

Exercise 2.6

Q1. Identify the following statements as true or false.

- (i) $\sqrt{-3} \times \sqrt{-3} = 3$ False
- (ii) $i^{73} = -i$ False
- (iii) $i^{10} = -1$ True
- (iv) Complex conjugate of $(-6i + i^2)$ is $(-1 + 6i)$ True
- (v) Difference of a complex number $z = a + bi$ and its conjugate is a real number. False
- (vi) If $(a-1) - (b+i)i = 5 + 8i$ then $a = 6$ and $b = -11$. True
- (vii) Product of a complex number and its conjugate is always a non-negative real number. True

Q2. Express each complex number in the standard form $a + bi$, where 'a' and 'b' are real numbers.

- (i) $(2 + 3i) + (7 - 2i)$
 $= 2 + 3i + 7 - 2i$
 $= (2 + 7) + (3 - 2)i$
 $= 9 + i$
- (ii) $2(5 + 4i) - 3(7 + 4i)$
 $= 10 + 8i - 21 - 12i$
 $= (10 - 21) + (8 - 12)i$
 $= -11 - 4i$
- (iii) $-1(-3 + 5i) - (4 + 9i)$
 $= 3 - 5i - 4 - 9i$
 $= (3 - 4) + (-5 - 9)i$
 $= -1 - 14i$
- (iv) $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$

$$\begin{aligned}
 &= 2(-1) + 6i^2 + 3(i^2)^8 - 6i^{18} + 4i^{24} \\
 &= -2 + 6(-1) + 3(-1)^8 - 6(i^2)^9 + 4(i^2)^{12} \\
 &= -2 - 6i + 3(1) - 6(-1)^9 + 4(-1)^{12} \\
 &= -2 - 6i + 3 - 6(-1)i + 4(1) \\
 &= -2 - 6i + 3 + 6i + 4 \\
 &= 1 + 4i
 \end{aligned}$$

Q3. Simplify and write your answer in the form $a + bi$

- (i) $(-7 + 3i)(-3 + 2i)$
 $= 21 - 14i - 9i + 6i^2$
 $= 21 - 23i + 6(-1)$
 $= 21 - 6 - 23i$
 $= 15 - 23i$
- (ii) $(2 - \sqrt{-4})(3 - \sqrt{-4})$
 $= (2 - \sqrt{4}\sqrt{-1})(3 - \sqrt{4}\sqrt{-1})$
 $= (2 - 2i)(3 - 2i)$
 $= 6 - 4i - 6i + 4i^2$
 $= 6 - 10i + 4(-1)$
 $= 6 - 10i - 4$
 $= 2 - 10i$
- (iii) $(\sqrt{5} - 3i)^2$
 $= (\sqrt{5})^2 + (3i)^2 - 2(\sqrt{5})(3i)$
 $= 5 + 9i^2 - 6\sqrt{5}i$
 $= 5 + 9(-1) - 6\sqrt{5}i$
 $= 5 - 9 - 6\sqrt{5}i$
 $= -4 - 6\sqrt{5}i$

$$\begin{aligned}
 \text{(iv)} \quad & (2-3i)(\overline{3-2i}) \\
 & = (2-3i)(3+2i) \\
 & = 6+4i-9i-6i^2 \\
 & = 6-5i-6(-1) \\
 & = 6-5i+6 \\
 & = 12-5i
 \end{aligned}$$

Q4. Simplify and write your answer in the form of $a+bi$

$$\begin{aligned}
 \text{(i)} \quad & \frac{-2}{1+i} \\
 & = \frac{-2}{1+i} \times \frac{1-i}{1-i} \\
 & = \frac{-2(1-i)}{(1)^2 - (i)^2} \\
 & = \frac{-2(1-i)}{1-i^2} \\
 & = \frac{-2(1-i)}{1-(-1)} \\
 & = \frac{-2(1-i)}{1+1} \\
 & = \frac{-\cancel{2}(1-i)}{\cancel{2}} \\
 & = -(1-i) \\
 & = -1+i
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \frac{2+3i}{4-i} \\
 & = \frac{2+3i}{4-i} \times \frac{4+i}{4+i} \\
 & = \frac{(2+3i)(4+i)}{(4)^2 - (i)^2}
 \end{aligned}$$

$$\begin{aligned}
 & = \frac{8+2i+12i+3i^2}{16-i^2} \\
 & = \frac{8+14i+3(-1)}{16-(-1)} \\
 & = \frac{8+14i-3}{16+1} \\
 & = \frac{5+14i}{17}
 \end{aligned}$$

$$= \frac{5}{17} + \frac{14}{17}i$$

$$\begin{aligned}
 \text{(iii)} \quad & \frac{9-7i}{3+i} \\
 & = \frac{9-7i}{3+i} \times \frac{3-i}{3-i} \\
 & = \frac{(9-7i)(3-i)}{(3)^2 - (i)^2}
 \end{aligned}$$

$$\begin{aligned}
 & = \frac{27-9i-21i+7i^2}{9-i^2} = \frac{27-30i+7(-1)}{9-(-1)} \\
 & = \frac{27-7-30i}{9+1} \\
 & = \frac{20-30i}{10}
 \end{aligned}$$

$$= \frac{20}{10} - \frac{30}{10}i$$

$$\begin{aligned}
 & = 2-3i \\
 \text{(iv)} \quad & \frac{2-6i}{3+i} - \frac{4+i}{3+i} \\
 & = \frac{(2-6i)-(4+i)}{3+i}
 \end{aligned}$$

$$= \frac{2-6i-4-i}{3+i}$$

$$= \frac{-2-7i}{3+i}$$

$$= \frac{-2-7i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{(-2-7i)(3-i)}{(3)^2 - (i)^2}$$

$$= \frac{-6+2i-21i+7i^2}{9-i^2}$$

$$= \frac{-6-19i+7(-1)}{9-(-1)}$$

$$= \frac{-6-7-19i}{9+1}$$

$$= \frac{-13-19i}{10}$$

$$= \frac{-13}{10} - \frac{19}{10}i$$

$$) \left(\frac{1+i}{1-i} \right)^2$$

$$= \frac{(1)^2 + (i)^2 + 2(1)(i)}{(1)^2 + (i)^2 - 2(1)(i)}$$

$$= \frac{1+i^2+2i}{1+i^2-2i}$$

$$= \frac{\cancel{1} - \cancel{1} + 2i}{\cancel{1} - \cancel{1} - 2i}$$

$$= \frac{2i}{-2i}$$

$$= -1$$

$$= -1 + 0i$$

$$\frac{1}{(2+3i)(1-i)}$$

$$= \frac{1}{2-2i+3i-3i^2}$$

$$= \frac{1}{2+i-3(-1)}$$

$$= \frac{1}{2+i+3}$$

$$= \frac{1}{5+i}$$

$$= \frac{1}{5+i} \times \frac{5-i}{5-i}$$

$$= \frac{5-i}{(5)^2 - (i)^2}$$

$$= \frac{5-i}{25-i^2}$$

$$= \frac{5-i}{25-(-1)}$$

$$= \frac{5-i}{25+1}$$

$$= \frac{5-i}{26}$$

$$= \frac{5}{26} - \frac{1}{26}i$$

Q5. Calculate (a) $\bar{z}$ (b) $z + \bar{z}$
 c) $z - \bar{z}$ (d) $z \cdot \bar{z}$ for each of the following.

(i) $z = 0 - i$

(a) $\bar{z} = 0 + i$

(b) $z + \bar{z} = 0 - i + 0 + i = 0$

(c) $z - \bar{z} = 0 - i - (0 + i)$

$$= 0 - i - 0 - i$$

$$= -2i$$

(d) $z \cdot \bar{z} = (0 - i)(0 + i)$

$$= (0)^2 - (i)^2 = 0 - (-1)$$

$$= 1$$

(ii) $z = 2 + i$

(a) $\bar{z} = 2 - i$

(b) $z + \bar{z} = 2 + \cancel{i} + 2 - \cancel{i}$

$$= 4$$

(c) $z - \bar{z} = (2 + i) - (2 - i)$

$$= \cancel{2} + i - \cancel{2} + i$$

$$= 2i$$

$$\begin{aligned}
 \text{(d)} \quad z \cdot \bar{z} &= (2+i)(2-i) \\
 &= (2)^2 - (i)^2 \\
 &= 4 - i^2 \\
 &= 4 - (-1) \\
 &= 4 + 1 \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad z &= \frac{1+i}{1-i} \\
 &= \frac{1+i}{1-i} \times \frac{1+i}{1+i} \\
 &= \frac{(1+i)^2}{(1)^2 - (i)^2} \\
 &= \frac{(1)^2 + (i)^2 + 2(1)(i)}{1 - i^2} \\
 &= \frac{1 + i^2 + 2i}{1 - (-1)} = \frac{1 - 1 + 2i}{1 + 1} \\
 &= \frac{2i}{2} = i \\
 z &= 0 + i
 \end{aligned}$$

$$\text{(a)} \quad \bar{z} = 0 - i$$

$$\text{(b)} \quad z + \bar{z} = 0 + i + 0 - i = 0$$

$$\begin{aligned}
 \text{(c)} \quad z - \bar{z} &= 0 + i - (0 - i) \\
 &= 0 + i - 0 + i \\
 &= 2i
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad z \cdot \bar{z} &= (0+i)(0-i) \\
 &= (0)^2 - (i)^2 = 0 - (-1) \\
 &= 0 + 1 = 1
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad z &= \frac{4-3i}{2+4i} \\
 &= \frac{4-3i}{2+4i} \times \frac{2-4i}{2-4i}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(4-3i)(2-4i)}{(2)^2 - (4i)^2} \\
 &= \frac{8-16i-6i+12i^2}{4-16i^2} \\
 &= \frac{8-22i+12(-1)}{4-16(-1)} \\
 &= \frac{8-12-22i}{4+16}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{-4-22i}{20} \\
 &= -\frac{4}{20} - \frac{22}{20}i \\
 z &= -\frac{1}{5} - \frac{11}{10}i
 \end{aligned}$$

$$\text{(a)} \quad \bar{z} = -\frac{1}{5} + \frac{11}{10}i$$

$$\begin{aligned}
 \text{(b)} \quad z + \bar{z} &= -\frac{1}{5} + \frac{11}{10}i - \frac{1}{5} + \frac{11}{10}i \\
 &= -\frac{2}{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad z - \bar{z} &= -\frac{1}{5} - \frac{11}{10}i - \left(-\frac{1}{5} + \frac{11}{10}i\right) \\
 &= -\frac{1}{5} - \frac{11}{10}i + \frac{1}{5} - \frac{11}{10}i \\
 &= -\frac{22}{10}i \\
 &= -\frac{11}{5}i
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad z \cdot \bar{z} &= \left(-\frac{1}{5} - \frac{11}{10}i\right) \left(-\frac{1}{5} + \frac{11}{10}i\right) \\
 &= \left(-\frac{1}{5}\right)^2 - \left(\frac{11}{10}i\right)^2 \\
 &= \frac{1}{25} - \frac{121}{100}i^2
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{25} - \frac{121}{100}(-1) \\
 &= \frac{1}{25} + \frac{121}{100} \\
 &= \frac{4+121}{100} \\
 &= \frac{125}{100} \\
 &= \frac{5}{4}
 \end{aligned}$$

Q6. If $z = 2 + 3i$ and $w = 5 - 4i$, show that:

(i) $\overline{z + w} = \overline{z} + \overline{w}$

Sol: L.H.S = $\overline{z + w}$

$$z + w = 2 + 3i + 5 - 4i$$

$$z + w = 7 - i$$

$$\overline{z + w} = 7 + i$$

Now R.H.S = $\overline{z} + \overline{w}$

$$\overline{z} = 2 - 3i$$

$$\overline{w} = 5 + 4i$$

$$\overline{z} + \overline{w} = 2 - 3i + 5 + 4i$$

$$= 7 + i$$

Hence $\overline{z + w} = \overline{z} + \overline{w}$

(ii) $\overline{z - w} = \overline{z} - \overline{w}$

Sol: L.H.S = $\overline{z - w}$

$$z - w = 2 + 3i - (5 - 4i)$$

$$= 2 + 3i - 5 + 4i$$

$$= -3 + 7i$$

$$\overline{z - w} = -3 - 7i$$

R.H.S = $\overline{z} - \overline{w}$

$$\overline{z} = 2 - 3i$$

$$\overline{w} = 5 + 4i$$

$$\overline{z} - \overline{w} = (2 - 3i) - (5 + 4i)$$

$$= 2 - 3i - 5 - 4i$$

$$= -3 - 7i$$

Hence $\overline{z - w} = \overline{z} - \overline{w}$

(iii) $\overline{z \cdot w} = \overline{z} \cdot \overline{w}$

L.H.S = $\overline{z \cdot w}$

$$z \cdot w = (2 + 3i)(5 - 4i)$$

$$= 10 - 8i + 15i - 12i^2$$

$$= 10 + 7i - 12(-1)$$

$$= 10 + 7i + 12$$

$$= 22 + 7i$$

$$\overline{z \cdot w} = 22 - 7i$$

R.H.S = $\overline{z} \cdot \overline{w}$

$$\overline{z} = 2 - 3i$$

$$\overline{w} = 5 + 4i$$

$$\overline{z} \cdot \overline{w} = (2 - 3i)(5 + 4i)$$

$$= 10 + 8i - 15i - 12i^2$$

$$= 10 - 7i - 12(-1)$$

$$= 10 - 7i + 12$$

$$\overline{z} \cdot \overline{w} = 22 - 7i$$

Hence $\overline{z \cdot w} = \overline{z} \cdot \overline{w}$

(iv) $\overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}$, where $w \neq 0$

LHS = $\overline{\left(\frac{z}{w}\right)}$

$$\frac{z}{w} = \frac{2 + 3i}{5 - 4i}$$

$$= \frac{2 + 3i}{5 - 4i} \times \frac{5 + 4i}{5 + 4i}$$

$$= \frac{(2 + 3i)(5 + 4i)}{(5)^2 - (4i)^2} = \frac{10 + 8i + 15i + 12i^2}{25 - 16i^2}$$

$$= \frac{10+23i+12(-1)}{25-16(-1)}$$

$$= \frac{10-12+23i}{25+16}$$

$$= \frac{-2+23i}{41}$$

$$= -\frac{2}{41} + \frac{23}{41}i$$

$$\left(\frac{z}{w}\right) = -\frac{2}{41} - \frac{23}{41}i$$

$$\text{R.H.S} = \frac{\bar{z}}{w}$$

$$\bar{z} = 2-3i$$

$$\bar{w} = 5+4i$$

$$\frac{\bar{z}}{\bar{w}} = \frac{2-3i}{5+4i}$$

$$= \frac{(2-3i)(5-4i)}{(5)^2 - (4i)^2}$$

$$= \frac{10-8i-15i+12i^2}{25-16i^2}$$

$$= \frac{10-23i+12(-1)}{25-16(-1)}$$

$$= \frac{10-12-23i}{25+16}$$

$$= \frac{-2-23i}{41}$$

$$= -\frac{2}{41} - \frac{23}{41}i$$

$$\text{Hence } \left(\frac{z}{w}\right) = \frac{\bar{z}}{w}$$

$$(v) \quad \frac{1}{2}(z+\bar{z}) \text{ is the real part of } z$$

$$\text{Sol: } z = 2+3i$$

$$\text{Now } \bar{z} = 2-3i$$

$$\frac{1}{2}(z+\bar{z}) = \frac{1}{2}(2+\cancel{3i}+2-\cancel{3i})$$

$$= \frac{1}{2}(4)$$

$$\frac{1}{2}(z+\bar{z}) = 2$$

$$\frac{1}{2}(z+\bar{z}) = \text{Re}(z)$$

Hence $\frac{1}{2}(z+\bar{z})$ is equal to the real part of z .

$$(vi) \quad \frac{1}{2i}(z-\bar{z}) \text{ is the real part of } z.$$

$$\text{Sol. } z = 2+3i$$

$$\text{Now } \bar{z} = 2-3i$$

$$\frac{1}{2i}(z-\bar{z}) = \frac{1}{2i}[(2+3i)-(2-3i)]$$

$$= \frac{1}{2i}(\cancel{2}+3i-\cancel{2}+3i)$$

$$= \frac{6i}{3i}$$

$$= 3$$

$$\frac{1}{2i}(z-\bar{z}) = \text{R}(z)$$

Hence proved that $\frac{1}{2i}(z-\bar{z})$ is equal to the real part of z .

Q7. Solve the following equation for real x and y

$$(i) \quad (2-3i)(x+yi) = 4+i$$

$$(x+yi) = \frac{4+i}{2-3i}$$

$$= \frac{4+i}{2-3i} \times \frac{2+3i}{2+3i}$$

$$\begin{aligned}
 &= \frac{(4+i)(2+3i)}{(2)^2 - (3i)^2} \\
 &= \frac{8+12i+2i+3i^2}{4-9i^2} \\
 &= \frac{8+14i+3(-1)}{4-9(-1)} \\
 &= \frac{8-3+14i}{4+9} \\
 &= \frac{5+14i}{13}
 \end{aligned}$$

$$(x+yi) = \frac{5}{13} + \frac{14}{13}i$$

$$\Rightarrow x = \frac{5}{13} \text{ and } y = \frac{14}{13}$$

$$(ii) (3-2i)(x+yi) = 2(x-2yi) + 2i - 1$$

$$3x+3yi-2xi-2yi^2 = 2x-4yi+2i-1$$

$$3x+(3y-2x)i-2y(-1) = 2x-1+(2-4y)i$$

$$(3x+2y)+(3y-2x)i = (2x-1)+(2-4y)i$$

$$\Rightarrow 3x+2y = 2x-1 \quad \dots\dots(i) \text{ and}$$

$$3y-2x = 2-4y \quad \dots\dots(ii)$$

$$\text{From (i)} \quad 3x-2x+2y = -1$$

$$x+2y = -1 \quad \dots\dots(iii)$$

$$\text{From (ii)} \quad -2x+3y+4y = 2$$

$$-2x+7y = 2 \quad \dots\dots(iv)$$

Multiplying (iii) by 2 and adding in (iv)

$$~~2x~~ + 4y = ~~-2~~$$

$$-~~2x~~ + 7y = ~~2~~$$

$$11y = 0$$

$$y = \frac{0}{11}$$

$$\boxed{y = 0}$$

Putting value of y in (iii)

$$x+2y = -1$$

$$x+2(0) = -1$$

$$x+0 = -1$$

$$\boxed{x = -1}$$

$$(iii) (3+4i)^2 - 2(x-yi) = x+yi$$

$$(3)^2 + (4i)^2 + 2(3)(4i) - 2x + 2yi = x+yi$$

$$9+16i^2+24i-2x+2yi = x+yi$$

$$9+16(-1)+24i-2x+2yi = x+yi$$

$$9-16+24i-2x+2yi = x+yi$$

$$-7-2x+(24+2y)i = x+yi$$

$$\Rightarrow x = -7-2x$$

$$x+2x = -7$$

$$3x = -7$$

$$\boxed{x = \frac{-7}{3}}$$

$$\text{and } 24+2y = y$$

$$2y-y = -24$$

$$\boxed{y = -24}$$

OBJECTIVE

Q. Select the correct answer.

1. $(27x^{-1})^{\frac{-2}{3}} = \underline{\hspace{2cm}}$

(a) $\frac{\sqrt[3]{x^2}}{9}$ (b) $\frac{\sqrt{x^3}}{9}$

(c) $\frac{\sqrt[3]{x^2}}{8}$ (d) $\frac{\sqrt{x^3}}{8}$

2. Write $\sqrt[7]{x}$ in exponential form

(a) x (b) x^7

(c) $x^{\frac{1}{7}}$ (d) $x^{\frac{7}{2}}$

3. Write $4^{\frac{2}{3}}$ with radical sign.....

(a) $\sqrt[3]{4^2}$ (b) $\sqrt{4^3}$

(c) $\sqrt[2]{4^3}$ (d) $\sqrt{4^6}$

4. In $\sqrt[3]{35}$ the radicand is

(a) 3 (b) $\frac{1}{3}$
(c) 35 (d) None of these

5. $\left(\frac{25}{16}\right)^{-\frac{1}{2}} = \underline{\hspace{2cm}}$

(a) $\frac{5}{4}$ (b) $\frac{4}{5}$

(c) $\frac{-5}{4}$ (d) $\frac{-4}{5}$

6. The conjugate of $5 + 4i$ is _____

(a) $-5+4i$ (b) $-5-4i$
(c) $5-4i$ (d) $5+4i$

7. The value of i^9 is _____

(a) 1 (b) -1
(c) i (d) $-i$

8. Every real number is _____

- (a) A positive integer
(b) A rational number
(c) A negative integer
(d) A complex number

9. Real part of $2ab(i+i^2)$ is _____

- (a) $2ab$ (b) $-2ab$
(c) $2abi$ (d) $-2abi$

10. Imaginary part of $-i(3i+2)$ is _____

- (a) -2 (b) 2
(c) 3 (d) -3

11. Which of the following sets have the closure property w.r.t. addition

- (a) $\{0\}$ (b) $\{0, -1\}$
(c) $\{0, 1\}$ (d) $\left\{1, \sqrt{2}, \frac{1}{2}\right\}$

12. Name the property of real numbers

used in $\left(\frac{-\sqrt{5}}{2}\right) \times 1 = \frac{-\sqrt{5}}{2} \times 1$

- (a) Additive identity
(b) Additive Inverse
(c) Multiplicative identity
(d) Multiplicative Inverse

13. If $z < 0$ then $x < y \Rightarrow$

- (a) $xz < yz$ (b) $xz > yz$
(c) $xz = yz$ (d) none of these

14. If $a, b \in \mathbb{R}$ then only one of $a = b$ or $a < b$ or $a > b$ holds is called...

- (a) Trichotomy property
(b) Transitive property
(c) Additive property
(d) Multiplicative property

15. A non-terminating, non-recurring decimal represents:
- A natural number
 - A rational number
 - An irrational number
 - A prime number
16. The union of the set of rational numbers and irrational numbers is known as set of ____
- Rational number
 - Irrational
 - Real number
 - Whole number
17. For each prime number A , $\sqrt{A}$ is an ____
- Irrational
 - Rational
 - Real
 - Whole
18. Square roots of all positive non-square integers are ____
- Irrational
 - Rational
 - Real
 - Whole
19. π is an ____ number.
- Irrational
 - Rational
 - Real
 - None
20. $\forall a, b, c \in \mathbb{R}$ then $a < b$ and $b < c \Rightarrow a < c$ is ____ property.
- Transitive
 - Trichotomy property
 - Additive property
 - Multiplicative property
21. Name the property of real numbers used in $x > y$ or $x = y$ or $x < y$.
- Trichotomy
 - Transitive
 - Additive
 - Multiplicative
22. Name the property of real numbers used in $\pi + (-\pi) = 0$.
- Additive inverse
 - Multiplicative inverse
 - Additive identity
 - Multiplicative identity
23. $\sqrt{3} \cdot \sqrt{3}$ is a ____ number.
- Rational
 - Irrational
 - Real
 - None
24. $\sqrt[3]{ab} = ______$
- $\sqrt[3]{a} \sqrt[3]{b}$
 - $\sqrt{a} \sqrt{b}$
 - $\sqrt[3]{a} \sqrt{b}$
 - $\sqrt{a} \sqrt[3]{b}$
25. $\sqrt[3]{-8} = ______$
- $(-8)^{\frac{1}{5}}$
 - $(-8)^5$
 - (-8)
 - $(8)^{\frac{1}{5}}$
26. The value of i^{10} is:
- 1
 - 1
 - i
 - i
27. The solution set of $x^2 + 1 = 0$ is:
- $\{i, i\}$
 - $\{i, -i\}$
 - $\{-i, -i\}$
 - None
28. The conjugate of $2 + 3i$ is ____
- $2 - 3i$
 - $-2 - 3i$
 - $-2 + 3i$
 - $2 + 3i$
29. Real part of $(-1 + \sqrt{-2})^2$ is:
- 1
 - $-2\sqrt{2}$
 - i
 - $2\sqrt{2}$
30. Imaginary part of $(-1 + \sqrt{-2})^2$ is:
- 1
 - $-2\sqrt{2}$
 - 1
 - $2\sqrt{2}$

31. Product of a complex number and its conjugate is always a non-negative__

- (a) Real (b) Irrational
(c) Rational (d) None

ANSWER KEY

1.	a	2.	c	3.	a	4.	c	5.	b
6.	c	7.	c	8.	d	9.	b	10.	a
11.	a	12.	c	13.	b	14.	a	15.	c
16.	c	17.	a	18.	a	19.	a	20.	a
21.	a	22.	a	23.	c	24.	a	25.	a
26.	a	27.	b	28.	a	29.	a	30.	b
31.	a								

REVIEW EXERCISE

3. Simplify: (i) $\sqrt[4]{81y^{-12}x^{-8}}$

$$\begin{aligned} &= (3^4 y^{-12} x^{-8})^{\frac{1}{4}} \\ &= (3^4)^{\frac{1}{4}} (y^{-12})^{\frac{1}{4}} (x^{-8})^{\frac{1}{4}} \\ &= 3y^{-3}x^{-2} \\ &= \frac{3}{x^2 y^3} \end{aligned}$$

(ii) $\sqrt{25x^{10n}y^{8m}}$

$$\begin{aligned} &= (5^2 x^{10n} y^{8m})^{\frac{1}{2}} \\ &= (5^2)^{\frac{1}{2}} (x^{10n})^{\frac{1}{2}} (y^{8m})^{\frac{1}{2}} \\ &= 5x^{5n}y^{4m} \end{aligned}$$

(iii) $\left(\frac{x^3 y^4 z^5}{x^{-2} y^{-1} z^{-5}}\right)^{\frac{1}{5}}$

$$\begin{aligned} &= (x^{3+2} y^{4+1} z^{5+5})^{\frac{1}{5}} \\ &= (x^5 y^5 z^{10})^{\frac{1}{5}} \\ &= (x^5)^{\frac{1}{5}} (y^5)^{\frac{1}{5}} (z^{10})^{\frac{1}{5}} \\ &= xyz^2 \end{aligned}$$

(iv) $\left(\frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}}\right)^{\frac{2}{5}}$

$$= \left(\frac{2^5 x^{-6} y^{-4} z}{5^4 x^4 y z^{-4}}\right)^{\frac{2}{5}}$$

$$\begin{aligned} &= \left(\frac{2^5 x^{-6-4} y^{-4-1} z^{1+4}}{5^4}\right)^{\frac{2}{5}} \\ &= \left(\frac{2^5 x^{-10} y^{-5} z^5}{5^4}\right)^{\frac{2}{5}} \\ &= \frac{(2^5)^{\frac{2}{5}} (x^{-10})^{\frac{2}{5}} (y^{-5})^{\frac{2}{5}} (z^5)^{\frac{2}{5}}}{(5^4)^{\frac{2}{5}}} \\ &= \frac{2^2 x^{-4} y^{-2} z^2}{5^5} \\ &= \frac{4z^2}{x^4 y^2 5.5^5} \end{aligned}$$

Q.4. Simplify: $\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

$$\begin{aligned} &= \left[\frac{(2^3 \times 3^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{-3}{2}}} \right]^{\frac{1}{2}} \\ &= \left[\frac{(2^3)^{\frac{2}{3}} \times (3^3)^{\frac{2}{3}} \times 5}{\left(\frac{100}{4}\right)^{\frac{3}{2}}} \right]^{\frac{1}{2}} \\ &= \left[\frac{2^2 \times 3^2 \times 5}{(25)^{\frac{3}{2}}} \right]^{\frac{1}{2}} = \left[\frac{2^2 \times 3^2 \times 5}{(5^2)^2} \right]^{\frac{1}{2}} \end{aligned}$$

REVIEW EXERCISE

3. Simplify: (i) $\sqrt[4]{81y^{-12}x^{-8}}$

$$\begin{aligned} &= (3^4 y^{-12} x^{-8})^{\frac{1}{4}} \\ &= (3^4)^{\frac{1}{4}} (y^{-12})^{\frac{1}{4}} (x^{-8})^{\frac{1}{4}} \\ &= 3y^{-3}x^{-2} \\ &= \frac{3}{x^2 y^3} \end{aligned}$$

(ii) $\sqrt{25x^{10n}y^{8m}}$

$$\begin{aligned} &= (5^2 x^{10n} y^{8m})^{\frac{1}{2}} \\ &= (5^2)^{\frac{1}{2}} (x^{10n})^{\frac{1}{2}} (y^{8m})^{\frac{1}{2}} \\ &= 5x^{5n}y^{4m} \end{aligned}$$

(iii) $\left(\frac{x^3 y^4 z^5}{x^{-2} y^{-1} z^{-5}}\right)^{\frac{1}{5}}$

$$\begin{aligned} &= (x^{3+2} y^{4+1} z^{5+5})^{\frac{1}{5}} \\ &= (x^5 y^5 z^{10})^{\frac{1}{5}} \\ &= (x^5)^{\frac{1}{5}} (y^5)^{\frac{1}{5}} (z^{10})^{\frac{1}{5}} \\ &= xyz^2 \end{aligned}$$

(iv) $\left(\frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}}\right)^{\frac{2}{5}}$

$$= \left(\frac{2^5 x^{-6} y^{-4} z}{5^4 x^4 y z^{-4}}\right)^{\frac{2}{5}}$$

$$\begin{aligned} &= \left(\frac{2^5 x^{-6-4} y^{-4-1} z^{1+4}}{5^4}\right)^{\frac{2}{5}} \\ &= \left(\frac{2^5 x^{-10} y^{-5} z^5}{5^4}\right)^{\frac{2}{5}} \\ &= \frac{(2^5)^{\frac{2}{5}} (x^{-10})^{\frac{2}{5}} (y^{-5})^{\frac{2}{5}} (z^5)^{\frac{2}{5}}}{(5^4)^{\frac{2}{5}}} \\ &= \frac{2^2 x^{-4} y^{-2} z^2}{5^{\frac{8}{5}}} \\ &= \frac{4z^2}{x^4 y^2 5.5^{\frac{3}{5}}} \end{aligned}$$

Q.4. Simplify: $\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

$$\begin{aligned} &= \left[\frac{(2^3 \times 3^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{-3}{2}}}\right]^{\frac{1}{2}} \\ &= \left[\frac{(2^3)^{\frac{2}{3}} \times (3^3)^{\frac{2}{3}} \times 5}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}\right]^{\frac{1}{2}} \\ &= \left[\frac{2^2 \times 3^2 \times 5}{(25)^2}\right]^{\frac{1}{2}} = \left[\frac{2^2 \times 3^2 \times 5}{(5^2)^2}\right]^{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned}
 &= \left[\frac{2^2 \times 3^2 \times 5}{5^3} \right]^{\frac{1}{2}} = \left[\frac{2^2 \times 3^2}{5^2} \right]^{\frac{1}{2}} \\
 &= \frac{(2^2)^{\frac{1}{2}} \times (3^2)^{\frac{1}{2}}}{(5^2)^{\frac{1}{2}}} = \frac{2 \times 3}{5} = \frac{6}{5}
 \end{aligned}$$

Q.5 Simplify:

$$\begin{aligned}
 &\left(\frac{a^p}{a^q} \right)^{p+q} \cdot \left(\frac{a^q}{a^r} \right)^{q+r} \div 5(a^p \cdot a^r)^{p-r} \\
 &= (a^{p-q})^{p+q} \cdot (a^{q-r})^{q+r} \div 5(a^{p+r})^{p-r} \\
 &= a^{p^2-q^2} \cdot a^{q^2-r^2} \div 5a^{p^2-r^2} \\
 &= \frac{a^{p^2-q^2} \cdot a^{q^2-r^2}}{5a^{p^2-r^2}} \\
 &= \frac{a^{p^2-q^2+q^2-r^2-p^2+r^2}}{5} \\
 &= \frac{a^0}{5} = \frac{1}{5}
 \end{aligned}$$

Q.6. Simplify:

$$\begin{aligned}
 &\left(\frac{a^{2l}}{a^{l+m}} \right) \left(\frac{a^{2m}}{a^{m+n}} \right) \left(\frac{a^{2n}}{a^{n+l}} \right) \\
 &= a^{2l-l-m} \times a^{2m-m-n} \times a^{2n-n-l} \\
 &= a^{l-m} \cdot a^{m-n} \cdot a^{n-l} \\
 &= a^{l-m+m-n+n-l} \\
 &= a^0 = 1
 \end{aligned}$$

Q.7 Simplify:

$$\begin{aligned}
 &\sqrt[3]{\frac{a^l}{a^m}} \times \sqrt[3]{\frac{a^m}{a^n}} \times \sqrt[3]{\frac{a^n}{a^l}} \\
 &= \left(\frac{a^l}{a^m} \right)^{\frac{1}{3}} \times \left(\frac{a^m}{a^n} \right)^{\frac{1}{3}} \times \left(\frac{a^n}{a^l} \right)^{\frac{1}{3}} \\
 &= \frac{a^{\frac{l}{3}}}{a^{\frac{m}{3}}} \times \frac{a^{\frac{m}{3}}}{a^{\frac{n}{3}}} \times \frac{a^{\frac{n}{3}}}{a^{\frac{l}{3}}} \\
 &= a^{\frac{l}{3} - \frac{m}{3} + \frac{m}{3} - \frac{n}{3} + \frac{n}{3} - \frac{l}{3}} \\
 &= a^{\frac{l}{3} - \frac{m}{3} + \frac{m}{3} - \frac{n}{3} + \frac{n}{3} - \frac{l}{3}} \\
 &= a^0 = 1
 \end{aligned}$$