

Exercise 3.2

Q1. Find the common logarithm of the following numbers.

i) 232.92

232.92 can be rounded off as 232.9

Characteristic = 2

Mantissa = .3672

Hence $\log 232.92 = 2.3672$

ii) 29.326

29.326 can be rounded off as 29.33

Characteristic = 1

Mantissa = .4673

Hence $\log 29.326 = 1.4673$

iii) 0.00032

Characteristic = $\bar{4}$

Mantissa = .5051

Hence $\log 0.0032 = \bar{4}.5051$

iv) 0.3206

Characteristic = $\bar{1}$

Mantissa = .5060

Hence $\log 0.3206 = \bar{1}.5060$

Q2. If $\log 31.09 = 1.4926$, find the values of following:

i) $\log 3.109$

Sol: $\log 3.109$

Characteristic = 0

Mantissa = .4926

So $\log 3.109 = 0.4926$

ii) $\log 310.9$

Sol: $\log 310.9$

Characteristic = 2

Mantissa = .4926

So $\log 310.9 = 2.4926$

iii) $\log 0.003109$

Sol: $\log 0.003109$

Characteristic = $\bar{3}$

Mantissa = .4926

So $\log 0.003109 = \bar{3}.4926$

iv) $\log 0.3109$

Sol: $\log 0.3109$

Characteristic = $\bar{1}$

Mantissa = .4926

So $\log 0.3109 = \bar{1}.4926$

Q3. Find the numbers whose common logarithms are:

i) 3.5621

let the number be x

$\log x = 3.5621$

Characteristic = 3

Mantissa = .5621

$x = \text{antilog } 3.5621 = 3648$

$x = 3648$

Hence 3648 is the required number

ii) $\bar{1}.7427$

Let the number be x

$\log x = \bar{1}.7427$

Characteristic = $\bar{1}$

Mantissa = .7427

$x = \text{antilog } \bar{1}.7427 = 0.5530$

$x = 0.5530$

Hence 0.5530 is the required number

Q4. What replacement for the unknown in each of following will make the statement true?

i) $\log_3 81 = L$

In exponential form

$$3^L = 81$$

$$3^L = 3^4$$

$\Rightarrow \boxed{L=4}$ Bases are equal so exponents are equal

ii) $\log_a 6 = 0.5$

In exponential form

$$a^{0.5} = 6$$

$$a^{\frac{1}{2}} = 6$$

Squaring both side

$$\left(a^{\frac{1}{2}}\right)^2 = (6)^2$$

$$\boxed{a = 36}$$

iii) $\log_5 n = 2$

In exponential form

$$5^2 = n$$

$$\Rightarrow \boxed{n = 25}$$

iv) $10^p = 40$

In logarithmic form

$$\log_{10} 40 = p$$

or $\log 40 = p$

$$\text{Characteristic} = 1$$

$$\text{Mantissa} = .6021$$

$$\text{So, } p = 1.6021$$

Q5. Evaluate

i) $\log_2 \frac{1}{128}$

$$\text{Let } x = \log_2 \frac{1}{128}$$

In exponential form

$$2^x = \frac{1}{128}$$

$$2^x = \frac{1}{2^7}$$

$$2^x = 2^{-7}$$

$$\Rightarrow \boxed{x = -7}$$

ii) $\log 512$ to the base $2\sqrt{2}$

Sol: $\log_{2\sqrt{2}} 512$

$$\text{Let } x = \log_{2\sqrt{2}} 512$$

In exponential form

$$(2\sqrt{2})^x = 512$$

$$\left(2 \times 2^{\frac{1}{2}}\right)^x = 2^9$$

$$\left(2^{1+\frac{1}{2}}\right)^x = 2^9$$

$$\left(2^{\frac{3}{2}}\right)^x = 2^9$$

$$2^{\frac{3}{2}x} = 2^9$$

$$\Rightarrow \frac{3}{2}x = 9$$

$$x = 9 \times \frac{2}{3}$$

$$\boxed{x = 6}$$

Q6. Evaluate the value of 'x' from the following statements.

i) $\log_2 x = 5$

In exponential form

$$2^5 = x$$

$$\Rightarrow \boxed{x = 32}$$

ii) $\log_{81} 9 = x$

In exponential form

$$81^x = 9$$

$$(9^2)^x = 9$$

$$9^{2x} = 9^1$$

$$\Rightarrow 2x = 1$$

or $\boxed{x = \frac{1}{2}}$

iii) $\log_{64} 8 = \frac{x}{2}$

In exponential form

$$(64)^{\frac{x}{2}} = 8$$

$$(8^2)^{\frac{x}{2}} = 8$$

$$8^{2 \times \frac{x}{2}} = 8$$

$$8^x = 8^1$$

$$\Rightarrow \boxed{x = 1}$$

iv) $\log_x 64 = 2$

In exponential form

$$x^2 = 64$$

$$x^2 = 8^2$$

$$\Rightarrow \boxed{x = 8}$$

v) $\log_3 x = 4$

In exponential form

$$3^4 = x$$

$$\Rightarrow \boxed{x = 81}$$

Laws of Logarithm

In this section we shall prove the laws of logarithm and then apply them to find products, quotients, powers and roots of numbers.

(i) $\log_a(mn) = \log_a m + \log_a n$

(ii) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

(iii) $\log_a m^n = n \log_a m$

(iv) $\log_a n = \log_b n \times \log_a b$

or $= \frac{\log_b n}{\log_b a}$

(i) $\log_a(mn) = \log_a m + \log_a n$

Proof

Let $\log_a m = x$ and $\log_a n = y$

Writing in exponential form

$$a^x = m \text{ and } a^y = n$$

$$\therefore a^x \times a^y = mn$$

$$\text{i.e., } a^{x+y} = mn$$

$$\text{or } \log_a(mn) = x + y = \log_a m + \log_a n$$

$$\text{Hence } \log_a(mn) = \log_a m + \log_a n$$

Note

$$(i) \log_a(mn) \neq \log_a m \times \log_a n$$

$$(ii) \log_a m + \log_a n \neq \log_a(m + n)$$

$$(iii) \log_a(mnp) = \log_a m + \log_a n + \log_a p + \dots$$

The rule given above is useful in finding the product of two or more numbers using logarithms.

Example

$$\text{Evaluate } 291.3 \times 42.36$$

Solution

$$\text{Let } x = 291.3 \times 42.36$$

$$\text{Then } \log x = \log(291.3 \times 42.36)$$

$$= \log 291.3 + \log 42.36$$

$$(\log_a mn = \log_a m + \log_a n)$$

$$= 2.4643 + 1.6269 = 4.0912$$

$$x = \text{antilog } 4.0912 = 12340$$

Example

$$\text{Evaluate } 0.2913 \times 0.004236.$$

Solution

$$\text{Let } y = 0.2913 \times 0.004236$$

$$\text{Then } \log y = \log 0.2913 + \log 0.004236$$

$$\log y = \bar{1}.4643 + \bar{3}.6269$$

$$\log y = \bar{3}.0912$$

$$y = \text{anti log } \bar{3}.0912$$

$$y = 0.001234$$

$$(ii) \log_a \left(\frac{m}{n} \right) = \log_a m - \log_a n$$

Proof

$$\text{Let } \log_a m = x \text{ and } \log_a n = y$$

$$\text{So that } a^x = m \text{ and } a^y = n$$

$$\therefore \frac{a^x}{a^y} = \frac{m}{n} \Rightarrow a^{x-y} = \frac{m}{n}$$

i.e.,

$$\log_a \left(\frac{m}{n} \right) = x - y = \log_a m - \log_a n$$

$$\text{Hence } \log_a \left(\frac{m}{n} \right) = \log_a m - \log_a n$$

Note

$$(i) \log_a \left(\frac{m}{n} \right) \neq \frac{\log_a m}{\log_a n}$$

$$(ii) \log_a m - \log_a n \neq \log_a(m - n)$$

$$(iii) \log_a \left(\frac{1}{n} \right) = \log_a 1 - \log_a n = -\log_a n \dots$$

$$(\because \log_a 1 = 0)$$

Example

$$\text{Evaluate } \frac{291.3}{42.36}$$

Solution

$$\text{Let } x = \frac{291.3}{42.36} \text{ so that } \log x = \log \frac{291.3}{42.36}$$

$$\text{Then } \log x = \log 291.3 - \log 42.36, \dots$$

$$(\log_a \frac{m}{n} = \log_a m - \log_a n)$$

$$\log x = 2.4643 - 1.6269 = 0.8374$$

$$\text{Thus } x = \text{antilog } 0.8374 = 6.877$$

Example

$$\text{Evaluate } \frac{0.0002913}{0.04236}$$

Solution

$$\text{Let } y = \frac{0.0002913}{0.04236} \text{ so that}$$

$$\log y = \log \left(\frac{0.0002913}{0.04236} \right)$$

$$\text{then } \log y = \log 0.0002913 - \log 0.04236$$

$$\begin{aligned}
 \log y &= \bar{3}.4643 - \bar{2}.6269 \\
 &= \bar{3} + (0.4643 - 0.6269) - \bar{2} \\
 &= \bar{3} - 0.1626 - \bar{2} \\
 &= \bar{3} + (1 - 0.1626) - 1 - \bar{2}, \\
 &\quad (\text{adding and subtracting 1}) \\
 &= \bar{2}.8374 \\
 [\therefore \bar{3} - 1 - \bar{2} &= -3 - 1 - (-2) = -2 = \bar{2}] \\
 \text{Therefore, } y &= \text{antilog } \bar{2}.8374 \\
 y &= 0.06877
 \end{aligned}$$

(iii) $\log_a(m^n) = n \log_a m$:

Proof

$$\begin{aligned}
 \text{Let } \log_a m^n &= x, \quad \text{i.e., } a^x = m^n \\
 \text{and } \log_a m &= y, \quad \text{i.e., } a^y = m \\
 \text{Then } a^x &= m^n = (a^y)^n \\
 \text{i.e., } a^x &= (a^y)^n = a^{yn} \Rightarrow x = ny \\
 \text{i.e., } \log_a m^n &= n \log_a m
 \end{aligned}$$

Example

Evaluate $\sqrt[4]{(0.0163)^3}$

Solution

$$\begin{aligned}
 \text{Let } y &= \sqrt[4]{(0.0163)^3} = (0.0163)^{3/4} \\
 \log y &= \frac{3}{4} (\log 0.0163) \\
 &= \frac{3}{4} \times \bar{2}.2122 \\
 &= \frac{\bar{6}.6366}{4} \\
 &= \frac{\bar{8} + 2.6366}{4} \\
 &= \bar{2} + 0.6592 = \bar{2}.6592 \\
 \text{Hence } y &= \text{antilog } \bar{2}.6592 \\
 &= 0.04562
 \end{aligned}$$

(iv) **Change of Base Formula:**

$$\log_a n = \log_b n \times \log_a b \quad \text{or} \quad \frac{\log_b n}{\log_b a}$$

Proof

$$\text{Let } \log_b n = x \quad \text{so that} \quad n = b^x$$

Taking log to the base a , we have

$$\log_a n = \log_a b^x = x \log_a b = \log_b n \log_a b$$

$$\text{Thus } \log_a n = \log_b n \log_a b \dots\dots (i)$$

Putting $n = a$ in the above result, we get

$$\log_b a \times \log_a b = \log_a a = 1$$

$$\text{or } \log_a b = \frac{1}{\log_b a}$$

Hence equation (i) gives

$$\log_a n = \frac{\log_b n}{\log_b a} \dots\dots (ii)$$

Using the above rule, a natural logarithm can be converted to a common logarithm and vice versa.

$$\log_e n = \log_{10} n \times \log_e 10 \quad \text{or} \quad \frac{\log_{10} n}{\log_{10} e}$$

$$\log_{10} n = \log_e n \times \log_{10} e \quad \text{or} \quad \frac{\log_e n}{\log_e 10}$$

The values of $\log_e 10$ and $\log_{10} e$ are available from the tables:

$$\log_e 10 = \frac{1}{0.4343} = 2.3026 \quad \text{and}$$

$$\log_{10} e = \log 2.718 = 0.4343$$

Example

$$\text{Calculate } \log_2 3 \times \log_3 8$$

Solution

We know that

$$\log_a n = \frac{\log_b n}{\log_b a}$$

$$\therefore \log_2 3 \times \log_3 8 = \frac{\log 3}{\log 2} \times \frac{\log 8}{\log 3}$$

$$= \frac{\log 8}{\log 2} = \frac{\log 2^3}{\log 2}$$

$$= \frac{3 \log 2}{\log 2} = 3$$