

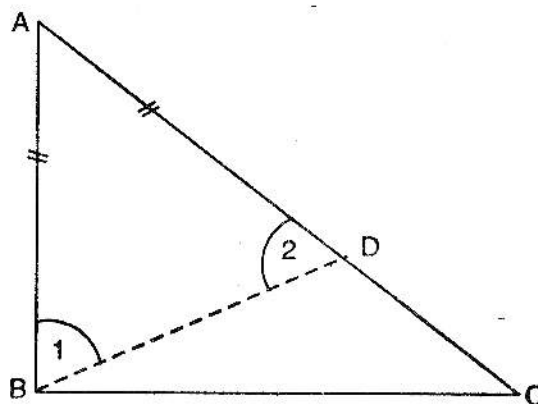
SIDES AND ANGLES OF A TRIANGLE

Theorem If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it.

Given In $\triangle ABC$, $\overline{AC} > \overline{AB}$

To Prove $m\angle ABC > m\angle ACB$

Construction On $\overline{AC}$ take a point D such that $\overline{AD} \cong \overline{AB}$. Join B to D so that $\triangle ADB$ is an isosceles triangle. Label $\angle 1$ and $\angle 2$ as shown in the given figure.

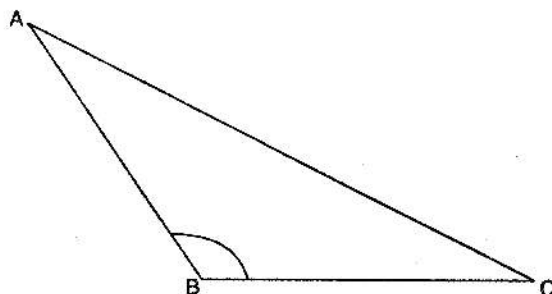


Proof

Statements	Reasons
In $\triangle ABD$	
$m\angle 1 = m\angle 2$... (i)	Angle opposite to congruent sides, (construction)
In $\triangle BCD$, $m\angle ACB < m\angle 2$	
i.e., $m\angle 2 > m\angle ACB$... (ii)	(An exterior angle of a triangle is greater than a non-adjacent interior angle).
$\therefore m\angle 1 > m\angle ACB$ (iii)	By (i) and (ii)
But	
$m\angle ABC = m\angle 1 + m\angle DBC$	Postulate of addition of angles.
$\therefore m\angle ABC > m\angle 1$ (iv)	
$\therefore m\angle ABC > m\angle 1 > m\angle ACB$	
Hence $m\angle ABC > m\angle ACB$	By (iii) and (iv) (Transitive property of inequality of real number)

Example Prove that in a scalene triangle, the angle opposite to the largest side is of measure greater than 60° . (i.e., two-third of a right-angle).

Given In $\triangle ABC$, $\overline{AC} > \overline{AB}$, $\overline{AC} > \overline{BC}$.



To Prove

$$m\angle B > 60^\circ.$$

Proof

Statements	Reasons
In $\triangle ABC$	
$m\angle B > m\angle C$	$\overline{mAC} > \overline{mAB}$ (given)
$m\angle B > m\angle A$	$\overline{mAC} > \overline{mBC}$ (given)
But $m\angle A + m\angle B + m\angle C = 180^\circ$	$\angle A, \angle B, \angle C$ are the angles of $\triangle ABC$
$\therefore m\angle B + m\angle B + m\angle B > 180^\circ$	$m\angle B > m\angle C, m\angle B > m\angle A$ (proved)
Hence $m\angle B > 60^\circ$	$180^\circ/3 = 60^\circ$.

Example In a quadrilateral ABCD, $\overline{AB}$ is the longest side and $\overline{CD}$ is the shortest side. Prove that $m\angle BCD > m\angle BAD$.

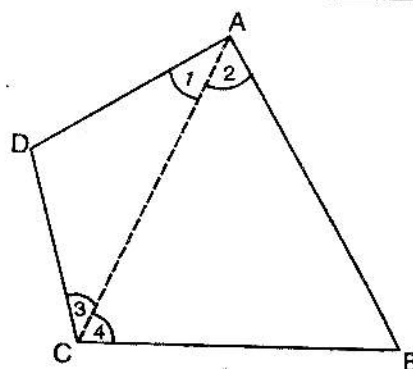
Given In quad. ABCD, $\overline{AB}$ is the longest side and $\overline{CD}$ is the shortest side.

To Prove $m\angle BCD > m\angle BAD$

Construction

Join A to C.

Name the angles $\angle 1, \angle 2, \angle 3$ and $\angle 4$ as shown in the figure.

**Proof**

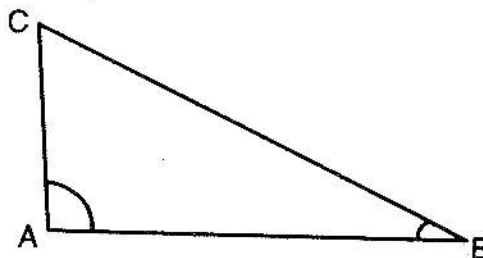
Statements	Reasons
In $\triangle ABC, m\angle 4 > \angle 2$(i)	$\overline{mAB} > \overline{mBC}$ (given)
In $\triangle ACD, m\angle 3 > m\angle 1$(ii)	$\overline{mAD} > \overline{mCD}$ (given)
$\therefore m\angle 4 + m\angle 3 > m\angle 2 + m\angle 1$	From I and II
Hence $m\angle BCD > m\angle BAD$	$\begin{cases} m\angle 4 + m\angle 3 = m\angle BCD \\ m\angle 2 + m\angle 1 = m\angle BAD \end{cases}$

Theorem:

If two angles of a triangle are unequal in measure, the side opposite to the greater angle is longer than the side opposite to the smaller angle.

Given In $\triangle ABC, m\angle A > m\angle B$

To Prove $\overline{mBC} > \overline{mAC}$



Proof

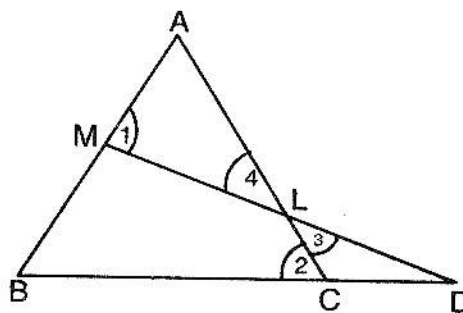
Statements	Reasons
If $m\overline{BC} > m\overline{AC}$, then either (i) $m\overline{BC} = m\overline{AC}$ or (ii) $m\overline{BC} < m\overline{AC}$	(Trichotomy property of real numbers)
From (i) if $m\overline{BC} = m\overline{AC}$, then $m\angle A = m\angle B$ which is not possible	(Angles opposite to congruent sides are congruent) Contrary to the given
From (ii) if $m\overline{BC} < m\overline{AC}$, then $m\angle A < m\angle B$ This is also not possible.	(The angle opposite to longer side is greater than angle opposite to smaller side) Contrary to the given
$\therefore m\overline{BC} \neq m\overline{AC}$ And $m\overline{BC} < m\overline{AC}$ Thus $m\overline{BC} > m\overline{AC}$	Trichotomy property of real numbers.

Note

- (i) The hypotenuse of a right angle triangle is longer than each of the other two sides.
- (ii) In an obtuse angled triangle, the side opposite to the obtuse angle is longer than each of the other two sides.

Example

ABC is an isosceles triangle with base $\overline{BC}$. On $\overline{BC}$ a point D is taken away from C. A line segment through D cuts $\overline{AC}$ at L and $\overline{AB}$ at M. Prove that $m\overline{AL} > m\overline{AM}$.

**Given**

In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$.

D is a point on $\overline{BC}$ away from C.

A line segment through D cuts $\overline{AC}$ at L and $\overline{AB}$ at M.

To Prove

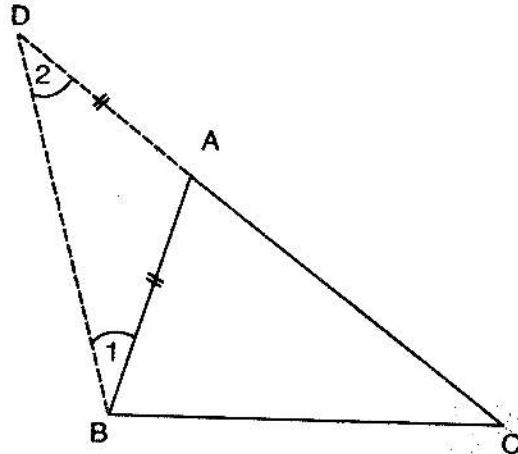
$m\overline{AL} > m\overline{AM}$

Proof

Statements			Reasons
In $\triangle ABC$			
$\angle B \cong \angle 2$	...I		$\overline{AB} \cong \overline{AC}$ (given)
In $\triangle MBD$			
$m\angle 1 > m\angle B$	...II		($\angle 1$ is an ext. $\angle$ and $\angle B$ is its internal opposite $\angle$)
$\therefore m\angle 1 > m\angle 2$	...III		From I and II
In $\triangle LCD$,			
$m\angle 2 > m\angle 3$	IV		($\angle 2$ is an ext. $\angle$ and $\angle 3$ is its internal opposite $\angle$)
$\therefore m\angle 1 > m\angle 3$	...V		From III and IV
But $\angle 3 \cong \angle 4$	...VI		Vertical angles
$\therefore m\angle 1 > m\angle 4$			From V and VI
Hence $m\overline{AL} > m\overline{AM}$			In $\triangle ALM$, $m\angle 1 > m\angle 4$ (proved)

Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

**To Prove**

- (i) $m\overline{AB} + m\overline{AC} > m\overline{BC}$
- (ii) $m\overline{AB} + m\overline{BC} > m\overline{AC}$
- (iii) $m\overline{BC} + m\overline{CA} > m\overline{AB}$

Construction

Take a point D on $\overline{CA}$ such that $\overline{AD} \cong \overline{AB}$. Join B to D and name the angles. $\angle 1$, $\angle 2$ as shown in the given figure.

Given

$\triangle ABC$

Proof

Statements			Reasons
In $\triangle ABD$,			
$\angle 1 \cong \angle 2$	...(i)		$\overline{AD} \cong \overline{AB}$ (construction)

$m\angle DBC > m\angle 1$(ii) $\therefore m\angle DBC > m\angle 2$(iii) In $\triangle DBC$, $m\overline{CD} > m\overline{BC}$ i.e., $m\overline{AD} + m\overline{AC} > m\overline{BC}$ Hence $m\overline{AB} + m\overline{AC} > m\overline{BC}$ Similarly, $m\overline{AB} + m\overline{BC} > m\overline{AC}$ And $m\overline{BC} + m\overline{CA} > m\overline{AB}$	$m\angle DBC = m\angle 1 + m\angle ABC$ From (i) and (ii) By (iii) $m\overline{CD} = m\overline{AD} + m\overline{AC}$ $m\overline{AD} = m\overline{AB}$ (construction)
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Example

Which of the following sets of lengths can be the lengths of the sides of a triangle.

(a) 2cm, 3cm, 5cm

(b) 3cm, 4cm, 5 cm

(c) 2cm, 4cm, 7cm

(a) $\therefore 2+3 = 5$

$\therefore$ This set of lengths cannot be those of the sides of a triangle.

(b) $\therefore 3+4 > 5, 3+5 > 4, 4+5 > 3$

$\therefore$ This set can form a triangle.

(c) $\therefore 2+4 < 7$

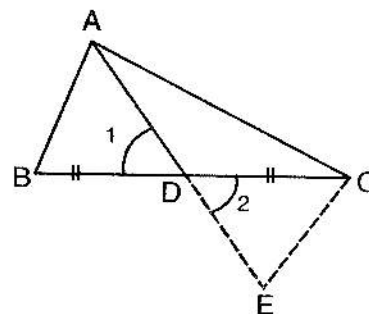
$\therefore$ This set of lengths cannot be the sides of a triangle.

Example Prove that the sum of the measures of two sides of a triangle is

Proof

Statements	Reasons
In $\triangle ABD \leftrightarrow \triangle ECD$	
$\overline{BD} \cong \overline{CD}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{AD} \cong \overline{ED}$	Construction
$\triangle ABD \cong \triangle ECD$	

greater than twice the measure of the median which bisects the third side.



Given

In $\triangle ABC$,

median AD bisects side $\overline{BC}$ at D.

To Prove

$m\overline{AB} + m\overline{AC} > 2m\overline{AD}$.

Construction On $\overline{AD}$, take a point E, such that $\overline{DE} \cong \overline{AD}$. Join C to E. Name the angles $\angle 1, \angle 2$ as shown in the figure.

$\overline{AB} \cong \overline{EC}$	I	S.A.S. Postulate
$m\overline{AC} + m\overline{EC} > m\overline{AE}$	II	Corresponding sides of $\cong \Delta$ s
$m\overline{AC} + m\overline{AB} > m\overline{AE}$		ACE is a triangle
Hence $m\overline{AC} + m\overline{AB} > 2m\overline{AD}$		From I and II
		$m\overline{AE} = 2m\overline{AD}$ (construction)

Example

Prove that the difference of measures of two sides of a triangle is less than the measure of the third side.

Given

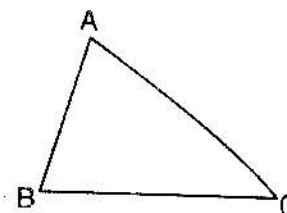
ΔABC

To Prove

$$m\overline{AC} - m\overline{AB} < m\overline{BC}$$

$$m\overline{BC} - m\overline{AB} < m\overline{AC}$$

$$m\overline{BC} - m\overline{AC} < m\overline{AB}$$



Proof

Statements	Reasons
$m\overline{AB} + m\overline{BC} > m\overline{AC}$	ABC is a triangle
$(m\overline{AB} + m\overline{BC} - m\overline{AB}) > (m\overline{AC} - m\overline{AB})$	Subtracting $m\overline{AB}$ from both sides.
$\therefore m\overline{BC} > (m\overline{AC} - m\overline{AB})$	
Or $m\overline{AC} - m\overline{AB} < m\overline{BC}$I	$a > b \Rightarrow b < a$
Similarly	
$\left. \begin{array}{l} m\overline{BC} - m\overline{AB} < m\overline{AC} \\ m\overline{BC} - m\overline{AC} < m\overline{AB} \end{array} \right\}$	Reason similar to I

Exercise 13.1

1. Two sides of a triangle measure 10 cm and 15 cm. Which of the following measure is possible for the third side?

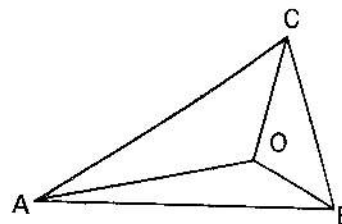
- (a) 5 cm (b) 20 cm
(c) 25 cm (d) 30 cm

Ans. 20cm.

2. O is an interior point of the ΔABC . Show that

$$m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2}(m\overline{AB} + m\overline{BC} + m\overline{CA})$$

Given: O is the interior point of ΔABC



To Prove:

$$m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2}(m\overline{AB} + m\overline{BC} + m\overline{CA})$$

Construction:

Join O with A, B and C.

Proof:

Statements	Reasons
$\triangle OAB$	
$m\overline{OA} + m\overline{OB} > m\overline{AB}$(i)	Sum of two sides > third side
Similarly	
$m\overline{OB} + m\overline{OC} > m\overline{BC}$(ii)	Sum of two sides > third side
and	
$m\overline{OC} + m\overline{OA} > m\overline{CA}$(iii)	
$2m\overline{OA} + 2m\overline{OB} + 2m\overline{OC} > m\overline{AB} + m\overline{BC} + m\overline{CA}$	Adding (i), (ii) and (iii)
$2(m\overline{OA} + m\overline{OB} + m\overline{OC}) > m\overline{AB} + m\overline{BC} + m\overline{CA}$	
$m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2}(m\overline{AB} + m\overline{BC} + m\overline{CA})$	

3. In the $\triangle ABC$, $m\angle B = 75^\circ$ and $m\angle C = 55^\circ$. Which of the sides of the triangle is longest and which is the shortest?

Ans: Given a $\triangle ABC$ in which

$$m\angle B = 75^\circ$$

$$m\angle C = 55^\circ$$

As $m\angle A + m\angle B + m\angle C = 180^\circ$

$$m\angle A + 75^\circ + 55^\circ = 180^\circ$$

$$m\angle A + 130^\circ = 180^\circ$$

$$m\angle A = 180^\circ - 130^\circ$$

$$m\angle A = 50^\circ$$

As we know in a triangle, the side opposite to greater angle is longer than the side opposite to smaller angle

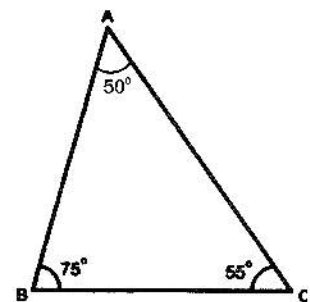
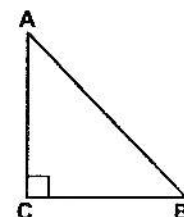
So $m\overline{AC} > m\overline{BC}$

Hence longest side is $\overline{AC}$

and shortest side is $\overline{BC}$

4. Prove that in a right-angled triangle, the hypotenuse is longer than each of the other two sides.

Ans.



Given: $\triangle ABC$ is a right angle triangle.

Hence AB is hypotenuse of $\triangle ABC$.

To prove:

$m\angle A > m\angle C$ and $m\angle A > m\angle B$

Proof:

As $\triangle ABC$ is a right angle triangle.

So $m\angle C = 90^\circ$ is the largest angle and the remaining angles $\angle A$ and $\angle B$ are acute.

So $m\angle C > m\angle A$ and $m\angle C > m\angle B$

As the side opposite to the greater angle is longer than the side opposite to the smaller angle.

Hence $m\angle A > m\angle C$ and $m\angle A > m\angle B$

Proof

Statements	Reasons
$\therefore$ in $\triangle ABC$ $\angle ACB > \angle ABC$ $\frac{1}{2} \angle ACB > \frac{1}{2} \angle ABC$	$\therefore \overline{AB} > \overline{AC}$
$\therefore \angle BCD > \angle CBD$ $\overline{BD} > \overline{CD}$	$\overline{CD}, \overline{BD}$ are bisectors of $\angle C, \angle B$. The bigger sides is opposite the bigger angle

Theorem From a point, outside a line, perpendicular is the shortest distance from the point to the line.

Given A line AB and a point C (not lying on $\overline{AB}$) and a point D on $\overline{AB}$ such that

$\overline{CD} \perp \overline{AB}$.

To Prove

$m\angle CDB$ is the shortest distance from the point C to $\overline{AB}$.

Construction

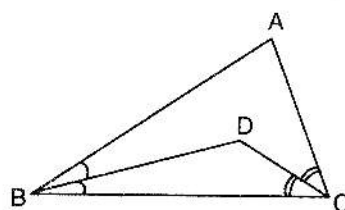
Take a point E on $\overline{AB}$. Join C and E to form a $\triangle CDE$

Proof:

Statements	Reasons
In $\triangle CDE$ $m\angle CDB > m\angle CED$	(An exterior angle of a triangle is greater

5. In the triangular figure, $\overline{AB} > \overline{AC}$.

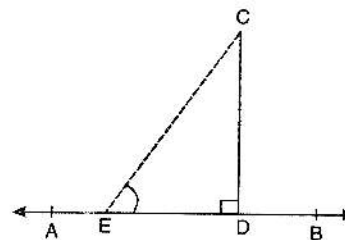
$\overline{BD}$ and $\overline{CD}$ are the bisectors of $\angle B$ and $\angle C$ respectively. Prove that $\overline{BC} > \overline{DC}$.



Given: $\overline{AB} > \overline{BC}$, $\overline{BD}$ and $\overline{CD}$ are the bisectors of the angles B and C

To Prove:

To prove = $\overline{BD} > \overline{CD}$



<p>But $m\angle CDB = m\angle CDE$ $\therefore m\angle CDE > m\angle CED$ or $m\angle CED < m\angle CDE$ or $m\overline{CD} < m\overline{CE}$ But E is any point on AB Hence $m\overline{CD}$ is the shortest distance from C to $\overline{AB}$.</p>	<p>than non adjacent interior angle). Supplement of right angle. $a > b \Rightarrow b < a$ Side opposite to greater angle is greater.</p>
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Note:

- (i) The distance between a line and a point not on it, is the length of the perpendicular line segment from the point to the line.
- (ii) The distance between a line and a point lying on it is zero