

Exercise 10.3

Q1. In the figure, $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$.

Prove that $\angle A \cong \angle C$, $\angle ABC \cong \angle ADC$.

Given

$$\overline{AB} \cong \overline{DC}$$

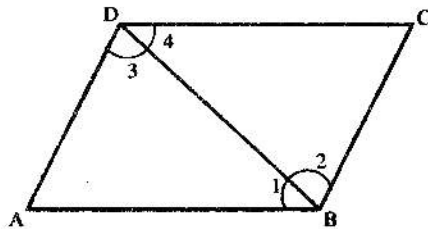
$$\overline{AD} \cong \overline{BC}$$

To prove

$$\angle A \cong \angle C$$

$$\angle ABC \cong \angle ADC$$

Proof



Statements		Reasons
In	$\triangle ABD \leftrightarrow \triangle CBD$	
	$\overline{AB} \cong \overline{DC}$	Given
	$\overline{AD} \cong \overline{BC}$	Given

$\overline{BD} \cong \overline{BD}$ $\therefore \triangle ABD \cong \triangle CBD$ $\angle A \cong \angle C$ $\angle 1 \cong \angle 4 \dots (i)$ $\angle 2 \cong \angle 3 \dots (ii)$ $\angle 1 + \angle 2 = \angle 3 + \angle 4$ $\angle ABC \cong \angle ADC$	Common S.S.S $\cong$ S.S.S Corresponding angles of congruent triangles Corresponding angles of congruent triangles Adding (i) and (ii)
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2. In the figure, $\overline{LN} \cong \overline{MP}$, $\overline{MN} \cong \overline{LP}$.
Prove that $\angle N \cong \angle P$, $\angle NML \cong \angle PLM$.

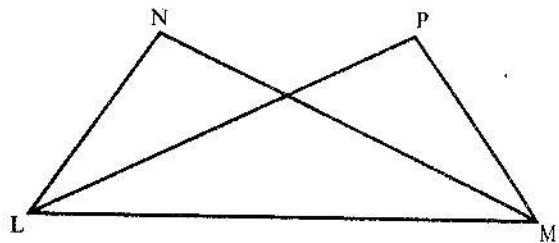
Given

$$\overline{LN} \cong \overline{MP}$$

$$\overline{LP} \cong \overline{MN}$$

To prove

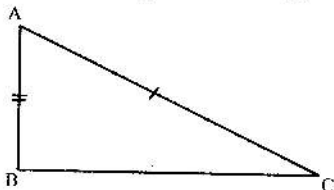
$$\angle N \cong \angle P, \quad \angle NML \cong \angle PLM$$



Statements	Reasons
In $\triangle LMN \leftrightarrow \triangle LPM$	
$\overline{LM} \cong \overline{MP}$	Given
$\overline{LP} \cong \overline{MN}$	Given
$\overline{LM} \cong \overline{LM}$	Common
$\triangle LMN \cong \triangle LPM$	S.S.S $\cong$ S.S.S
$\angle N \cong \angle P$	Corresponding angles of congruent triangles
$\angle NML \cong \angle PLM$	Corresponding angles of congruent triangles

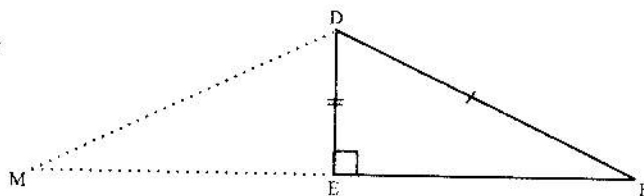
Theorem

If in the correspondence of the two right-angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other, then the triangles are congruent. (H.S $\cong$ H.S)



Given

$$\text{In } \triangle ABC \leftrightarrow \triangle DEF$$



$$\angle B \cong \angle E \text{ (right angles)}$$

$$\overline{CA} \cong \overline{FD}, \quad \overline{AB} \cong \overline{DE}$$

To Prove $\triangle ABC \cong \triangle DEF$

Construction

Produce $\overline{FE}$ to a point M such that $\overline{EM} \cong \overline{BC}$ and join the points D and M.

Proof

Statements	Reasons
In $m\angle DEF + m\angle DEM = 180^\circ \dots (i)$	(Supplementary angles)
Now $m\angle DEF = 90^\circ \dots (ii)$	(Given)
$\therefore m\angle DEM = 90^\circ$	{from (i) and (ii)}
In $\triangle ABC \leftrightarrow \triangle DEM$	
$\overline{BC} \cong \overline{EM}$	(construction)
$\angle ABC \cong \angle DEM$	(each $\angle$ equal to 90°)
$\overline{AB} \cong \overline{DE}$	(given)
$\therefore \triangle ABC \cong \triangle DEM$	S.A.S. postulate
And $\angle C \cong \angle M$	(Corresponding angles of congruent triangles)
$\overline{CA} \cong \overline{MD}$	(Corresponding sides of congruent triangles)
But $\overline{CA} \cong \overline{FD}$	(given)
$\therefore \overline{MD} \cong \overline{FD}$	
In $\triangle DMF$	Each is congruent to $\overline{CA}$
$\angle F \cong \angle M$	$\overline{FD} \cong \overline{MD}$ (Proved)
But $\angle C \cong \angle M$	(proved)
$\angle C \cong \angle F$	(each is congruent to $\angle M$)
In $\triangle ABC \leftrightarrow \triangle DEF$	
$\overline{AB} \cong \overline{DE}$	(given)
$\angle ABC \cong \angle DEF$	(given)
$\angle C \cong \angle F$	(proved)
Hence $\triangle ABC \cong \triangle DEF$	(S.A.A $\cong$ S.A.A)

Example

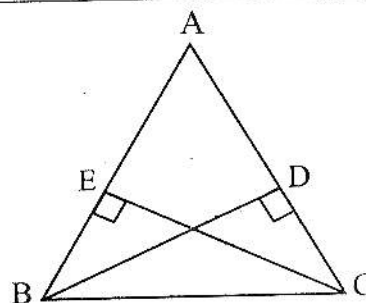
If perpendiculars from two vertices of a triangle to the opposite sides are congruent, then the triangle is isosceles.

Given

In $\triangle ABC$, $\overline{BD} \perp \overline{AC}$, $\overline{CE} \perp \overline{AB}$

Such that $\overline{BD} \cong \overline{CE}$

To Prove $\overline{AB} \cong \overline{AC}$



Proof

Statements	Reasons
<p>In $\triangle BCD \leftrightarrow \triangle CBE$ $\angle BDC \cong \angle BEC$</p> <p>$\overline{BC} \cong \overline{BC}$ $\overline{BD} \cong \overline{CE}$ $\triangle BCD \cong \triangle CBE$ $\angle BCD \cong \angle CBE$</p> <p>Thus $\angle BCA \cong \angle CBA$</p> <p>Hence $\overline{AB} \cong \overline{AC}$</p>	<p>$\overline{BD} \perp \overline{AC}, \overline{CE} \perp \overline{AB}$ (given) $\Rightarrow$ each angle = 90° Common hypotenuse Given H.S. $\cong$ H.S. Corresponding angles of $\cong \Delta$s.</p> <p>In $\triangle ABC, \angle BCA \cong \angle CBA$</p>