

Exercise 10.4

1. In $\triangle PAB$ of figure, $\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$, prove that $\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$.

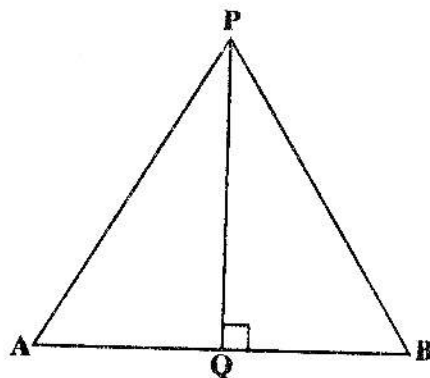
Given

In $\triangle PAB$, $\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$

To Prove

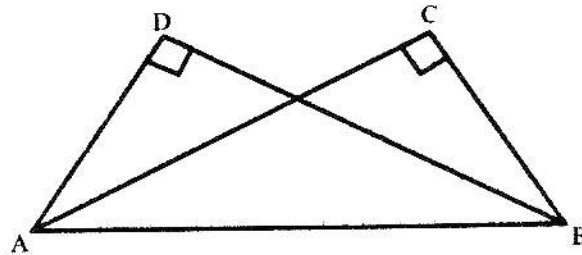
$\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$

Proof



Statements	Reasons
<p>In $\triangle APQ \leftrightarrow \triangle BPQ$</p> <p>$\overline{PA} \cong \overline{PB}$</p> <p>$\overline{PQ} \cong \overline{PQ}$</p> <p>$\therefore \triangle PAQ \cong \triangle PBQ$</p> <p>$\therefore \overline{AQ} \cong \overline{BQ}$</p> <p>$\angle APQ \cong \angle BPQ$</p>	<p>Given</p> <p>Common</p> <p>H.S $\cong$ H.S</p> <p>Corresponding sides of congruent triangles</p> <p>Corresponding angles of the congruent triangles.</p>

2. In the figure, $m\angle C = m\angle D = 90^\circ$ and $\overline{BC} \cong \overline{AD}$. Prove that $\overline{AC} \cong \overline{BD}$ and $\angle BAC \cong \angle ABD$.



Given

$$m\angle C = m\angle D = 90^\circ$$

$$\overline{BC} \cong \overline{AD}$$

To Prove

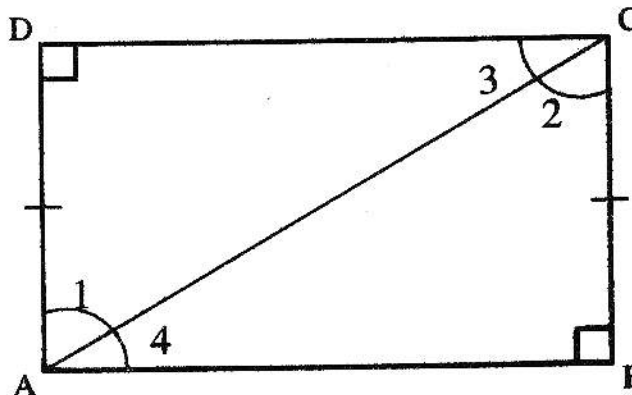
$$\overline{AC} \cong \overline{BD}$$

$$\angle BAC \cong \angle ABD$$

Proof

Statements	Reasons
<p>In $\triangle ABC \leftrightarrow \triangle ABD$</p> <p>$m\angle C \cong m\angle D$</p> <p>$\overline{BC} \cong \overline{AD}$</p> <p>$\overline{AB} \cong \overline{AB}$</p> <p>$\therefore \triangle ABC \cong \triangle ABD$</p> <p>$\overline{AC} \cong \overline{BD}$</p> <p>$\therefore \angle BAC \cong \angle ABD$</p>	<p>Each of 90°</p> <p>Given</p> <p>Common</p> <p>H.S $\cong$ H.S</p> <p>Corresponding sides of congruent triangles</p> <p>Corresponding angles of the congruent triangles</p>

3. In the figure, $m\angle B = m\angle D = 90^\circ$ and $\overline{AD} \cong \overline{BC}$. Prove that ABCD is a rectangle.



Given

$$m\angle B = m\angle D = 90^\circ, \overline{AD} \cong \overline{BC}$$

Proof

ABCD is a rectangle

Statements	Reasons
In $\triangle ABC \leftrightarrow \triangle ADC$	
$m\angle B \cong m\angle D$	Each of 90°
$\overline{AD} \cong \overline{BC}$	Given
$\overline{AC} \cong \overline{AC}$	Common
$\therefore \triangle ABC \cong \triangle ADC$	H.S $\cong$ H.S
$\overline{AB} \cong \overline{DC}$	
$\angle 1 \cong \angle 2 \quad \dots(i)$	
$\angle 4 \cong \angle 3 \quad \dots(ii)$	
$\angle 1 + \angle 4 = \angle 2 + m\angle 3$	
$\angle A = \angle C = 90^\circ$	
ABCD is a rectangle	By (i) and (ii)

4. Which of the following are true and which are false?

- (i) A ray has two end points.
- (ii) In a triangle, there can be only one right angle.
- (iii) Three points are said to be collinear if they lie on same line.
- (iv) Two parallel lines intersect at a point.
- (v) Two lines can intersect only in one point.
- (vi) A triangle of congruent sides has non-congruent angles.

Answers

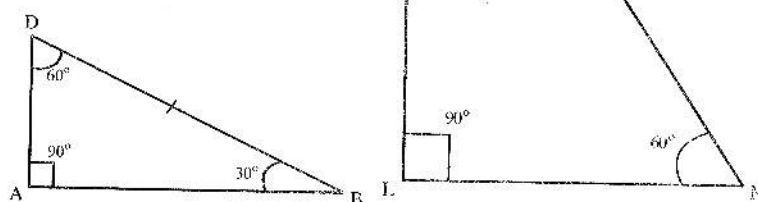
- (i) False (ii) True (iii) True
- (iv) False (v) True (vi) False

5. If $\triangle ABC \cong \triangle LMN$, then

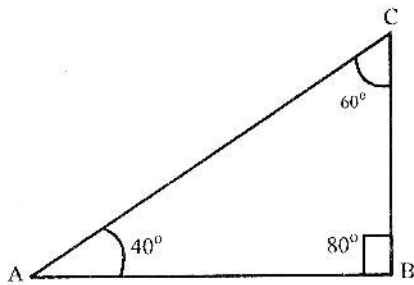
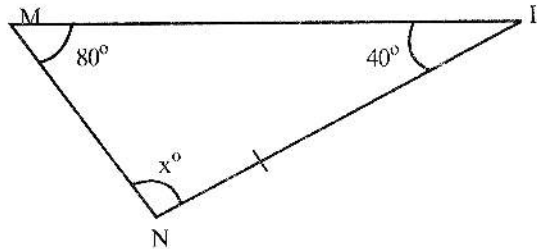
- (i) $m\angle M \cong \dots\dots\dots$
- (ii) $m\angle N \cong \dots\dots\dots$
- (iii) $m\angle A \cong \dots\dots\dots$

Answers

- (i) $m\angle M \cong m\angle B$
- (ii) $m\angle N \cong m\angle C$
- (iii) $m\angle A \cong m\angle L$



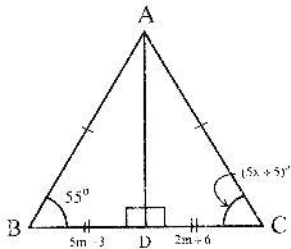
6. If $\triangle ABC \cong \triangle LMN$, then find the unknown x .



Answers

$$x = 60^\circ$$

7. Find the value of unknowns for the given congruent triangles.



$$\triangle ABD \cong \triangle ACD$$

$$\overline{BD} \cong \overline{DC}$$

$$\Rightarrow 5m - 3 = 2m + 6$$

$$5m - 2m = 3 + 6$$

$$3m = 9$$

$$m = \frac{9}{3} = 3$$

Also

$$\angle ACD \cong \angle ABD \Rightarrow$$

Angles opposite to congruent sides are congruent

$$5x + 5 = 55$$

$$5x = 55 - 5$$

$$5x = 50$$

$$x = \frac{50}{5}$$

$$x = 10$$

8. If $\triangle PQR \cong \triangle ABC$

, then find the unknowns.

$$\triangle PQR \cong \triangle ABC$$

$$\overline{PQ} \cong \overline{AB}$$

$$x = 3$$

$$\overline{BC} \cong \overline{QR}$$

$$\Rightarrow z = 4 \text{ cm}$$

$$\overline{AC} \cong \overline{PR}$$

$$y - 1 = 5$$

$$y = 5 + 1$$

$$y = 6 \text{ cm}$$

$$\therefore x = 3 \text{ cm}, y = 6 \text{ cm}, z = 4 \text{ cm}$$

