

REAL AND COMPLEX NUMBERS

Define the following:

Natural Numbers

The numbers 1, 2, 3, Which we use for counting certain objects are called natural numbers or positive integers. The set of natural numbers is denoted by N.

$$\text{i.e. } N = \{1, 2, 3, \dots\}$$

Whole Numbers

If we include 0 in the set of natural number, the resulting set is the set of whole numbers, denoted by W,

$$\text{i.e. } W = \{0, 1, 2, 3, \dots\}$$

Integers

The set of integers consist of positive integers, 0 and negative integers and is denoted by Z

$$\text{i.e. } Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

Rational Numbers

All numbers of the form p/q where p, q are integers and q is not zero are called rational numbers. The set of rational numbers is denoted by Q,

$$\text{i.e. } Q = \left\{ \frac{p}{q} \mid p, q \in Z \wedge q \neq 0, (p, q) = 1 \right\} \text{ or}$$

$$Q = \left\{ x \mid x = \frac{p}{q}, p, q \in Z \wedge q \neq 0 \right\}$$

Irrational Numbers

The numbers which cannot be expressed as quotient of integers are called irrational numbers.

The set of irrational numbers is denoted by Q' ,

$$\text{i.e., } Q' = \left\{ x \mid x \neq \frac{p}{q}, p, q \in Z \wedge q \neq 0 \right\}$$

For example, the numbers $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π and e are all irrational numbers.

Decimal form of Rational and Irrational number

a) Rational Numbers

The Decimal representation of rational numbers are of two types terminating and recurring

(i) Terminating Decimal Fractions:

The decimal fraction in which there are finite number of digits in its decimal part is called a terminating decimal fraction.

$$\text{For example } \frac{2}{5} = 0.4 \text{ and } \frac{3}{8} = 0.375.$$

(ii) Recurring and Non-terminating Decimal Fractions

The decimal fraction (non-terminating) in which some digits are repeated again and again in the same order in its decimal part is called a recurring decimal fraction.

$$\text{For example } \frac{2}{9} = 0.2222 \dots \text{ and } \frac{4}{11} = 0.363636 \dots$$

b) Irrational Numbers

The decimal representations for irrational numbers are neither terminating nor repeating in blocks. The decimal form

of an irrational number would continue forever and never begin to repeat the same block of digits e.g., $\sqrt{2} = 1.414213562 \dots$

Real Number

The Union of the set of rational numbers and irrational numbers is known as the set of real numbers it is denoted by R .

$$R = Q \cup Q'$$

Hence Q and Q' are both subsets of R and

$$Q \cap Q' = \phi$$

Example

Express the following decimals in the form $\frac{p}{q}$, where $p, q \in Z$ and $q \neq 0$

(a) $0.\bar{3} = 0.333 \dots$

(b) $0.\bar{23} = 0.232323 \dots$

Solution

(a) Let $x = 0.\bar{3}$, which can be rewritten as

$$x = 0.3333 \dots \quad (i)$$

Example

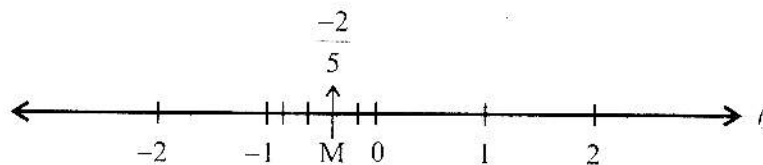
Represent the following numbers on the number line.

(i) $-\frac{2}{5}$ (ii) $\frac{15}{7}$

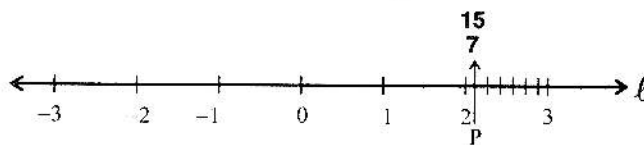
(iii) $-1\frac{7}{9}$

Solution

(i) For representing the rational number $-\frac{2}{5}$, on the number line ℓ , divide the unit length between 0 and -1 into five equal parts and take the end of the second part from 0 to its left side. The point M in the following figure represents the rational number $-\frac{2}{5}$.



(ii) $\frac{15}{7} = 2 + \frac{1}{7}$. It lies between 2 and 3.



We multiply both sides of (i) by 10, and obtain

$$10x = (0.3333 \dots) \times 10$$

$$\text{or } 10x = 3.3333 \dots \quad (ii)$$

Subtracting (i) from (ii), we have

$$10x - x = (3.3333 \dots) - (0.3333 \dots)$$

$$\text{or } 9x = 3.0000 \Rightarrow x = \frac{1}{3}$$

$$\text{Hence } 0.\bar{3} = \frac{1}{3}$$

(b) Let $x = 0.\bar{23} = 0.23232323 \dots$

We multiply both sides of (i) by 100.

$$\text{Then } 100x = (0.232323 \dots) \times 100$$

$$100x = 23.232323 \dots \quad (ii)$$

Subtracting (i) from (ii), we get

$$100x - x = (23.232323 \dots) - (0.232323 \dots)$$

$$99x = 23$$

$$x = \frac{23}{99}$$

$$\Rightarrow \text{Thus } 0.\bar{23} = \frac{23}{99} \text{ is a rational number.}$$

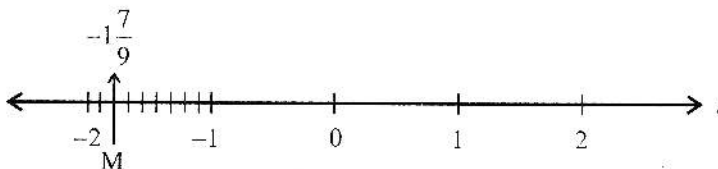
The point P represents the

point $\frac{15}{7} = 2\frac{1}{7}$.

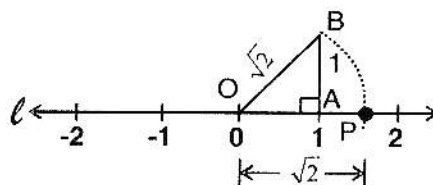
- (iii) For representing the rational number, $-1\frac{7}{9}$,

divide the unit length between -1 and -2 into

nine equal parts. Take the end of the 7th part from -1 . The point M in the following figure represents the rational number, $-1\frac{7}{9}$.



- (iv) Irrational number such as $\sqrt{2}$ can be located on the line l by geometric construction the point corresponding to $\sqrt{2}$ may be constructed by forming a right $\triangle AOB$ with sides each of length 1 as shown in the figure.



By Pythagoras theorem, $OB = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$

By drawing an arc with centre at O and radius $OB = \sqrt{2}$ we get point P representing $\sqrt{2}$ on the number line.

Exercise 2.1

Q1. Identify which of the following are relational and irrational numbers.

- (i) $\sqrt{3}$ Irrational Number
- (ii) $\frac{1}{6}$ Rational Number
- (iii) π Irrational Number
- (iv) $\frac{15}{2}$ Rational Number
- (v) 7.25 Rational Number
- (vi) $\sqrt{29}$ Irrational Number

Q2. Convert the following fractions into decimal fraction.

- (i) $\frac{17}{25}$

Sol: $\frac{17}{25} = 0.68$

(ii) $\frac{19}{4}$

Sol: $\frac{19}{4} = 4.75$

(iii) $\frac{57}{8}$

Sol: $\frac{57}{8} = 7.125$

(iv) $\frac{205}{18}$

Sol: $\frac{205}{18} = 11.3889$

(v) $\frac{5}{8}$

Sol: $\frac{5}{8} = 0.625$

(vi) $\frac{25}{38}$

Sol: $\frac{25}{38} = 0.65789$

Q2. Which of the following statements are true and which are false?

(i) $\frac{2}{3}$ is an irrational number. False

(ii) π is an irrational number. True

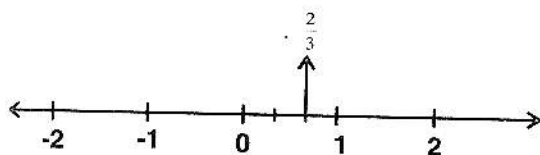
(iii) $\frac{1}{9}$ is a terminating fraction. False

(iv) $\frac{3}{4}$ is a terminating fraction. True

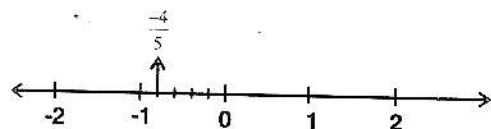
(v) $\frac{4}{5}$ is a recurring fraction. False

Q4. Represent the following numbers on the number line.

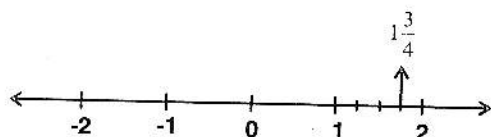
(i) $\frac{2}{3}$



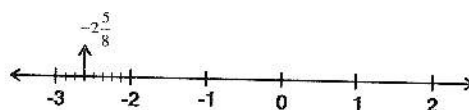
(ii) $-\frac{4}{5}$



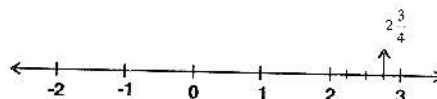
(iii) $1\frac{3}{4}$



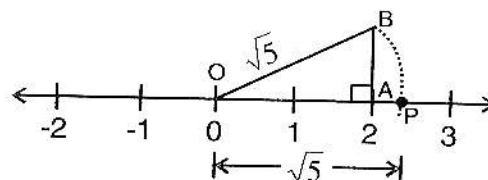
(iv) $-2\frac{5}{6}$



(v) $2\frac{3}{4}$



(vi) $\sqrt{5}$



By Pythagoras theorem

$$OB = \sqrt{(2)^2 + (1)^2} = \sqrt{4+1} = \sqrt{5}$$

By drawing an arc with centre at O and radius $OB = \sqrt{5}$ we get point P representing $\sqrt{5}$ on the number line.

Q5. Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$.

Ans. The required rational number is the mean of two given numbers, so the required number

$$\begin{aligned} & \frac{\frac{3}{4} + \frac{5}{9}}{2} \\ &= \frac{1}{2} \left(\frac{3}{4} + \frac{5}{9} \right) \\ &= \frac{1}{2} \left(\frac{27+20}{36} \right) \\ &= \frac{47}{72} \end{aligned}$$

Q6. Express the following recurring decimals as the rational number $\frac{p}{q}$,

where p, q are integers and $q \neq 0$

(i) $0.\overline{5}$

Sol: Let $x = 0.\overline{5}$

$$x = 0.55555\ldots \quad (i)$$

Multiplying both sides by 10

$$10x = 10(0.5555\ldots)$$

$$10x = 5.5555\ldots \quad (ii)$$

Subtracting (i) from (ii)

$$10x - x = (5.5555\ldots) - (0.5555\ldots)$$

$$9x = 5$$

$$x = \frac{5}{9}$$

$$\text{Hence } 0.\overline{5} = \frac{5}{9}$$

(ii) $0.\overline{13}$

Sol: Let $x = 0.\overline{13}$

$$x = 0.131313\ldots \quad (i)$$

Multiplying both sides by 100

$$100x = 100(0.131313\ldots)$$

$$100x = 13.131313\ldots \quad (ii)$$

Subtracting (i) from (ii)

$$100x - x = (13.1313\ldots) - (0.1313\ldots)$$

$$99x = 13$$

$$x = \frac{13}{99}$$

$$\text{Hence } 0.\overline{13} = \frac{13}{99}$$

(iii) $0.\overline{67}$

Let $x = 0.\overline{67}$

$$x = 0.676767\ldots \quad (i)$$

Multiplying both sides by 100

$$100x = 100(0.676767\ldots)$$

$$100x = 67.676767\ldots \quad (ii)$$

Subtracting (i) from (ii)

$$100x - x = (67.676767\ldots) - (0.676767\ldots)$$

$$99x = 67$$

$$x = \frac{67}{99}$$

$$\text{Hence } 0.\overline{67} = \frac{67}{99}$$

Properties of Real numbers with respect to Addition and Multiplication

a. Properties of real numbers under addition are as follows:

(i) **Closure Property**

$$a + b \in \mathbb{R}, \forall a, b \in \mathbb{R}$$

e.g., if -3 and $5 \in \mathbb{R}$

$$\text{then } -3 + 5 = 2 \in \mathbb{R}$$

(ii) **Commutative Property**

$$a + b = b + a, \forall a, b \in \mathbb{R}$$

e.g., if $2, 3 \in \mathbb{R}$

$$\text{then } 2 + 3 = 3 + 2$$

$$\text{or } 5 = 5$$

(iii) **Associative Property**

$$(a + b) + c = a + (b + c), \forall a, b, c \in \mathbb{R}$$

e.g., if $5, 7, 3 \in \mathbb{R}$

$$\text{then } (5 + 7) + 3 = 5 + (7 + 3)$$

$$\text{or } 12 + 3 = 5 + 10$$

$$\text{or } 15 = 15$$

(iv) **Additive Identity**

There exists a unique real number 0 called additive identity such that

$$a + 0 = a = 0 + a, \quad \forall a \in \mathbb{R}$$

(v) **Additive Inverse**

For every $a \in \mathbb{R}$, there exists a unique real number $-a$ called the additive inverse of a such that

$$a + (-a) = 0 = (-a) + a$$

e.g., additive inverse of 3 is -3

$$\text{since } 3 + (-3) = 0 = (-3) + (3)$$

b. Properties of real numbers under multiplication are as follows:

(i) Closure Property

$$ab \in \mathbb{R}, \quad \forall a, b \in \mathbb{R}$$

e.g., if $-3, 5 \in \mathbb{R}$

then $(-3)(5) \in \mathbb{R}$

or $-15 \in \mathbb{R}$

(ii) Commutative Property:

$$ab = ba, \quad \forall a, b \in \mathbb{R}$$

e.g., if $\frac{1}{3}, \frac{3}{2} \in \mathbb{R}$

$$\text{then } \left(\frac{1}{3}\right)\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)\left(\frac{1}{3}\right)$$

$$\text{or } \frac{1}{2} = \frac{1}{2}$$

(iii) Associative Property:

$$(ab)c = a(bc), \quad \forall a, b, c \in \mathbb{R}$$

e.g., if $2, 3, 5 \in \mathbb{R}$

then $(2 \times 3) \times 5 = 2 \times (3 \times 5)$

or $6 \times 5 = 2 \times 15$

or $30 = 30$

(iv) Multiplicative Identity:

There exists a unique real number 1, called the multiplicative identity such that

$$a.1 = a = 1.a \quad \forall a \in \mathbb{R}$$

(v) Multiplicative Inverse

For every non-zero real number, there exists a unique real number a^{-1} or $\frac{1}{a}$, called multiplicative inverse of a , such that

$$aa^{-1} = 1 = a^{-1}a$$

$$\text{or } a \times \frac{1}{a} = 1 = \frac{1}{a} \times a$$

e.g., if $5 \in \mathbb{R}$, then $\frac{1}{5} \in \mathbb{R}$

such that

$$5 \times \frac{1}{5} = 1 = \frac{1}{5} \times 5$$

So, 5 and $\frac{1}{5}$ are multiplicative inverse of each other.

(vi) Multiplication is Distributive over Addition and Subtraction

For all $a, b, c \in \mathbb{R}$

$$a(b + c) = ab + ac \quad (\text{Left distributive law})$$

$$(a + b)c = ac + bc \quad (\text{Right distributive law})$$

e.g., if $2, 3, 5 \in \mathbb{R}$, then

$$2(3 + 5) = 2 \times 3 + 2 \times 5$$

$$\text{or } 2 \times 8 = 6 + 10$$

$$\text{or } 16 = 16$$

And for all $a, b, c \in \mathbb{R}$

$$a(b - c) = ab - ac \quad (\text{Left distributive law})$$

$$(a - b)c = ac - bc \quad (\text{Right distributive law})$$

e.g., if $2, 5, 3 \in \mathbb{R}$, then

$$2(5 - 3) = 2 \times 5 - 2 \times 3$$

$$\text{or } 2 \times 2 = 10 - 6$$

$$\text{or } 4 = 4$$

(b) Properties of Equality of Real Numbers:

Properties of equality of real numbers are as follows:

(i) Reflexive Property

$$a = a, \quad \forall a \in \mathbb{R}$$

(ii) Symmetric Property

$$\text{If } a = b, \text{ then } b = a, \quad \forall a, b \in \mathbb{R}$$

(iii) Transitive Property

$$\text{If } a = b \text{ and } b = c, \text{ then } a = c, \quad \forall a, b, c \in \mathbb{R}$$

(iv) Additive Property

$$\text{If } a = b, \text{ then } a + c = b + c, \quad \forall a, b, c \in \mathbb{R}$$

(v) Multiplicative Property

$$\text{If } a = b, \text{ then } ac = bc, \quad \forall a, b, c \in \mathbb{R}$$

(vi) Cancellation Property for Addition

If $a+c=b+c$, then $a=b$, $\forall a, b, c \in \mathbb{R}$

(vii) Cancellation property for Multiplication

If $ac = bc$, $c \neq 0$ then $a = b$, $\forall a, b, c \in \mathbb{R}$

(c) Properties of Inequalities of Real numbers

Properties of inequalities of real numbers are as follows:

(i) **Trichotomy Property**

$\forall a, b \in \mathbb{R}$

$a < b$ or $a = b$ or $a > b$

(ii) **Transitive Property**

$\forall a, b, c \in \mathbb{R}$

(a) $a < b$ and $b < c \Rightarrow a < c$

(b) $a > b$ and $b > c \Rightarrow a > c$

(iii) **Multiplicative Property**

(a) $\forall a, b, c \in \mathbb{R}$ and $c > 0$

(i) $a > b \Rightarrow ac > bc$ (ii) $a < b \Rightarrow ac < bc$

(i) $a > b \Rightarrow ca > cb$ (ii) $a < b \Rightarrow ca < cb$

(b) $\forall a, b, c \in \mathbb{R}$ and $c < 0$

(i) $a > b \Rightarrow ac < bc$ (ii) $a < b \Rightarrow ac > bc$

(i) $a > b \Rightarrow ca < cb$ (ii) $a < b \Rightarrow ca > cb$

(iv) **Multiplicative Inverse Property:**

$\forall a, b \in \mathbb{R}$ and $a \neq 0, b \neq 0$

(a) $a < b \Leftrightarrow \frac{1}{a} > \frac{1}{b}$

(b) $a > b \Leftrightarrow \frac{1}{a} < \frac{1}{b}$

(v) **Additive property:**

$\forall a, b, c \in \mathbb{R}$

(a) $a < b \Rightarrow a + c < b + c$

$a < b \Rightarrow c + a < c + b$

(b) $a > b \Rightarrow a + c > b + c$

$a > b \Rightarrow c + a > c + b$