                   | Proved                                        |
| Hence $\triangle ABC \cong \triangle DEF$                  | Each $\triangle \cong \triangle MEF$ (Proved) |

## **Example**

If two isosceles triangles are formed on the same side of their common base, the line through their vertices would be the right bisector of their common base.

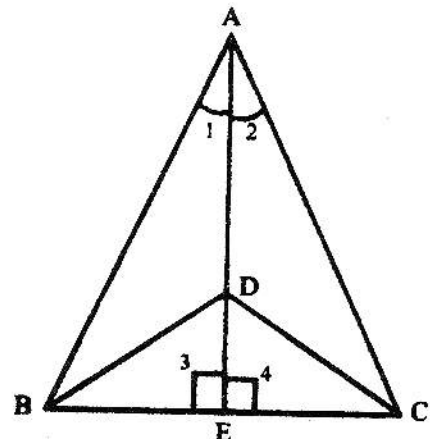
### **Given**

$\triangle ABC$  and  $\triangle DBC$  are formed on the same side of  $\overline{BC}$  such that

$\overline{AB} \cong \overline{AC}$ ,  $\overline{DB} \cong \overline{DC}$ ,  $\overline{AD}$  meets  $\overline{BC}$  at  $E$ .

### **To prove**

$\overline{BE} \cong \overline{CE}$ ,  $\overline{AE} \perp \overline{BC}$



**Proof**

| Statements                                                                                                                                                            | Reasons                                    |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------|
| In $\triangle ADB \leftrightarrow \triangle ADC$<br>$\overline{AB} \cong \overline{AC}$<br>$\overline{DB} \cong \overline{DC}$<br>$\overline{AD} \cong \overline{AD}$ | Given<br>Given<br>Common                   |
| $\therefore \triangle ADB \cong \triangle ADC$                                                                                                                        | S.S.S. $\cong$ S.S.S.                      |
| $\therefore \angle 1 \cong \angle 2$                                                                                                                                  | Corresponding angles of $\cong$ $\Delta$ s |
| In $\triangle ABE \leftrightarrow \triangle ACE$<br>$\overline{AB} \cong \overline{AC}$<br>$\angle 1 \cong \angle 2$<br>$\overline{AE} \cong \overline{AE}$           | Given<br>Proved<br>Common                  |
| $\therefore \triangle ABE \cong \triangle ACE$                                                                                                                        | S.A.S. postulate                           |
| $\therefore \overline{BE} \cong \overline{CE}$                                                                                                                        | Corresponding sides of $\cong$ $\Delta$ s  |
| $\angle 3 \cong \angle 4$ .....I                                                                                                                                      | Corresponding angles of $\cong$ $\Delta$ s |
| $m\angle 3 + m\angle 4 = 180^\circ$ .....II                                                                                                                           | Supplementary angles Postulate             |
| $\therefore m\angle 3 = m\angle 4 = 90^\circ$                                                                                                                         | From I and II                              |
| Hence $\overline{AE} \perp \overline{BC}$                                                                                                                             |                                            |

**Corollary: An equilateral triangle is an equiangular triangle.**