

ALGEBRAIC EXPRESSIONS AND ALGEBRAIC FORMULAS

Define the following terms

Algebraic Expressions

When operations of addition and subtraction are applied to algebraic terms we obtain an algebraic expression. For

Polynomials

A polynomial in the variable x is an algebraic expression of the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0, a_n \neq 0, \dots (i)$$

Where n , the highest power of x , is a non-negative integer called the degree of the polynomial and each coefficient a_n is a real number. The coefficient a_n of the highest power of x is called the leading coefficient of the polynomial. $2x^4 y^3 + x^2 y^2 + 8x$ is a polynomial in two variables x and y and has degree 7.

Rational Expression

The quotient $\frac{p(x)}{q(x)}$ of two polynomials $p(x)$ and $q(x)$, where $q(x)$ is a non-zero polynomial, is called a rational expression.

For example, $\frac{2x+1}{3x+8}$, $3x+8 \neq 0$ is a rational expression.

In the rational expression $\frac{p(x)}{q(x)}$, $p(x)$ is called the numerator and $q(x)$ is known as the denominator of the rational expression $\frac{p(x)}{q(x)}$. The rational expression

$\frac{p(x)}{q(x)}$ need not be a polynomial.

instance, $5x^2 - 3x + \frac{2}{\sqrt{x}}$ and

$3xy + \frac{3}{x} (x \neq 0)$ are algebraic expressions.

Example

Reduce the following algebraic fractions to their lowest forms.

$$(i) \quad \frac{lx+mx-ly-my}{3x^2-3y^2} \quad (ii)$$

$$\frac{3x^2+18x+27}{5x^2-45}$$

Solution

$$\begin{aligned} (i) \quad & \frac{lx+mx-ly-my}{3x^2-3y^2} \\ &= \frac{x(l+m)-y(l+m)}{3(x^2-y^2)} \\ &= \frac{(l+m)(x-y)}{3(x+y)(x-y)} \\ &= \frac{l+m}{3(x+y)} \end{aligned}$$

Which is in the lowest forms.

$$(ii) \quad \frac{3x^2 + 18x + 27}{5x^2 - 45} = \frac{3(x^2 + 6x + 9)}{5(x^2 - 9)}$$

$$\frac{3(x+3)(x+3)}{5(x+3)(x-3)}$$

$$\frac{3(x+3)}{5(x-3)}$$

Which is in the lowest forms

Example

Simplify (i) $\frac{1}{x-y} - \frac{1}{x+y} + \frac{2x}{x^2 - y^2}$

(ii) $\frac{2x^2}{x^4 - 16} - \frac{x}{x^2 - 4} + \frac{1}{x+2}$

Solution

$$(i) \quad \frac{1}{x-y} - \frac{1}{x+y} + \frac{2x}{x^2 - y^2}$$

$$= \frac{1}{x-y} - \frac{1}{x+y} + \frac{2x}{(x+y)(x-y)}$$

$$= \frac{x+y - (x-y) + 2x}{(x+y)(x-y)}$$

(L.C.M of denominators)

$$= \frac{\cancel{x} + y - \cancel{x} + y + 2x}{(x+y)(x-y)}$$

$$= \frac{2x + 2y}{(x+y)(x-y)}$$

$$= \frac{2(x+y)}{(x+y)(x-y)} = \frac{2}{x-y}$$

$$(ii) \quad \frac{2x^2}{x^4 - 16} - \frac{x}{x^2 - 4} + \frac{1}{x+2}$$

$$= \frac{2x^2}{(x^2 + 4)(x^2 - 4)} - \frac{x}{x^2 - 4} + \frac{1}{x+2}$$

$$= \frac{2x^2}{(x^2 + 4)(x+2)(x-2)} - \frac{x}{(x+2)(x-2)} + \frac{1}{x+2}$$

$$= \frac{2x^2 - x(x^2 + 4) + (x^2 + 4)(x-2)}{(x^2 + 4)(x+2)(x-2)} = \frac{\cancel{2x^2} - \cancel{x^3} - \cancel{4x} + \cancel{x^3} + \cancel{4x} - \cancel{2x^2} - 8}{(x^2 + 4)(x+2)(x-2)}$$

$$= \frac{-8}{(x^2+4)(x+2)(x-2)}$$

$$= \frac{-8}{(x^2+4)(x^2-4)} = \frac{-8}{x^4-16}$$

Example

Find the product $\frac{x+2}{2x-3y} \cdot \frac{4x^2-9y^2}{xy+2y}$

Solution

$$\frac{x+2}{2x-3y} \cdot \frac{4x^2-9y^2}{xy+2y} = \frac{(x+2)[(2x)^2-(3y)^2]}{(2x-3y)(x+2)y}$$

$$= \frac{\cancel{(x+2)}(2x+3y)(\cancel{2x-3y})}{y(\cancel{x+2})(\cancel{2x-3y})}$$

$$= \frac{2x+3y}{y}$$

Example

Simplify $\frac{7xy}{x^2-4x+4} \div \frac{14y}{x^2-4}$

Solution

$$\frac{7xy}{x^2-4x+4} \div \frac{14y}{x^2-4}$$

$$= \frac{7xy}{x^2-4x+4} \times \frac{x^2-4}{14y}$$

$$= \frac{7xy}{(x-2)(\cancel{x-2})} \times \frac{(x+2)(\cancel{x-2})}{14y}$$

$$= \frac{x(x+2)}{2(x-2)}$$

Example

Evaluate $\frac{3x^2\sqrt{y}+6}{5(x+y)}$ if $x = -4$ and $y = 9$

Solution

We have, by putting $x = -4$ and $y = 9$,

$$\frac{3x^2\sqrt{y}+6}{5(x+y)} = \frac{3(-4)^2\sqrt{9}+6}{5(-4+9)} = \frac{3(16)(3)+6}{5(5)} = \frac{150}{25} = 6$$

Exercise 4.1

1. Identify whether the following algebraic expression are polynomials (yes or no).

(i) $3x^2 + \frac{1}{x} - 5$ No

(ii) $3x^3 - 4x^2 - x\sqrt{x} + 3$ No

(iii) $x^2 - 3x + \sqrt{2}$ Yes

(iv) $\frac{3x}{2x-1} + 8$ No

2. State whether each of the following expression is a rational expression or not.

(i) $\frac{3\sqrt{x}}{3\sqrt{x}+5}$ No

(ii) $\frac{x^3 - 2x^2 + \sqrt{3}}{2+3x-x^2}$ Yes

(iii) $\frac{x^2+6x+9}{x^2-9}$ Yes

$$(iv) \quad \frac{2\sqrt{x+3}}{2\sqrt{x-3}} \quad \text{No}$$

3. Reduce the following rational expression to the lowest forms.

$$(i) \quad \frac{120x^2y^3z^5}{30x^3yz^2}$$

$$= 4x^{2-3}y^{3-1}z^{5-2}$$

$$= 4x^{-1}y^2z^3$$

$$= \frac{4y^2z^3}{x}$$

$$(ii) \quad \frac{8a(x+1)}{2(x^2-1)} = \frac{4a(\cancel{x+1})}{(x-1)(\cancel{x+1})} = \frac{4a}{x-1}$$

$$(iii) \quad \frac{(x+y)^2 - 4xy}{(x-y)^2} = \frac{x^2 + y^2 + 2xy - 4xy}{(x-y)(x-y)}$$

$$= \frac{x^2 + y^2 - 2xy}{(x-y)(x-y)}$$

$$= \frac{(x-y)^2}{(x-y)(x-y)}$$

$$= \frac{(\cancel{x-y})^2}{(\cancel{x-y})(\cancel{x-y})} = 1$$

$$(iv) \quad \frac{(x^3 - y^3)(x^2 - 2xy + y^2)}{(x-y)(x^2 + xy + y^2)}$$

$$= \frac{(\cancel{x^3 - y^3})(x-y)^2}{\cancel{x^3 - y^3}} = (x-y)^2$$

$$(v) \quad \frac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$$

$$= \frac{(\cancel{x+2})(x-1)(\cancel{x+1})}{(\cancel{x+1})(x-2)(\cancel{x+2})} = \frac{x-1}{x-2}$$

$$(vi) \quad \frac{x^2 - 4x + 4}{2x^2 - 8} = \frac{(x-2)^2}{2(x^2-4)}$$

$$= \frac{(x-2)^2}{2(x-2)(x+2)}$$

$$= \frac{(\cancel{x-2})(x-2)}{2(\cancel{x-2})(x+2)}$$

$$= \frac{x-2}{2(x+2)}$$

$$(vii) \quad \frac{64x^5 - 64x}{(8x^2 + 8)(2x + 2)}$$

$$= \frac{64x(x^4 - 1)}{8(x^2 + 1) \cdot 2(x + 1)}$$

$$= \frac{64x(x^4 - 1)}{16(x^2 + 1)(x + 1)}$$

$$= \frac{4x(x^2 + 1)(x^2 - 1)}{(x^2 + 1)(x + 1)}$$

$$= \frac{4x(\cancel{x^2 + 1})(x-1)(\cancel{x+1})}{(\cancel{x^2 + 1})(\cancel{x+1})}$$

$$= 4x(x-1)$$

$$\frac{9x^2 - (x^2 - 4)^2}{4 + 3x - x^2} = \frac{(3x)^2 - (x^2 - 4)^2}{4 + 3x - x^2}$$

$$= \frac{(3x + x^2 - 4)(\cancel{3x - x^2} + 4)}{(\cancel{4 + 3x - x^2})}$$

$$= 3x + x^2 - 4$$

$$= x^2 + 3x - 4$$

4. Evaluate (a) $\frac{x^3y - 2z}{xz}$ for (i) $x = 3$

$y = -1, z = -2.$

$$(a) \quad \frac{(3)^3(-1) - 2(-2)}{3(-2)} = \frac{-27 + 4}{-6}$$

$$= \frac{-23}{-6} = \frac{23}{6} = 3\frac{5}{6}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{x^2y^3 - 5z^4}{xyz} \text{ for } x = 4, y = -2, z = -1 \\
 &= \frac{(4)^2(-2)^3 - 5(-1)^4}{(4)(-2)(-1)} = \frac{-16(8) - 5}{8} \\
 &= \frac{-128 - 5}{8} = \frac{-133}{8} = -16\frac{5}{8}
 \end{aligned}$$

5. Perform the indicated operation and simplify

$$\begin{aligned}
 \text{(i)} \quad & \frac{15}{2x-3y} - \frac{4}{3y-2x} \\
 &= \frac{15(3y-2x) - 4(2x-3y)}{(2x-3y)(3y-2x)} \\
 &= \frac{45y - 30x - 8x + 12y}{(2x-3y)(3y-2x)} \\
 &= \frac{57y - 38x}{(2x-3y)(3y-2x)} \\
 &= \frac{19(3y-2x)}{(2x-3y)(3y-2x)} = \frac{19}{2x-3y}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \frac{1+2x}{1-2x} - \frac{1-2x}{1+2x} \\
 &= \frac{(1+2x)^2 - (1-2x)^2}{(1-2x)(1+2x)} \\
 &= \frac{(1+4x^2+4x) - (1+4x^2-4x)}{(1-2x)(1+2x)} \\
 &= \frac{1 + \cancel{4x^2} + 4x - 1 - \cancel{4x^2} + 4x}{(1-2x)(1+2x)} \\
 &= \frac{8x}{(1-2x)(1+2x)} = \frac{8x}{1-4x^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & \frac{x^2-25}{x^2-36} - \frac{x+5}{x+6} \\
 &= \frac{(x-5)(x+5)}{(x-6)(x+6)} - \frac{x+5}{x+6} \\
 &= \frac{(x-5)(x+5) - (x+5)(x-6)}{(x-6)(x+6)}
 \end{aligned}$$

$$= \frac{(x-5)(x+5) - (x+5)(x-6)}{(x+6)(x-6)}$$

$$= \frac{(x+5)[(x-5) - (x-6)]}{(x+6)(x-6)}$$

$$= \frac{(x+5)(\cancel{x-5} - \cancel{x} + 6)}{(x+6)(x-6)}$$

$$= \frac{(x+5)(1)}{(x+6)(x-6)} = \frac{x+5}{x^2-36}$$

$$\begin{aligned}
 \text{(iv)} \quad & \frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{x^2-y^2} \\
 &= \frac{x(x+y) - y(x-y)}{(x-y)(x+y)} - \frac{2xy}{x^2-y^2} \\
 &= \frac{x^2 + \cancel{xy} - \cancel{xy} + y^2}{x^2-y^2} - \frac{2xy}{x^2-y^2} \\
 &= \frac{x^2+y^2}{x^2-y^2} - \frac{2xy}{x^2-y^2} \\
 &= \frac{x^2+y^2-2xy}{(x^2-y^2)}
 \end{aligned}$$

$$= \frac{(x-y)^2}{(\cancel{x-y})(x+y)} = \frac{x-y}{x+y}$$

$$\begin{aligned}
 \text{(v)} \quad & \frac{x-2}{x^2+6x+9} - \frac{x+2}{2x^2-18} \\
 &= \frac{x-2}{x^2+3x+3x+9} - \frac{x+2}{2(x^2-9)} \\
 &= \frac{x-2}{x(x+3)+3(x+3)} - \frac{x+2}{2(x-3)(x+3)} \\
 &= \frac{x-2}{(x+3)(x+3)} - \frac{x+2}{2(x-3)(x+3)} \\
 &= \frac{2(x-3)(x-2) - (x+3)(x+2)}{2(x-3)(x+3)(x+3)} \\
 &= \frac{2(x^2-2x-3x+6) - (x^2+2x+3x+6)}{2(x-3)(x+3)^2}
 \end{aligned}$$

$$= \frac{2(x^2 - 5x + 6) - (x^2 + 5x + 6)}{2(x-3)(x+3)^2}$$

$$= \frac{2x^2 - 10x + 12 - x^2 - 5x - 6}{2(x-3)(x+3)^2}$$

$$= \frac{x^2 - 15x + 6}{2(x-3)(x+3)^2}$$

$$(vi) \quad \frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{x+1-(x-1)}{(x-1)(x+1)} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{\cancel{x+1} - \cancel{x-1}}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{2}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{2(x^2+1) - 2(x^2-1)}{(x^2-1)(x^2+1)} - \frac{4}{x^4-1}$$

$$= \frac{\cancel{2x^2} + 2 - \cancel{2x^2} + 2}{x^4-1} - \frac{4}{x^4-1}$$

$$= \frac{4}{x^4-1} - \frac{4}{x^4-1}$$

$$= \frac{4-4}{x^4-1}$$

$$= \frac{0}{x^4-1}$$

$$= 0$$

6. Perform the indicated operation and simplify:

$$(i) \quad (x^2 - 49) \frac{5x+2}{x+7}$$

$$= (x-7)(\cancel{x+7}) \frac{5x+2}{\cancel{x+7}}$$

$$= (x-7)(5x+2)$$

$$(ii) \quad \frac{4x-12}{x^2-9} \div \frac{18-2x^2}{x^2+6x+9}$$

$$= \frac{4(x-3)}{(x-3)(x+3)} \div \frac{2(9-x^2)}{x^2+3x+3x+9}$$

$$= \frac{4(x-3)}{(x-3)(x+3)} \div \frac{2(3-x)(3+x)}{x(x+3)+3(x+3)}$$

$$= \frac{4(x-3)}{(x-3)(x+3)} \div \frac{2(3-x)(3+x)}{(x+3)(x+3)}$$

$$= \frac{4(x-3)}{(x+3)(x-3)} \times \frac{(x+3)(x+3)}{2(3+x)(3-x)}$$

$$= \frac{2}{3-x}$$

$$(iii) \quad \frac{x^6-y^6}{x^2-y^2} \div (x^4+x^2y^2+y^4)$$

$$= \frac{(x^3)^2 - (y^3)^2}{x^2-y^2} \div (x^4+x^2y^2+y^4)$$

$$= \frac{(x^3-y^3)(x^3+y^3)}{x^2-y^2} \div (x^4+x^2y^2+y^4)$$

$$= \frac{(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)}{x^2-y^2}$$

$$\times \frac{1}{x^4+x^2y^2+y^4}$$

$$= \frac{(\cancel{x^2-y^2})(x^2+xy+y^2)(x^2-xy+y^2)}{\cancel{x^2-y^2}}$$

$$\times \frac{1}{x^4+x^2y^2+y^4}$$

$$= \frac{x^4+x^2y^2+y^4}{x^4+x^2y^2+y^4} = 1$$

$$(iv) \quad \frac{x^2-1}{x^2+2x+1} \cdot \frac{x+5}{1-x}$$

$$= \frac{-(\cancel{x-1})(x+1)}{x^2+x+x+1} \cdot \frac{x+5}{(x-1)}$$

$$= \frac{-(x+1)(x+5)}{x(x+1)+1(x+1)}$$

$$= \frac{\cancel{-(x+1)}(x+5)}{(x+1)\cancel{(x+1)}} = -\frac{x+5}{x+1}$$

$$\begin{aligned} \text{(v)} \quad & \frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} + \frac{x^2-x}{xy-2y} \\ &= \frac{x\cancel{(x+y)}}{y\cancel{(x+y)}} \cdot \frac{\cancel{x}(x+y)}{\cancel{y}(x+y)} \times \frac{\cancel{y}(x-2)}{\cancel{x}(x-1)} \\ &= \frac{x(x-2)}{y(x-1)} \end{aligned}$$

Example

If $a+b=7$ and $a-b=3$, then find the value of (a) a^2+b^2 (b) ab

Solution

We are given that $a+b=7$ and $a-b=3$

(a) To find the value of (a^2+b^2) , we use the formula

$$(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$$

Substituting the values $a+b=7$ and $a-b=3$, we get

$$\begin{aligned} (7)^2 + (3)^2 &= 2(a^2+b^2) \\ \Rightarrow 49+9 &= 2(a^2+b^2) \\ \Rightarrow 58 &= 2(a^2+b^2) \\ \Rightarrow 29 &= a^2+b^2, \end{aligned}$$

(b) To find the value of ab , we make use of the formula

$$\begin{aligned} (a+b)^2 - (a-b)^2 &= 4ab \\ \Rightarrow (7)^2 - (3)^2 &= 4ab, \end{aligned}$$

$$\begin{aligned} \Rightarrow 49-9 &= 4ab \\ \Rightarrow 40 &= 4ab, \\ \Rightarrow 10 &= ab, \end{aligned}$$

Hence $a^2+b^2=29$ and $ab=10$.

Example

If $a^2+b^2+c^2=43$ and $ab+bc+ca=3$, then find the value of $a+b+c$.

Solution

We know that

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

$$\Rightarrow (a+b+c)^2 = 43+2 \times 3$$

$$(\text{Putting } a^2+b^2+c^2=43 \text{ and } ab+bc+ca=3)$$

$$\Rightarrow (a+b+c)^2 = 49$$

$$\Rightarrow a+b+c = \pm\sqrt{49}$$

$$\text{Hence } a+b+c = \pm 7$$

Example

If $a+b+c=6$ and $a^2+b^2+c^2=24$ then find the value of $ab+bc+ca$.

Solution

We have

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

$$(6)^2 = 24+2(ab+bc+ca)$$

$$\Rightarrow 36=24+2(ab+bc+ca)$$

$$\Rightarrow 12=2(ab+bc+ca)$$

$$\text{Hence } ab+bc+ca=6$$

Example

If $a+b+c=7$ and $ab+bc+ca=9$, then find the value of $a^2+b^2+c^2$.

Solution

We know that

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$\Rightarrow (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$$

$$\Rightarrow (7)^2 = a^2 + b^2 + c^2 + 2(9)$$

$$\Rightarrow 49 = a^2 + b^2 + c^2 + 18$$

$$\Rightarrow 31 = a^2 + b^2 + c^2$$

$$\text{Hence } a^2 + b^2 + c^2 = 31$$

Example

If $2x - 3y = 10$ and $xy = 2$, then find the value of $8x^3 - 27y^3$.

Solution

We are given that $2x - 3y = 10$

$$\Rightarrow (2x - 3y)^3 = (10)^3$$

$$\Rightarrow 8x^3 - 27y^3 - 3 \times 2x \times 3y(2x - 3y) = 1000$$

$$\Rightarrow 8x^3 - 27y^3 - 18xy(2x - 3y) = 1000$$

$$\Rightarrow 8x^3 - 27y^3 - 18 \times 2 \times 10 = 1000$$

$$\Rightarrow 8x^3 - 27y^3 - 360 = 1000$$

Hence

$$8x^3 - 27y^3 = 1360$$

Example

If $x + \frac{1}{x} = 8$, then find the value of $x^3 + \frac{1}{x^3}$

Solution

We have been given $x + \frac{1}{x} = 8$

$$\Rightarrow \left(x + \frac{1}{x}\right)^3 = (8)^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x} \left(x + \frac{1}{x}\right) = 512$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \left(x + \frac{1}{x}\right) = 512$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 8 = 512$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 24 = 512$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 512 - 24$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 488$$

Example

If $x - \frac{1}{x} = 4$, then find $x^3 - \frac{1}{x^3}$

Solution

We have $x - \frac{1}{x} = 4$

$$\Rightarrow \left(x - \frac{1}{x}\right)^3 = (4)^3$$

$$\Rightarrow x^3 - \frac{1}{x^3} - 3x \times \frac{1}{x} \left(x - \frac{1}{x}\right) = 64$$

$$\Rightarrow x^3 - \frac{1}{x^3} - 3(4) = 64$$

$$\Rightarrow x^3 - \frac{1}{x^3} - 12 = 64$$

$$\Rightarrow x^3 - \frac{1}{x^3} = 64 + 12$$

$$\Rightarrow x^3 - \frac{1}{x^3} = 76$$

Example

Factorize $64x^3 + 343y^3$

Solution

We have

$$64x^3 + 343y^3 = (4x)^3 + (7y)^3$$

$$\begin{aligned}
 &= (4x+7y)[(4x)^2 - (4x)(7y) + (7y)^2] \\
 &= (4x+7y)(16x^2 - 28xy + 49y^2)
 \end{aligned}$$

Example

Factorize $125x^3 - 1331y^3$

Solution

We have

$$\begin{aligned}
 125x^3 - 1331y^3 &= (5x)^3 - (11y)^3 \\
 &= (5x-11y)[(5x)^2 + (5x)(11y) + (11y)^2] \\
 &= (5x-11y)(25x^2 + 55xy + 121y^2)
 \end{aligned}$$

Example

Factorize

$$\left(\frac{2}{3}x + \frac{3}{2x}\right)\left(\frac{4}{9}x^2 - 1 + \frac{9}{4x^2}\right)$$

Solution

$$\begin{aligned}
 &\left(\frac{2}{3}x + \frac{3}{2x}\right)\left(\frac{4}{9}x^2 - 1 + \frac{9}{4x^2}\right) \\
 &= \left(\frac{2}{3}x + \frac{3}{2x}\right)\left[\left(\frac{2}{3}x\right)^2 - \left(\frac{2}{3}x\right)\left(\frac{3}{2x}\right) + \left(\frac{3}{2x}\right)^2\right] \\
 &= \left(\frac{2}{3}x\right)^3 + \left(\frac{3}{2x}\right)^3 \\
 &= \frac{8}{27}x^3 + \frac{27}{8x^3}
 \end{aligned}$$

Example

Find the product $\left(\frac{4}{5}x - \frac{5}{4x}\right)$

$$\left(\frac{16}{25}x^2 + \frac{25}{16x^2} + 1\right)$$

Solution

$$\begin{aligned}
 &\left(\frac{4}{5}x - \frac{5}{4x}\right)\left(\frac{16}{25}x^2 + \frac{25}{16x^2} + 1\right) \\
 &= \left(\frac{4}{5}x - \frac{5}{4x}\right)\left(\frac{16x^2}{25} + 1 + \frac{25}{16x^2}\right) \\
 &\text{(rearranging)} \\
 &= \left(\frac{4}{5}x - \frac{5}{4x}\right)\left[\left(\frac{4}{5}x\right)^2 + \left(\frac{4}{5}x\right)\left(\frac{5}{4x}\right) + \left(\frac{5}{4x}\right)^2\right] \\
 &= \left(\frac{4}{5}x\right)^3 - \left(\frac{5}{4x}\right)^3 = \frac{64}{125}x^3 - \frac{125}{64x^3}
 \end{aligned}$$

Example

Find the continued product of $(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$

Solution

$$\begin{aligned}
 &(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2) \\
 &= (x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2) \\
 &= (x^3+y^3)(x^3-y^3) = (x^3)^2 - (y^3)^2 = x^6 - y^6
 \end{aligned}$$