

Exercise 2.3

Q1. Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify.

(i) $\sqrt[3]{-64} = (-64)^{1/3}$

(ii) $2^{3/5} = (2^3)^{1/5} = \sqrt[5]{2^3}$

(iii) $-7^{1/3} = -\sqrt[3]{7}$

(iv) $y^{-2/3} = (y^{-2})^{1/3} = \sqrt[3]{y^{-2}}$

Q2. Tell whether the following statements are true or false?

(i) $5^{1/5} = \sqrt{5}$ **False**

(ii) $2^{2/3} = \sqrt[3]{4}$ **True**

(iii) $\sqrt{49} = \sqrt{7}$ **False**

(iv) $\sqrt[3]{x^{27}} = x^3$ **False**

Q3. Simplify the following radical expressions.

(i) $\sqrt[3]{-125} = (-125)^{1/3}$
 $= [(-5)^3]^{1/3} = (-5)^{3 \times \frac{1}{3}}$
 $= -5$

(ii) $\sqrt[4]{32} = \sqrt[4]{16 \times 2}$
 $= \sqrt[4]{16} \times \sqrt[4]{2}$

$$\begin{aligned}
 &= (2^4)^{1/4} \sqrt[4]{2} \\
 &= 2^{4 \times \frac{1}{4}} \sqrt[4]{2} \\
 &= 2(\sqrt[4]{2}) \\
 \text{(iii)} \quad &\sqrt[5]{\frac{3}{32}} \\
 &= \frac{\sqrt[5]{3}}{\sqrt[5]{32}} \\
 &= \frac{\sqrt[5]{3}}{(2^5)^{1/5}} \\
 &= \frac{\sqrt[5]{3}}{2^{5 \times \frac{1}{5}}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\sqrt[5]{3}}{2} \\
 \text{(iv)} \quad &\sqrt[3]{\frac{-8}{27}} \\
 &= \left(\frac{-8}{27}\right)^{1/3} \\
 &= \left[\left(\frac{-2}{3}\right)^3\right]^{1/3} \\
 &= \left(\frac{-2}{3}\right)^{3 \times \frac{1}{3}} = \frac{-2}{3}
 \end{aligned}$$

Example

Use rules of exponents to simplify each expression and write the answer in terms of positive exponents.

$$\begin{aligned}
 \text{(i)} \quad &\frac{x^{-2}x^{-3}y^7}{x^{-3}y^4} = \frac{x^{-5}y^7}{x^{-3}y^4} = x^{-5+3}y^{7-4} = x^{-2}y^3 = \frac{y^3}{x^2} \\
 \text{(ii)} \quad &\left(\frac{4a^3b^0}{9a^{-5}}\right)^{-2} = \left(\frac{4a^{3+5} \times 1}{9}\right)^{-2} = \left(\frac{4a^8}{9}\right)^{-2} = \left(\frac{9}{4a^8}\right)^2 = \frac{81}{16a^{16}}
 \end{aligned}$$

Example

Simplify the following by using laws of indices:

$$\begin{aligned}
 \text{(i)} \quad &\left(\frac{8}{125}\right)^{-4/3} \\
 \text{(ii)} \quad &\frac{4(3)^n}{3^{n+1}-3^n}
 \end{aligned}$$

Solution

Using Laws of Indices.

$$\begin{aligned}
 \text{(i)} \quad &\left(\frac{8}{125}\right)^{-4/3} = \left(\frac{125}{8}\right)^{4/3} = \frac{(125)^{4/3}}{(8)^{4/3}} = \frac{(5^3)^{4/3}}{(2^3)^{4/3}} = \frac{5^4}{2^4} = \frac{625}{16} \\
 \text{(ii)} \quad &\frac{4(3)^n}{3^{n+1}-3^n} = \frac{4(3)^n}{3^n[3-1]} = \frac{4(3)^n}{2(3^n)} = \frac{4}{2} = 2
 \end{aligned}$$

