

Exercise 3.4

Q1. Use log tables to find the values of

i) 0.8176×13.64

Sol: Let $x = 0.8176 \times 13.64$
Taking log of both sides
 $\log x = \log 0.8176 \times 13.64$

$$\begin{aligned}\log x &= \log 0.8176 + \log 13.64 \\ &= \bar{1}.9125 + 1.1348 \\ &= -1 + 0.9125 + 1.1348\end{aligned}$$

$$\log x = 1.0473$$

$$\begin{aligned}\text{Characteristics} &= -1 \\ \text{Mantissa} &= .0473 \\ x = \text{antilog } 1.0473 &= 11.15\end{aligned}$$

ii) $(789.5)^{\frac{1}{8}}$

Sol: Let $x = (789.5)^{\frac{1}{8}}$
Taking log of both sides

$$\begin{aligned}\log x &= \log (789.5)^{\frac{1}{8}} \\ &= \frac{1}{8} \log (789.5) \\ &= \frac{1}{8} (2.8974)\end{aligned}$$

$$\log x = 0.3622$$

$$\begin{aligned}\text{Characteristics} &= 0 \\ \text{Mantissa} &= .3622 \\ x = \text{antilog } 0.3622 &= 2.302\end{aligned}$$

iii) $\frac{0.678 \times 9.01}{0.0234}$

$$\text{Let } x = \frac{0.678 \times 9.01}{0.0234}$$

Taking log of both sides

$$\log x = \log \frac{0.678 \times 9.01}{0.0234}$$

$$= \log 0.678 + \log 9.01 - \log 0.0234$$

$$= \bar{1}.8312 + 0.9547 - (\bar{2}.3692)$$

$$= -1 + 0.8312 + 0.9547 - (-2 + 0.3692)$$

$$= -1 + 0.8312 + 0.9547 + 2 - 0.3692$$

$$\log x = 2.4167$$

$$\text{Characteristics} = 2$$

$$\text{Mantissa} = .4167$$

$$x = \text{antilog } 2.4167 = 261.0$$

iv) $\sqrt[3]{2.709} \times \sqrt[7]{1.239}$

Sol: Let $x = \sqrt[3]{2.709} \times \sqrt[7]{1.239}$

Taking log of both sides

$$\log x = \log (2.709)^{\frac{1}{3}} \times (1.239)^{\frac{1}{7}}$$

$$= \log (2.709)^{\frac{1}{3}} + \log (1.239)^{\frac{1}{7}}$$

$$= \frac{1}{3} \log (2.709) + \frac{1}{7} \log (1.239)$$

$$= \frac{1}{3} (0.4328) + \frac{1}{7} (0.0931)$$

$$= 0.0866 + 0.0133$$

$$\log x = 0.0999$$

$$\text{Characteristics} = 0$$

$$\text{Mantissa} = .0999$$

$$x = \text{antilog } 0.0999$$

$$x = 1.259$$

v) $\frac{(1.23)(0.6975)}{(0.0075)(1278)}$

Sol: Let $x = \frac{(1.23)(0.6975)}{(0.0075)(1278)}$

Taking log of both sides

$$\log x = \log \frac{(1.23)(0.6975)}{(0.0075)(1278)}$$

$$= \log 1.23 + \log 0.6975 - \log 0.0075 - \log 1278$$

$$= 0.0899 + \bar{1}.8435 - \bar{3}.8751 - 3.1065$$

$$= 0.0899 - 1 + 0.8435 + 3 - 0.8751 - 3.1065$$

$$\log x = -1.0482$$

$$= -2 + 2 - 1.0482$$

$$= -2 + 0.9518$$

$$\log x = \bar{2}.9518$$

$$\text{Characteristics} = \bar{2}$$

$$\text{Mantissa} = .9518$$

$$x = \text{antilog } \bar{2}.9518 = 0.0895$$

$$\text{vi)} \quad \sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$$

$$\text{Let } x = \sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$$

$$x = \left(\frac{0.7214 \times 20.37}{60.8} \right)^{\frac{1}{3}}$$

Taking log of both sides

$$\log x = \log \left(\frac{0.7214 \times 20.37}{60.8} \right)^{\frac{1}{3}}$$

$$= \frac{1}{3} \log \left(\frac{0.7214 \times 20.37}{60.8} \right)$$

$$= \frac{1}{3} (\log 0.7214 + \log 20.37 - \log 60.8)$$

$$= \frac{1}{3} (\bar{1}.8582 + 1.3090 - 1.7839)$$

$$= \frac{1}{3} (-1 + 0.8582 + 1.3090 - 1.7839)$$

$$= \frac{1}{3} (-0.6167)$$

$$\log x = -0.2056$$

$$= -1 + 1 - 0.2056$$

$$= -1 + 0.7944$$

$$\log x = \bar{1}.7944$$

$$\text{Characteristics} = \bar{1}$$

$$\text{Mantissa} = .7944$$

$$x = \text{antilog } \bar{1}.7944$$

$$= 0.6229$$

$$\text{vii)} \quad \frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$$

$$\text{Sol: Let } x = \frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$$

$$x = \frac{83 \times (92)^{\frac{1}{3}}}{127 \times (246)^{\frac{1}{5}}}$$

Taking log of both sides

$$\log x = \log \frac{83 \times (92)^{\frac{1}{3}}}{127 \times (246)^{\frac{1}{5}}}$$

$$= \log 83 + \log (92)^{\frac{1}{3}} - \log 127 - \log (246)^{\frac{1}{5}}$$

$$= \log 83 + \frac{1}{3} \log (92) - \log 127 - \frac{1}{5} \log (246)$$

$$= 1.9191 + \frac{1}{3} (1.9638) - 2.1038 - \frac{1}{5} (2.391)$$

$$= 1.9191 + 0.6546 - 2.1038 - 0.4782$$

$$\log x = -0.0083$$

$$= -1 + 1 - 0.0083$$

$$= -1 + 0.9917$$

$$\log x = \bar{1}.9917$$

$$\text{Characteristics} = \bar{1}$$

$$\text{Mantissa} = .9917$$

$$x = \text{antilog } \bar{1}.9917 = 0.9811$$

$$\text{viii)} \quad \frac{(438)^3 \sqrt{0.056}}{(388)^4}$$

$$\text{Sol: Let } x = \frac{(438)^3 \sqrt{0.056}}{(388)^4}$$

$$x = \frac{(438)^3 \times (0.056)^{\frac{1}{2}}}{(388)^4}$$

Taking log of both sides

$$\begin{aligned}\log x &= \log \frac{(438)^3 \times (0.056)^{\frac{1}{2}}}{(388)^4} \\&= \log (438)^3 + \log (0.056)^{\frac{1}{2}} - \log (388)^4 \\&= 3 \log (438) + \frac{1}{2} \log (0.056) - 4 \log (388) \\&= 3(2.6415) + \frac{1}{2}(\bar{2}.7482) - 4(2.5888) \\&= 3(2.6415) + \frac{1}{2}(-2 + 0.7482) - 4(2.5888) \\&= 7.9245 + \frac{1}{2}(-1.2518) - 10.3552 \\&= 7.9245 - 0.6259 - 10.3552 \\&\log x = -3.0566 \\&= -4 + 4 - 3.0566 \\&= -4 + 0.9434 \\&\log x = \bar{4}.9434\end{aligned}$$

$$\begin{aligned}\text{Characteristic} &= \bar{4} \\ \text{Mantissa} &= .9434\end{aligned}$$

$$x = \text{antilog } \bar{4}.9434 = 0.0008778$$

Q2. A gas is expanding according to the law $PV^n = C$. Find C when $P=80$, $V=3.1$

and $n = \frac{5}{4}$.

Sol: $PV^n = C$

Taking log of both sides:

$$\begin{aligned}\log PV^n &= \log C \\ \log P + \log V^n &= \log C \\ \log C &= \log P + n \log V\end{aligned}$$

Putting $P = 80$, $V = 3.1$ and $n = \frac{5}{4}$

$$\log C = \log 80 + \frac{5}{4} \log 3.1$$

$$= 1.9031 + \frac{5}{4}(0.4914)$$

$$= 1.9031 + 0.6143$$

$$\log C = 2.5174$$

$$\text{Characteristic} = 2$$

$$\text{Mantissa} = .5174$$

$$C = \text{antilog } 2.5174$$

$$C = 329.2 \text{ unit}$$

Q3. The formula $p = 90(5)^{-\frac{q}{10}}$ applies to the demand of a product, where 'q' is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs. 18.00?

Sol: $p = 90(5)^{-\frac{q}{10}}$
 $q = ?$ and $p = \text{Rs. } 18.00$

As $p = 90(5)^{-\frac{q}{10}}$

$$18 = 90(5)^{-\frac{q}{10}}$$

Taking log of both sides

$$\log 18 = \log 90(5)^{-\frac{q}{10}}$$

$$\log 18 = \log 90 + \log (5)^{-\frac{q}{10}}$$

$$\log 18 - \log 90 = \frac{-q}{10} \log 5$$

$$10(\log 18 - \log 90) = -q \log 5$$

$$10(1.2553 - 1.9542) = -q(0.6990)$$

$$-6.989 = -q(0.6990)$$

$$\Rightarrow q(0.6990) = 6.989$$

$$q = \frac{6.989}{0.6990}$$

$$q = 9.998$$

$$q = 10 \text{ approximately}$$

So 10 units will be demanded
OR

$$p = 90 (5)^{-\frac{q}{10}}$$

Taking log of both sides

$$\log p = \log 90 (5)^{-\frac{q}{10}}$$

$$\log p = \log 90 + \log (5)^{-\frac{q}{10}}$$

$$\log p = \log 90 - \frac{q}{10} \log 5$$

$$\frac{q}{10} \log 5 = \log 90 - \log p$$

$$\frac{q}{10} \log 5 = \log 90 - \log 18$$

$$\frac{q}{10} \log 5 = \log \frac{90}{18}$$

$$\frac{q}{10} \log 5 = \log 5$$

$$\frac{q}{10} = \frac{\log 5}{\log 5}$$

$$\frac{q}{10} = 1$$

$$q = 10 \text{ Units}$$

Q4. If $A = \pi r^2$

$$\pi = \frac{22}{7}, r = 15, A = ?$$

$$\text{As } A = \pi r^2$$

Taking log of both sides

$$\log A = \log \pi r^2$$

$$= \log \pi + \log r^2$$

$$= \log \pi + 2 \log r$$

$$= \log \frac{22}{7} + 2 \log 15$$

$$= \log 22 - \log 7 + 2 \log 15$$

$$= 1.3424 - 0.8451 + 2(1.1761)$$

$$= 1.3424 - 0.8451 + 2.3522$$

$$\log A = 2.8495$$

$$\text{Characteristics} = 2$$

$$\text{Mantissa} = .8495$$

$$A = \text{antilog } 2.8495$$

$$A = 707.1$$

Q5. If $v = \frac{1}{3} \pi r^2 h$, find v when

$$\pi = \frac{22}{7}, r = 2.5 \text{ and } h = 4.2$$

$$\text{Sol: } v = \frac{1}{3} \pi r^2 h$$

$$\pi = \frac{22}{7}, r = 2.5 \text{ and } h = 4.2, v = ?$$

$$\text{As } v = \frac{1}{3} \pi r^2 h$$

Taking log of both sides

$$\log v = \log \frac{1}{3} \pi r^2 h$$

$$= \log \frac{1}{3} + \log \pi + \log r^2 + \log h$$

$$= \log \frac{1}{3} + \log \frac{22}{7} + 2 \log r + \log h$$

$$= \log 1 - \log 3 + \log 22 - \log 7 + 2 \log 2.5 + \log 4.2$$

$$= 0 - 0.4771 + 1.3424 - 0.8451 + 2(0.3979) + 0.6232$$

$$\log v = 1.4392$$

$$\text{Characteristics} = 1$$

$$\text{Mantissa} = .4392$$

$$v = \text{antilog } 1.4392$$

$$v = 27.49$$

Review Exercise 3

Q3. Find the value of 'x' in the following.

i) $\log_3 x = 5$

Sol. $\log_3 x = 5$
In exponential form

$$x = 3^5$$

$\Rightarrow x = 243$

ii) $\log_4 256 = x$

Sol. $\log_4 256 = x$
In exponential form

$$4^x = 256$$

$$4^x = 4^4$$

$\Rightarrow x = 4$

iii) $\log_{625} 5 = \frac{1}{4}x$

Sol. $\log_{625} 5 = \frac{1}{4}x$
In exponential form

$$(625)^{\frac{1}{4}x} = 5$$

$$(5^4)^{\frac{1}{4}x} = 5$$

$$5^{4 \times \frac{1}{4}x} = 5$$

$$5^x = 5^1$$

$\Rightarrow x = 1$

iv) $\log_{64} x = -\frac{2}{3}$

Sol. $\log_{64} x = -\frac{2}{3}$

In exponential form

$$x = 64^{-\frac{2}{3}}$$

$$x = (4^3)^{-\frac{2}{3}}$$

$$= 4^{\cancel{3} \left(-\frac{2}{\cancel{3}} \right)}$$

$$x = 4^{-2}$$

$$x = \frac{1}{4^2}$$

$$x = \frac{1}{16}$$

Q4. Find the value of 'x' in the following.

i) $\log x = 2.4543$

Characteristic = 2

Mantissa = .4543

$x = \text{antilog } 2.4543$
 $= 284.6$

ii) $\log x = 0.1821$

Characteristic = 0

Mantissa = .1821

$x = \text{antilog } 0.1821$
 $= 1.521$

iii) $\log x = 0.0044$

Characteristic = 0

Mantissa = .0044

$x = \text{antilog } 0.0044$
 $x = 1.010$

iv) $\log x = \bar{1}.6238$

Characteristic = $\bar{1}$

Mantissa = .6238

$x = \text{antilog } \bar{1}.6238$
 $x = 0.4205$

Q5. If $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 5 = 0.6990$, then find the values of the following.

i) $\log 45$

Sol. $\log 45$

$$\begin{aligned}
 &= \log 3^2 \times 5 \\
 &= \log 3^2 + \log 5 \\
 &= 2\log 3 + \log 5 \\
 &= 2(0.4771) + 0.6990 \\
 &= 0.9542 + 0.6990 \\
 &= 1.6532
 \end{aligned}$$

$$\begin{aligned}
 \text{ii)} \quad &\log \frac{16}{15} \\
 &= \log \frac{2^4}{3 \times 5} \\
 &= \log 2^4 - \log 3 - \log 5 \\
 &= 4\log 2 - \log 3 - \log 5 \\
 &= 4(0.3010) - 0.4771 - 0.6990 \\
 &= 1.2040 - 0.4771 - 0.6990 \\
 &= 0.0279
 \end{aligned}$$

$$\begin{aligned}
 \text{iii)} \quad &\log 0.048 \\
 &= \log \frac{48}{1000} \\
 &= \log \frac{16 \times 3}{10^3} \\
 &= \log \frac{2^4 \times 3}{2^3 \times 5^3} \\
 &= \log \frac{2 \times 3}{5^3} \\
 &= \log 2 + \log 3 - \log 5^3 \\
 &= \log 2 + \log 3 - 3\log 5 \\
 &= 0.3010 + 0.4771 - 3(0.6990) \\
 &= -1.3189 \\
 &= -2 + 2 - 1.3189 \\
 &= -2 + 0.6811 \\
 &= \bar{2}.6811
 \end{aligned}$$

Q6. Simplify the following:

$$\text{i)} \quad \sqrt[3]{25.47}$$

$$\begin{aligned}
 \text{Sol.} \quad &\text{Let } x = (25.47)^{\frac{1}{3}} \\
 &\text{Taking log of both sides} \\
 &\log x = \log (25.47)^{\frac{1}{3}} \\
 &= \frac{1}{3} \log (25.47) \\
 &= \frac{1}{3} (1.4060)
 \end{aligned}$$

$$\begin{aligned}
 &\log x = 0.4687 \\
 &\text{Characteristic} = 0 \\
 &\text{Mantissa} = .4687 \\
 &x = \text{antilog } 0.4687 \\
 &x = 2.942
 \end{aligned}$$

$$\text{ii)} \quad \sqrt[5]{342.2}$$

$$\begin{aligned}
 \text{Sol.} \quad &\text{Let } x = (342.2)^{\frac{1}{5}} \\
 &\text{Taking log of both sides} \\
 &\log x = \log (342.2)^{\frac{1}{5}} \\
 &= \frac{1}{5} \log (342.2) \\
 &= \frac{1}{5} (2.5343)
 \end{aligned}$$

$$\begin{aligned}
 &\log x = 0.5069 \\
 &\text{Characteristic} = 0 \\
 &\text{Mantissa} = .5069 \\
 &x = \text{antilog } 0.5069 \\
 &x = 3.213
 \end{aligned}$$

$$\text{iii)} \quad \frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$$

$$\begin{aligned}
 \text{Sol:} \quad &\text{Let } x = \frac{(8.97)^3 \times (3.95)^2}{(15.37)^{\frac{1}{3}}} \\
 &\text{Taking log of both sides} \\
 &\log x = \log \frac{(8.97)^3 \times (3.95)^2}{(15.37)^{\frac{1}{3}}}
 \end{aligned}$$

$$\begin{aligned}
 &= \log(8.97)^3 + \log(3.95)^2 - \log(15.37)^{\frac{1}{3}} \\
 &= 3\log(8.97) + 2\log(3.95) - \frac{1}{3}\log(15.37) \\
 &= 3(0.9528) + 2(0.5966) - \frac{1}{3}(1.1867)
 \end{aligned}$$

$$= 2.8584 + 1.1932 - 0.3956$$

$$\log x = 3.6560$$

$$\text{Characteristic} = 3$$

$$\text{Mantissa} = .6560$$

$$x = \text{antilog } 3.6560$$

$$x = 4529$$

Objective

- If $a^x = n$, then ____
 (a) $a = \log_x n$ (b) $x = \log_n a$
 (c) $x = \log_a n$ (d) $a = \log_n x$
- The relation of $y = \log_z x$ implies
 (a) $x^y = z$ (b) $z^y = x$
 (c) $x^z = y$ (d) $y^z = x$
- The logarithm of unity to any base is ____
 (a) 1 (b) 10
 (c) e (d) 0
- The logarithm of any number to itself as base is ____
 (a) 1 (b) 0
 (c) -1 (d) 10
- $\log e = \underline{\hspace{1cm}}$ where $e \approx 2.718$
 (a) 0 (b) 0.4343
 (c) ∞ (d) 1
- The value of $\log\left(\frac{p}{q}\right)$ is ____
 (a) $\log p - \log q$
 (b) $\frac{\log p}{\log q}$
 (c) $\log p + \log q$
 (d) $\log q - \log p$
- $\log m^n$ can be written as
 (a) $(\log m)^n$ (b) $m \log n$
 (c) $n \log m$ (d) $\log(mn)$
- $\log_b a \times \log_c b$ can be written as ____
 (a) $\log_c a$ (b) $\log_a c$
 (c) $\log_a b$ (d) $\log_b c$
- $\text{Log}_y x$ will be equal to ____
 (a) $\frac{\log_z x}{\log_y z}$ (b) $\frac{\log_x z}{\log_y z}$
 (c) $\frac{\log_z x}{\log_z y}$ (d) $\frac{\log_z y}{\log_z x}$
- For common logarithm, the base is ____
 (a) 2 (b) 10
 (c) e (d) 1
- For natural logarithm, the base is ____
 (a) 10 (b) e
 (c) 2 (d) 1
- The integral part of the common logarithm of a number is called the ____
 (a) Characteristic (b) Mantissa
 (c) Logarithm (d) None
- The decimal part of the common logarithm of a number is called the ____:
 (a) Characteristic (b) Mantissa
 (c) Logarithm (d) None

14. If $x = \log y$, then y is called the ____ of x .
 (a) Antilogarithm (b) Logarithm
 (c) Characteristic (d) None
15. If the characteristic of the logarithm of a number is $\bar{2}$, that number will have zero (s) immediately after the decimal point.
 (a) One (b) Two
 (c) Three (d) Four
16. If the characteristic of the logarithm of a number is 1, that number will have ____ digits in its integral part
 (a) 2
 (b) 3
 (c) 4
 (d) 5
17. The value of x in $\log_3 x = 5$ is ____
 (a) 243 (b) 143
 (c) 200 (d) 144
18. The value of x in $\log x = 2.4543$ is
 (a) 284.6 (b) 1.521
 (c) 1.1010 (d) 0.4058
19. The number corresponding to a given logarithm is known as ____
 (a) Logarithm (b) Antilogarithm
 (c) Characteristic (d) None
20. 30600 in scientific notation is ____
 (a) 3.06×10^4 (b) 3.006×10^4
 (c) 30.6×10^4 (d) 306×10^4
21. 6.35×10^6 in ordinary notation is ____
 (a) 6350000 (b) 635000
 (c) 6350 (d) 63500
22. A number written in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer is called ____
 (a) Scientific notation
 (b) Ordinary notation
 (c) Logarithm notation
 (d) None
23. $\log p - \log q$ is same as
 (a) $\log \left(\frac{q}{p} \right)$
 (b) $\log (p - q)$
 (c) $\frac{\log p}{\log q}$
 (d) $\log \left(\frac{p}{q} \right)$

ANSWER KEY

1.	c	2.	b	3.	d	4.	a	5.	b
6.	a	7.	c	8.	a	9.	c	10.	b
11.	b	12.	a	13.	b	14.	a	15.	a
16.	a	17.	a	18.	a	19.	b	20.	a
21.	a	22.	a	23.	d				