

LOGARITHMS

Scientific Notation

A number written in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer, is called the scientific notation.

Example

Write each of the following ordinary numbers in scientific notation

- (i) 30600 (ii) 0.000058

Solution

(i) $30600 = 3.06 \times 10^4$
(move decimal point four places to the left)

(ii) $0.000058 = 5.8 \times 10^{-5}$

(move decimal point five places to the right)

Example

Change each of the following numbers from scientific notation to ordinary notation.

- (i) 6.35×10^6 (ii) 7.61×10^{-4}

Solution

(i) $6.35 \times 10^6 = 6350000$
(move the decimal point six places to the right)

(ii) $7.61 \times 10^{-4} = 0.000761$
(move the decimal point four places to the left)

Exercise 3.1

Q1. Express each of the following numbers in scientific notation.

i) 5700

Sol: $5700 = 5.7 \times 10^3$ (move decimal point three places to left)

ii) 49,800,000

Sol: $49,800,000 = 4.98 \times 10^7$ (move decimal point seven places to left)

iii) 96,000,000

Sol: $96,000,000 = 9.6 \times 10^7$ (move decimal point seven places to left)

iv) 416.9

Sol: $416.9 = 4.169 \times 10^2$ (move decimal point two places to left)

v) 83,000

Sol: $83,000 = 8.3 \times 10^4$ (move decimal point four places to left)

vi) 0.00643

Sol: $0.00643 = 6.43 \times 10^{-3}$ (move decimal point three places to right)

vii) 0.0074

Sol: $0.0074 = 7.4 \times 10^{-3}$ (move decimal point three places to right)

viii) 60,000,000

Sol: $60,000,000 = 6.0 \times 10^7$ (move decimal point seven places to left)

ix) 0.00000000395

Sol: $0.00000000395 = 3.95 \times 10^{-9}$
(move decimal point nine places to right)

$$\text{x) } \frac{275,000}{0.0025}$$

$$\text{Sol: } \frac{275,000}{0.0025}$$

$$= \frac{2.75 \times 10^5}{2.5 \times 10^{-3}} \quad \begin{array}{l} \text{(move decimal point five places to left)} \\ \text{(move decimal point three places to right)} \end{array}$$

Q2. Express the following numbers in ordinary notation.

i) 6×10^{-4}

Sol: $6 \times 10^{-4} = 0.0006$ (move decimal point four places to left)

ii) 5.06×10^{10}

Sol: $5.06 \times 10^{10} = 50,600,000,000$
(move decimal point ten places to right)

iii) 9.018×10^{-6}

Sol: $9.018 \times 10^{-6} = 0.000009018$ (move decimal point six places to left)

iv) 7.865×10^8

Sol: $7.865 \times 10^8 = 786,500,000$ (move decimal point eight places to right)

Logarithm of a Real Number

If $a^x = y$ then x is called the logarithm of y to the base ' a ' and is written as $\log_a y = x$, where $a > 0$, $a \neq 1$ and $y > 0$

i.e., the logarithm of a number y to the base ' a ' is the index x of the power to which a must be raised to get that number y .

The relations $a^x = y$ and $\log_a y = x$ are equivalent. When one relation is given, it can be converted into the other. Thus

$$a^x = y \Leftrightarrow \log_a y = x$$

Example

Find $\log_4 2$, i.e., find log of 2 to the base 4.

Solution

Let $\log_4 2 = x$

Then its exponential form is $4^x = 2$

i.e., $2^{2x} = 2^1 \Rightarrow 2x = 1$

$\therefore x = \frac{1}{2} \Rightarrow \log_4 2 = \frac{1}{2}$

Deductions from Definition of Logarithm

1. Since $a^0 = 1$, $\log_a 1 = 0$

2. Since $a^1 = a$, $\log_a a = 1$

Common Logarithm or Brigg's Logarithm

If the base of logarithm is taken as 10 then logarithm is called Common Logarithm.

Characteristic

The integral part of the logarithm of any number is called the characteristic.

Characteristic of Logarithm of a number > 1

The characteristic of the logarithm of a number greater than 1 is always one less than the number of digits in the integral part of the number.

When a number b is written in the scientific notation, i.e., in the form $b = a \times 10^n$ where $1 \leq a < 10$, the power of 10 i.e., n will give the characteristic of $\log b$.

Examples

Number	Scientific Notation	Characteristic of the Logarithm
1.02	1.02×10^0	0
99.6	9.96×10^1	1
102	1.092×10^2	2
1662.4	1.6624×10^3	3

Characteristic of Logarithm of a Number < 1

The characteristic of the logarithm of a number less than 1, is always negative and one more than the number of zeroes immediately after the decimal point of the number.

Example

Write the characteristic of the log of following numbers by expressing them in scientific notation and noting the power of 10.

0.872, 0.02, 0.00345

Number	Scientific Notation	Characteristic of the Logarithm
0.872	8.72×10^{-1}	-1
0.02	2.0×10^{-2}	-2
0.00345	3.45×10^{-3}	-3

Mantissa

The fractional part of the logarithm of a number is called the mantissa. Mantissa is always positive

Example

Find the mantissa of the logarithm of 43.254

Solution

Rounding off 43.254 we consider only the four significant digits 4325.

- We first locate the row corresponding to 43 in the log tables and
- Proceed horizontally till we reach the column corresponding to 2. The number at the intersection is 6355.

- Again proceeding horizontally till the mean difference column corresponding to 5 intersects this row, we get the number 5 at the intersection.
- Adding the two numbers 6355 and 5 we get .6360 as the mantissa of the logarithm of 43.25.

Example

Find the mantissa of the logarithm of 0.002347

Solution

Here also, we consider only the four significant digits 2347

We first locate the row corresponding to 23 in the logarithm tables and proceed as before.

Along the same row to its intersection with the column corresponding to 4 the resulting number is 3692. The number at the intersection of this row and the mean difference column corresponding to 7 is 13. Hence the sum of 3692 and 13 gives the mantissa of the logarithm of 0.0023476 as 0.3705

Example

- Find (i) $\log 278.23$
(ii) $\log 0.07058$

Solution

- 278.23 can be rounded off as 278.2

The characteristic is 2 and the mantissa, using log tables, is .4443

$$\therefore \log 278.23 = 2.4443$$

- The characteristic of $\log 0.07058$ is -2 which is written as $\bar{2}$ by convention.

Using log tables the mantissa is .8487, so that

$$\text{Log } 0.07053 = \bar{2}.8487$$

Example

Find the numbers whose logarithms are

$$(i) \quad 1.3247 \quad (ii) \quad \bar{2}.1324$$

Solution

$$(i) \quad 1.3247$$

Reading along the row corresponding to .32 (as mantissa = 0.3247), we get 2109 at the intersection of this row with the column corresponding to 4. The number at the intersection of this row and the mean difference column

corresponding to 7 is 3. Adding 2109 and 3 we get 2112.

Since the characteristic is 1, it is increased by 1 (because there should be two digits in the integral part) and therefore the decimal point is fixed after two digits from left in 2112.

Hence antilog of 1.3247 is 21.12.

$$(ii) \quad \bar{2}.1324$$

Proceeding as in (i) the significant figures corresponding to the mantissa 0.1324 are 1356. Since the characteristic is $\bar{2}$, its numerical value 2 is decreased by 1. Hence there will be one zero after the decimal point.

Hence antilog of $\bar{2}.1324$ is 0.01356.