

EXERCISE 11.3

- (1) Prove that the line-segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Given

ABCD is a quadrilateral.

P, Q, R, S are the mid-points of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$ respectively.

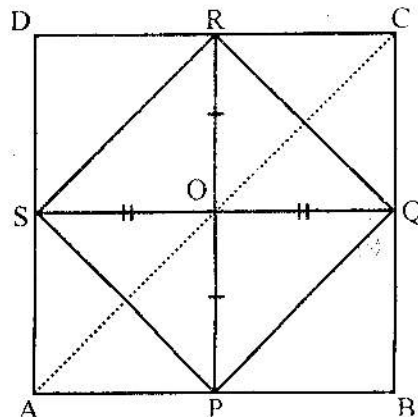
P is joined to R, Q is joined to S. $\overline{SQ}$, $\overline{PR}$ intersect at point "O"

To Prove

$$\overline{OP} \cong \overline{OR}, \overline{OS} \cong \overline{OQ}$$

Construction Join P, Q, R, S in order, join A to C.

Proof



Statements		Reasons
$\overline{SR} \parallel \overline{AC}$	(i)	In $\triangle ADC$, S, R are mid-points of $\overline{AD}$, $\overline{DC}$.
$m\overline{SR} = \frac{1}{2} m\overline{AC}$	(ii)	

And $\overline{PQ} \parallel \overline{AC}$ (iii)	In $\triangle ABC$; P, Q are mid-points of $\overline{AB}, \overline{BC}$
$m\overline{PQ} = \frac{1}{2} m\overline{AC}$ (iv)	
$\therefore \overline{PQ} \parallel \overline{SR}$ (v)	from (i), and (iii)
$m\overline{PQ} = m\overline{SR}$ (vi)	From (ii) and (iv)
Similarly $\overline{PS} \parallel \overline{QR}$	
$m\overline{PS} = m\overline{QR}$	
Hence PQRS is a parallelogram	
Now $\overline{PR}, \overline{SQ}$ are the diagonals of PQRS that intersect at point O.	
$\therefore \overline{OP} \cong \overline{OR}$	
$\therefore \overline{OS} \cong \overline{OQ}$	
	Diagonals of a parallelogram Bisect each other.

(2) Prove that the line-segments joining the mid-points of the opposite sides of a rectangle are the right-bisectors of each other.

Given

ABCD is a rectangle.

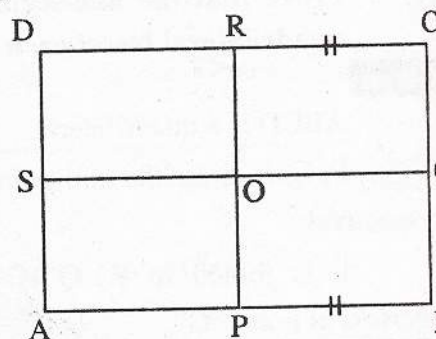
and P, Q, R, S are the mid-points of sides $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$, respectively.

P is joined to R, S to Q These intersect at "O"

To Prove

$\overline{OQ} \cong \overline{OS}, \overline{OR} \cong \overline{OP}$ and $\overline{RP} \perp \overline{SQ}$

Proof



Statements	Reasons
$\overline{AB} \parallel \overline{CD}$	opposite sides of rectangle
$\overline{AP} = \overline{DR}$ (i)	
$m\overline{AB} = m\overline{CD}$	
$\frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$	
$m\overline{AP} = m\overline{DR}$ (ii)	
$\therefore$ APRD is rectangle	

$$\therefore \overline{OR} \cong \overline{OP}$$

Similarly $\overline{OQ} \cong \overline{OS}$

Now In rectangle APRD

$$m\overline{DA} = m\overline{RP}$$

$$\frac{1}{2}m\overline{DA} = m\overline{RP}$$

$$m\overline{DS} = m\overline{RO}$$

$$\therefore \overline{DS} \parallel \overline{RO},$$

Hence SORD is rectangle.

$$\therefore m\angle SOR = 90^\circ, \overline{RP} \perp \overline{SQ}.$$

$$\text{As } m\angle A = m\angle D = 90^\circ$$

[Note: Diagonals of a rectangle are congruent.]

Q) Prove that the line-segment passing through the mid-point of one side and parallel to another side of a triangle also bisects the third side.

Given In $\triangle ABC$, D is mid-point

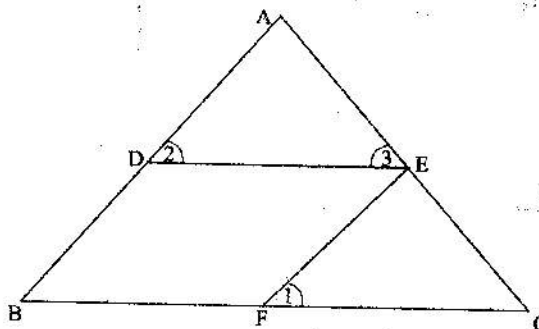
of $\overline{AB}$, $\overline{DE} \parallel \overline{BC}$ which meets $\overline{AC}$ at E.

To Prove E is mid-point of

$\overline{AC}$ and $\overline{EA} \cong \overline{EC}$

Construction

Take $\overline{EF} \parallel \overline{AB}$ which meets $\overline{BC}$ at F.



Statements	Reasons
Now BDEF is parallelogram	$\overline{DE} \parallel \overline{BF}$ given, $\overline{EF} \parallel \overline{DB}$ const.
$\therefore \overline{EF} \cong \overline{DB}$ (i)	Opposite sides of parallelogram
$\overline{EF} \cong \overline{AD}$ (ii)	Given
$\angle 1 \cong \angle B$	Corresponding angles.
$\angle 2 \cong \angle B$ (iii)	Corresponding angles.
$\therefore \angle 1 \cong \angle 2$ (iv)	Form (iii)
Now In $\triangle ADE \leftrightarrow \triangle EFC$	
$\angle 1 \cong \angle 2$	Form (iv)
$\angle 3 \cong \angle C$	Corresponding angles.
$\overline{AD} \cong \overline{EF}$	Form (ii)
Hence $\triangle ADE \cong \triangle EFC$	A.A.S $\cong$ A.A.S

$\therefore \overline{AE} \cong \overline{CE}$	Corresponding sides of congruent triangles.
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Theorem

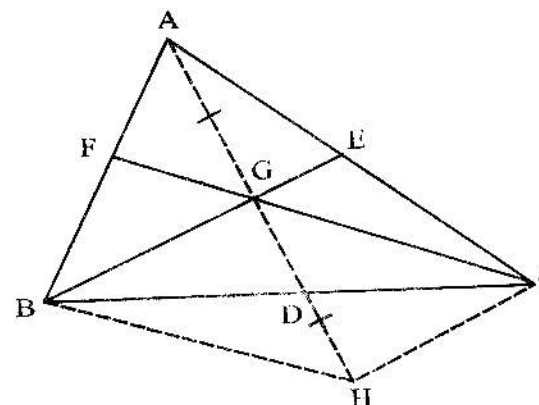
The medians of a triangle are concurrent and their point of concurrency is the point of trisection of each median.

Given

$\triangle ABC$

To Prove

The medians of the $\triangle ABC$ are concurrent and the point of concurrency is the point of trisection of each median.



Construction

Draw two medians $\overline{BE}$ and $\overline{CF}$ of the $\triangle ABC$ which intersect each other at point G. Join A to G and produce it to point H such that $\overline{AG} \cong \overline{GH}$. Join H to the points B and C. $\overline{AH}$ intersects $\overline{BC}$ at the point D.

Proof

Statements	Reasons
In $\triangle ACH$, $\overline{GE} \parallel \overline{HC}$,	G and E are mid-points of sides $\overline{AH}$ and $\overline{AC}$ respectively
or $\overline{BE} \parallel \overline{HC}$(i)	$\overline{AC}$ respectively G is a point of $\overline{BE}$
Similarly $\overline{CF} \parallel \overline{HB}$(ii)	
$\therefore$ BHCG is a parallelogram	from (i) and (ii)
and $m\overline{GD} = \frac{1}{2} m\overline{GH}$(iii)	(Diagonals $\overline{BC}$ and $\overline{GH}$ of a parallelogram BHCG intersect each other at point D).
$\overline{BD} \cong \overline{CD}$	
$\overline{AD}$ is a median of $\triangle ABC$	
Medians $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ pass through the point G	(G is the intersecting point of $\overline{BE}$ and $\overline{CF}$ and $\overline{AD}$ pass through it.)
Now $\overline{GH} \cong \overline{AG}$(iv)	Construction

$$\therefore \quad m\overline{GD} = \frac{1}{2}m\overline{AG}$$

and G is the point of trisection of $\overline{AD}$ —(v)
 similarly it can be proved that G is also
 the point of trisection of $\overline{CF}$ and $\overline{BE}$.

from (iii) and (iv)