

Exercise 2.4

Q1. Use laws of exponents to simplify

$$\begin{aligned}
 \text{(i)} \quad & \frac{(243)^{-2/3} (32)^{-1/5}}{\sqrt{(196)^{-1}}} \\
 &= \frac{\sqrt{196}}{(243)^{2/3} (32)^{1/5}} \\
 &= \frac{\sqrt{14 \times 14}}{(3 \times 3 \times 3 \times 3 \times 3)^{2/3} (2 \times 2 \times 2 \times 2 \times 2)^{1/5}} \\
 &= \frac{\sqrt{(14)^2}}{(3^3 \times 3^2)^{2/3} (2^5)^{1/5}} \\
 &= \frac{14}{3^{3 \times \frac{2}{3}} \times 3^{2 \times \frac{2}{3}} \times 2^{5 \times \frac{1}{5}}} \\
 &= \frac{14}{3^2 \times 3^{2 \times \frac{2}{3}} \times 2} \\
 &= \frac{7}{3^2 \times 3^{\frac{4}{3}}} \\
 &= \frac{7}{3^2 \times 3^{\frac{3+1}{3}}} \\
 &= \frac{7}{3^2 \times 3^{\frac{3}{3} + \frac{1}{3}}} \\
 &= \frac{7}{3^2 \times 3 \times 3^{\frac{1}{3}}} \\
 &= \frac{7}{3^3 \times \sqrt[3]{3}} \\
 &= \frac{7}{27(\sqrt[3]{3})}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & (2x^5y^{-4})(-8x^{-3}.y^2) \\
 &= 2(-8)x^{5-3}.y^{-4+2} \\
 &= -16x^2.y^{-2} \\
 &= -16 \frac{x^2}{y^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & \left(\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0} \right)^{-3} \\
 &= (x^{-2-4}.y^{-1+3}.z^{-4+0})^{-3} \\
 &= (x^{-6}.y^2.z^{-4})^{-3} \\
 &= x^{-6(-3)}.y^{2(-3)}.z^{-4(-3)} \\
 &= x^{18}.y^{-6}.z^{12} \\
 &= \frac{x^{18}.z^{12}}{y^6}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & \frac{(81)^n.3^5 - (3)^{4n-1}(243)}{(9^{2n})(3^3)} \\
 &= \frac{(3^4)^n.3^5 - (3)^{4n-1}(3^5)}{(3^2)^{2n}(3^3)} \\
 &= \frac{3^{4n+5} - 3^{4n-1+5}}{3^{4n+3}} \\
 &= \frac{3^{4n+3+2} - 3^{4n+4}}{3^{4n+3}} \\
 &= \frac{3^{4n+3+2} - 3^{4n+3+1}}{3^{4n+3}} \\
 &= \frac{3^{4n+3}.3^2 - 3^{4n+3}.3^1}{3^{4n+3}}
 \end{aligned}$$

$$= \frac{3^{4n+3} (3^2 - 3^1)}{3^{4n+3}}$$

$$= 9 - 3$$

$$= 6$$

Q2. Show that

$$\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = 1$$

Sol: L.H.S

$$= \left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$$

$$= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a}$$

$$= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$$

$$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$$

$$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$= x^0$$

$$= 1$$

$$= \text{R.H.S}$$

Q3. Simplify

$$(i) \frac{2^{1/3} \times (27)^{1/3} \times (60)^{1/2}}{(180)^{1/2} \times (4)^{-1/3} \times (9)^{1/4}}$$

$$= \frac{2^{1/3} \times (3^3)^{1/3} \times (2^2 \times 3 \times 5)^{1/2}}{(2^2 \times 3^2 \times 5)^{1/2} \times (2^2)^{-1/3} \times (3^2)^{1/4}}$$

$$= \frac{2^{1/3} \times 3^{3 \times \frac{1}{3}} \times 2^{2 \times \frac{1}{2}} \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{2^{2 \times \frac{1}{2}} \times 3^{2 \times \frac{1}{2}} \times 5^{\frac{1}{2}} \times 2^{2 \left(-\frac{1}{3}\right)} \times 3^{2 \times \frac{1}{4}}}$$

$$= \frac{2^{1/3} \times 3 \times 2 \times 3^{1/2} \times 5^{1/2}}{2 \times 3 \times 5^{1/2} \times 2^{-2/3} \times 3^{1/2}}$$

$$= 2^{1+1-1+\frac{2}{3}} \times 3^{1+\frac{1}{2}-1-\frac{1}{2}} \times 5^{\frac{1}{2}-\frac{1}{2}}$$

$$= 2^{2/3} \times 3^0 \times 5^0$$

$$= 2 \times 1 \times 1$$

$$= 2$$

$$(ii) \sqrt{\frac{(216)^{2/3} \times (25)^{1/2}}{(.04)^{-1/2}}}$$

$$= \sqrt{\frac{(6^3)^{2/3} \times (5^2)^{1/2}}{\left(\frac{4}{100}\right)^{-1/2}}}$$

$$= \sqrt{\frac{6^{2 \times \frac{2}{3}} \times 5^{2 \times \frac{1}{2}}}{\left(\frac{100}{4}\right)^{-1/2}}} = \sqrt{\frac{6^2 \times 5}{(25)^{-1/2}}}$$

$$= \sqrt{\frac{6^2 \times 5}{5^{2 \times \frac{1}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{5}}$$

$$= \sqrt{6^2}$$

$$= 6$$

$$(iii) 5^{2^3} \div (5^2)^3$$

$$= 5^8 \div 5^6$$

$$= \frac{5^8}{5^6}$$

$$= 5^{8-6}$$

$$= 5^2$$

$$= 25$$

$$\begin{aligned}
 \text{iv)} \quad & (x^3)^2 \div x^{3^2} \\
 &= x^6 \div x^9 \\
 &= \frac{x^6}{x^9} \\
 &= \frac{1}{x^{9-6}} = \frac{1}{x^3}
 \end{aligned}$$

Definition of a Complex Number

A number of the form $z = a + bi$ where a and b are real numbers and $i = \sqrt{-1}$, is called a complex number and is represented by z i.e., $z = a + ib$

Set of Complex Numbers

The set of all complex numbers is denoted by C and

$$C = \{ z \mid z = a + bi, \text{ where } a, b \in \mathbb{R} \text{ and } i = \sqrt{-1} \}$$

The numbers a and b , called the real and imaginary parts of z , are denoted as $a = \text{Re}(z)$ and $b = \text{Im}(z)$ respectively.

Conjugate of a Complex Number

If we change i to $-i$ in $z = a + bi$, we obtain another complex number $a - bi$ called the complex conjugate of z and is denoted by $\bar{z}$ (read z bar).

Thus, if $z = -1 - i$, then $\bar{z} = -1 + i$.

The number $a + bi$ and $a - bi$ are called conjugates of each other.

Equality of Complex Numbers and its Properties

For all $a, b, c, d \in \mathbb{R}$,

$a + bi = c + di$ if and only if $a = c$ and $b = d$.

$$\text{e.g., } 2x + y^2i = 4 + 9i$$

if and only if

$$2x = 4 \text{ and } y^2 = 9, \text{ i.e., } x = 2 \text{ and } y = \pm 3$$

Properties of real numbers $\mathbb{R}$ are also valid for the set of complex numbers.

(i) $Z_1 = Z_2$, (Reflexive Law)

(ii) If $Z_1 = Z_2$, then $Z_2 = Z_1$ (Symmetric law)

(iii) If $Z_1 = Z_2$, and $Z_2 = Z_3$ then $Z_1 = Z_3$

(transitive law)