

Exercise 3.3

Q1. Write the following into sum or difference.

i) $\log(A \times B)$

Sol: $\log(A \times B) = \log A + \log B$

ii) $\log \frac{15.2}{30.5}$

Sol: $\log \frac{15.2}{30.5} = \log 15.2 - \log 30.5$

iii) $\log \frac{21 \times 5}{8}$

Sol: $\log \frac{21 \times 5}{8} = \log 21 + \log 5 - \log 8$

iv) $\log \sqrt[3]{\frac{7}{15}}$

Sol: $\log \sqrt[3]{\frac{7}{15}} = \log \left(\frac{7}{15} \right)^{\frac{1}{3}} = \frac{1}{3} \log \left(\frac{7}{15} \right)$
 $= \frac{1}{3} (\log 7 - \log 15)$

v) $\log \frac{(22)^{\frac{1}{3}}}{5^3}$

Sol: $\log \frac{(22)^{\frac{1}{3}}}{5^3} = \log (22)^{\frac{1}{3}} - \log 5^3$
 $= \frac{1}{3} \log 22 - 3 \log 5$

vi) $\log \frac{25 \times 47}{29}$

$= \log 25 + \log 47 - \log 29$

Q2. Express

$\log x - 2 \log x + 3 \log(x+1) - \log(x^2-1)$
as a single logarithm

Sol:

$$\begin{aligned} & \log x - 2 \log x + 3 \log(x+1) - \log(x^2-1) \\ &= \log x - \log x^2 + \log(x+1)^3 - \log(x^2-1) \\ &= \log x + \log(x+1)^3 - \log x^2 - \log(x^2-1) \\ &= \log \frac{x(x+1)^3}{x^2(x^2-1)} \\ &= \log \frac{(x+1)^3}{x(x-1)(x+1)} \\ &= \log \frac{(x+1)^2}{x(x-1)} \end{aligned}$$

Q3. Write the following in the form of a single logarithm.

i) $\log 21 + \log 5$

Sol: $\log 21 + \log 5$
 $= \log 21 \times 5$

ii) $\log 25 - 2 \log 3$
 $= \log 25 - \log 3^2$
 $= \log \frac{25}{3^2} = \log \frac{25}{9}$

iii) $2 \log x - 3 \log y$

Sol: $2 \log x - 3 \log y$
 $= \log x^2 - \log y^3$
 $= \log \frac{x^2}{y^3}$

iv) $\log 5 + \log 6 - \log 2$

Sol: $\log 5 + \log 6 - \log 2$
 $= \log \frac{5 \times 6}{2}$

Q4. Calculate the following:

i) $\log_3 2 \times \log_2 81$

Sol: As we know that $\log_a n = \frac{\log_b n}{\log_b a}$

$$\therefore \log_3 2 \times \log_2 81 = \frac{\log 2}{\log 3} \times \frac{\log 81}{\log 2}$$

$$= \frac{\log 81}{\log 3}$$

$$= \frac{\log 3^4}{\log 3}$$

$$= \frac{4 \log 3}{\log 3}$$

$$= 4$$

ii) $\log_5 3 \times \log_3 25$

Sol: As we know that

$$\log_a n = \frac{\log_b n}{\log_b a}$$

$$\log_5 3 \times \log_3 25 = \frac{\log 3}{\log 5} \times \frac{\log 25}{\log 3}$$

$$= \frac{\log 25}{\log 5}$$

$$= \frac{\log 5^2}{\log 5}$$

$$= \frac{2 \log 5}{\log 5}$$

$$= 2$$

Q5. If $\log 2 = 0.3010$, $\log 3 = 0.4771$,

$\log 5 = 0.6990$, then find the values of the following.

i) $\log 32$

Sol: $\log 32$

$$= \log 2^5$$

$$= 5 \log 2$$

$$= 5(0.3010)$$

$$= 1.5050$$

ii) $\log 24$

$$= \log 8 \times 3$$

$$= \log 2^3 \times 3$$

$$= \log 2^3 + \log 3$$

$$= 3 \log 2 + \log 3$$

$$= 3(0.3010) + 0.4771$$

$$= 0.9030 + 0.4771$$

$$= 1.3801$$

iii) $\log \sqrt{3 \frac{1}{3}}$

$$= \log \sqrt{\frac{10}{3}}$$

$$= \log \left(\frac{2 \times 5}{3} \right)^{\frac{1}{2}}$$

$$= \frac{1}{2} \log \left(\frac{2 \times 5}{3} \right) = \frac{1}{2} (\log 2 + \log 5 - \log 3)$$

$$= \frac{1}{2} (0.3010 + 0.6990 - 0.4771)$$

$$= \frac{1}{2} (0.5229)$$

$$= 0.2615$$

iv) $\log \frac{8}{3}$

$$= \log \frac{2^3}{3}$$

$$= \log 2^3 - \log 3$$

$$= 3 \log 2 - \log 3$$

$$= 3(0.3010) - 0.4771$$

$$= 0.4259$$

v) $\log 30$

$$= \log 2 \times 3 \times 5$$

$$= \log 2 + \log 3 + \log 5$$

$$= 0.3010 + 0.4771 + 0.6990$$

$$= 1.4771$$

Applications of logarithm

Example

Show that

$$7 \log \frac{16}{15} + 5 \log \frac{25}{24} + \log \frac{81}{80} = \log 2.$$

Solution

$$\begin{aligned} \text{L.H.S} &= 7 \log \frac{16}{15} + 5 \log \frac{25}{24} + \log \frac{81}{80} \\ &= 7[\log 16 - \log 15] + 5[\log 25 - \log 24] \\ &\quad + 3[\log 81 - \log 80] \\ &= 7[\log 2^4 - \log (3 \times 5)] + 5[\log 5^2 - \log (2^3 \times 3)] \\ &\quad + 3[\log 3^4 - \log (2^4 \times 5)] \\ &= 7[4 \log 2 - \log 3 - \log 5] + 5[2 \log 5 - 3 \log 2 - \log 3] \\ &\quad + 3[4 \log 3 - 4 \log 2 - \log 5] \\ &= (28 - 15 - 12) \log 2 + (-7 - 5 + 12) \log 3 \\ &\quad + (-7 + 10 - 3) \log 5 \\ &= \log 2 + 0 + 0 = \log 2 = \text{R.H.S} \end{aligned}$$

Example

Evaluate:

$$\sqrt[3]{\frac{0.07921 \times (18.99)^2}{(5.79)^4 \times 0.9474}}$$

Let $y =$

$$\sqrt[3]{\frac{0.07921 \times (18.99)^2}{(5.79)^4 \times 0.9474}} =$$

$$\left(\frac{0.07921 \times (18.99)^2}{(5.79)^4 \times 0.9474} \right)^{1/3}$$

Log $y =$

$$\frac{1}{3} \log \left(\frac{0.07921 \times (18.99)^2}{(5.79)^4 \times 0.9474} \right)$$

$$\begin{aligned} &= \frac{1}{3} [\log \{0.07921 \times (18.99)^2\} - \log \{(5.79)^4 \times 0.9474\}] \\ &= \frac{1}{3} [\log 0.07921 + 2 \log 18.99 - 4 \log 5.79 - \log 0.9474] \\ &= \frac{1}{3} [2.8988 + 2(1.2786) - 4(0.7627) - 1.9765] \\ &= \frac{1}{3} [2.8988 + 2.5572 - 3.0508 - 1.9765] \\ &= \frac{1}{3} [-2 + 0.8988 + 2.5572 - 3.0508 + 1 - 0.9765] \\ &= \frac{1}{3} (-2.4287) \end{aligned}$$

$$= \frac{1}{3} (-3 + 1.4287)$$

$$= -1 + 0.4762 = -1.4762$$

$$y = \text{antilog } -1.4762 = 0.2993$$

Example

Given $A = A_0 e^{-kd}$. If $k = 2$, what should be the value of d to make $A = \frac{A_0}{2}$?

Solution

$$\text{Given that } A = A_0 e^{-kd} \Rightarrow$$

$$\frac{A}{A_0} = e^{-kd}$$

$$\text{Substituting } k = 2 \text{ and } A = \frac{A_0}{2},$$

$$\text{we get } \frac{1}{2} = e^{-2d}$$

Taking common log on both sides,

$$\log_{10} 1 - \log_{10} 2 = -2d \log_{10} e,$$

where $e = 2.718$

$$0 - 0.3010 = -2d (0.4343)$$

$$d = \frac{0.3010}{2 \times 0.4343} = 0.3465$$

