

EXERCISE 11.2

(1) Prove that a quadrilateral is a parallelogram if its

(a) Opposite angles are congruent.

(b) Diagonals bisect each other.

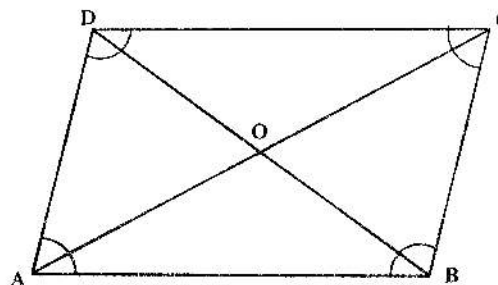
Given Given ABCD is a quadrilateral.

$$m\angle A = m\angle C,$$

$$m\angle B = m\angle D$$

To prove ABCD is a parallelogram.

Proof



Statements	Reasons
$m\angle A = m\angle C$ (i)	Given
$m\angle B = m\angle D$ (ii)	Given
Now	
$m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$	Angles of a quad.
$m\angle A + m\angle B + m\angle A + m\angle B = 360^\circ$	From (i), (ii)
$m\angle A + m\angle A + m\angle B + m\angle B = 360^\circ$	Rearranging
$2m\angle A + 2m\angle B = 360^\circ$	
$(m\angle A + m\angle B) = 360^\circ / 2 = 180^\circ$	Dividing by 2
$\therefore \overline{AD} \parallel \overline{BC}$	As $m\angle A + m\angle B = 180^\circ$ (sum of interior angles)
Similarly it can be	
Proved that $\overline{AB} \parallel \overline{CD}$	
Hence ABCD is a parallelogram.	

(2) prove that a quadrilateral is a parallelogram if its opposite sides are congruent.

Given

In quadrilateral

$$ABCD, \overline{AB} \cong \overline{DC},$$

$$\overline{AD} \cong \overline{BC}$$

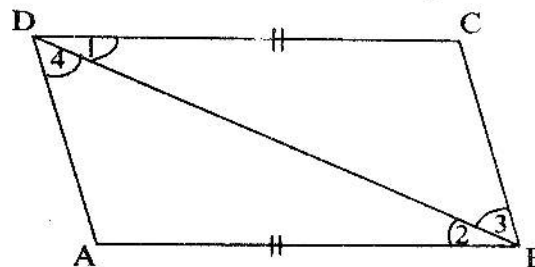
Required

ABCD is a || gm

$$\overline{AB} \parallel \overline{CD}, \overline{AD} \parallel \overline{BC}$$

Construction

Join point B to D and name the angles $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$



Proof

Statements	Reasons
$\triangle ABD \leftrightarrow \triangle CDB$	
$\overline{AD} \cong \overline{CB}$	Given
$\overline{AB} \cong \overline{CD}$	Given
$\overline{BD} \cong \overline{BD}$	Common
$\therefore \triangle ABD \cong \triangle CDB$	S.S.S $\cong$ S.S.S
So $\angle 2 \cong \angle 1$ (i)	Corresponding angles of Congruent triangles
$\angle 4 \cong \angle 3$ (ii)	Alternate angles
Hence $\overline{AB} \parallel \overline{CD}$ (iii)	$\angle 2$ and $\angle 1$ are congruent
Similarly $\overline{BC} \parallel \overline{AD}$ (iv)	Alternate angles $\angle 3, \angle 4$ congruent
$\therefore$ ABCD is a parallelogram.	From iii, iv

Theorem

The line segment, joining the mid-points of two sides of a triangle, is parallel to the third side and is equal to one half of its length.

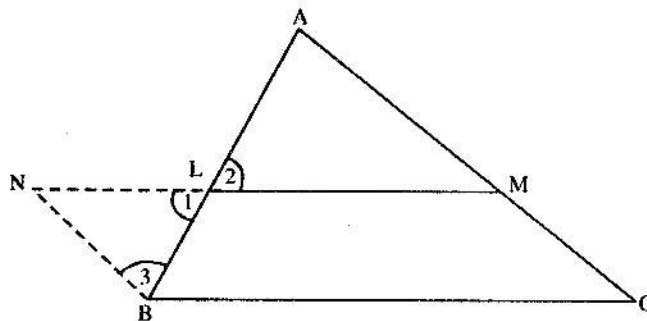
Given In $\triangle ABC$, the mid-points of $\overline{AB}$ and $\overline{AC}$ are L and M respectively.

To Prove

$$\overline{LM} \parallel \overline{BC} \text{ and } m\overline{LM} = \frac{1}{2} m\overline{BC}$$

Construction

Join M to L and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$. Join N to B. and in the figures name the angles $\angle 1, \angle 2, \angle 3$ and $\angle 4$ as shown.

**Proof**

Statements	Reasons
In $\triangle BLN \leftrightarrow \triangle ALM$	
$\overline{BL} \cong \overline{AL}$,	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{NL} \cong \overline{ML}$	Construction

$\therefore \triangle BNL \cong \triangle ALM$	S.A.S. postulate
$\angle A \cong \angle 3$(i)	(corresponding angles of congruent triangles)
and $\overline{NB} \cong \overline{AM}$(ii)	(corresponding sides of congruent triangles)
But $\overline{NB} \parallel \overline{AM}$	
Thus $\overline{NB} \parallel \overline{MC}$(iii)	From (i), alternate $\angle$ s
$\overline{MC} \cong \overline{AM}$(iv)	(M is a point of $\overline{AC}$)
$\overline{NB} \cong \overline{MC}$... (v)	Given
$\therefore$ BCMN is a parallelogram	{from (ii) and (iv)}
$\therefore \overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$	From (iii) and (v)
$\overline{BC} \cong \overline{NM}$(vi)	(Opposite sides of a parallelogram BCMN)
$m\overline{LM} = \frac{1}{2} m\overline{NM}$(vii)	(Opposite sides of parallelogram)
Hence $m\overline{LM} = \frac{1}{2} m\overline{BC}$	Construction
	{from (vi) and (vii)}

Example

The line segments, joining the mid-points of the sides of a quadrilateral, taken in order, form a parallelogram.

Given

A quadrilateral ABCD, in which P is the mid-point of $\overline{AB}$, Q is the mid-point of $\overline{BC}$, R is the mid-point of $\overline{CD}$, S is the mid-point of $\overline{DA}$.

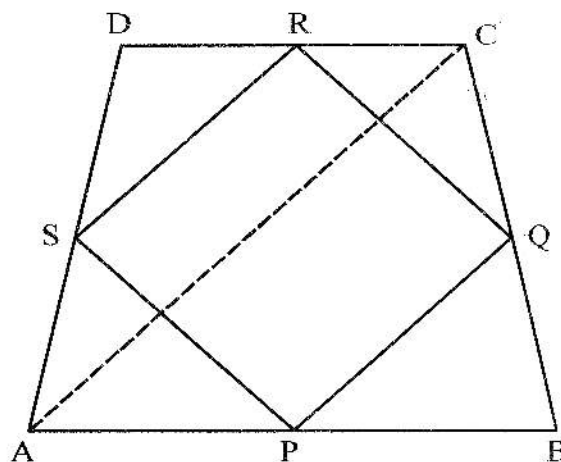
P is joined to Q, Q is joined to R. R is joined to S and S is joined to P.

To prove

PQRS is a parallelogram.

Construction

Join A to C.



Proof

Statements	Reasons
<p>In $\triangle DAC$,</p> $\left. \begin{array}{l} \overline{SR} \parallel \overline{AC} \\ m\overline{SR} = \frac{1}{2} m\overline{AC} \end{array} \right\}$	<p>S is the mid-point of $\overline{DA}$ R is the mid-point of $\overline{CD}$</p>
<p>In $\triangle BAC$,</p> $\left. \begin{array}{l} \overline{PQ} \parallel \overline{AC} \\ m\overline{PQ} = \frac{1}{2} m\overline{AC} \end{array} \right\}$ <p>$\overline{SR} \parallel \overline{PQ}$</p> <p>$m\overline{SR} = m\overline{PQ}$</p> <p>Thus PQRS is a parallelogram</p>	<p>P is the mid-point of $\overline{AB}$ Q is the mid-point of $\overline{BC}$</p> <p>Each $\parallel \overline{AC}$</p> <p>Each $= \frac{1}{2} m\overline{AC}$</p> <p>$\overline{SR} \parallel \overline{PQ}, m\overline{SR} = m\overline{PQ}$ (proved)</p>