

Exercise 4.2

1.(i) If $a + b = 10$ and $a - b = 6$ then find value of $a^2 + b^2$.

Solution:

$$2(a^2 + b^2) = (a+b)^2 + (a-b)^2$$

$$2(a^2 + b^2) = (10)^2 + (6)^2$$

$$2(a^2 + b^2) = 100 + 36$$

$$a^2 + b^2 = \frac{136}{2} = 68$$

(ii) If $a + b = 5$, $a - b = \sqrt{17}$ then find value of ab .

Solution:

$$4ab = (a+b)^2 - (a-b)^2$$

$$4ab = (5)^2 - (\sqrt{17})^2$$

$$4ab = 25 - 17$$

$$4ab = 8$$

$$ab = \frac{8}{4} = 2$$

2. If $a^2 + b^2 + c^2 = 45$ and $a + b + c = -1$ find value of $ab + bc + ca$.

Solution:

$$a+b+c = -1$$

Squaring

$$(a+b+c)^2 = (-1)^2$$

$$a^2+b^2+c^2+2ab+2bc+2ca = 1$$

$$a^2+b^2+c^2+2(ab+bc+ca) = 1$$

$$45 + 2(ab + bc + ca) = 1$$

$$2(ab + bc + ca) = 1 - 45$$

$$2(ab + bc + ca) = -44$$

$$ab + bc + ca = \frac{-44}{2} = -22$$

3. If $m+n+p = 10$, $mn + np + pm = 27$ find value of $m^2+n^2+p^2$.

Solution:

$$m+n+p=10$$

Squaring both sides

$$(m+n+p)^2 = (10)^2$$

$$m^2+n^2+p^2+2mn+2np+2mp=100$$

$$m^2+n^2+p^2+2(mn+np+pm)=100$$

$$m^2+n^2+p^2+2(27)=100$$

$$m^2+n^2+p^2+54 = 100$$

$$m^2+n^2+p^2 = 100 - 54$$

$$m^2+n^2+p^2=46$$

4. If $x^2 + y^2 + z^2 = 78$ and $y+yz+zx=59$ find $x + y + z$.

Solution:

$$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$$

$$= x^2+y^2+z^2+2(xy+yz+zx)$$

$$= 78+2(59)$$

$$= 78 + 118$$

$$= 196$$

$$\sqrt{(x+y+z)^2} = \sqrt{196} = \sqrt{(\pm 14)^2}$$

$$x + y + z = \pm 14$$

5. If $x + y + z = 12$ and $x^2 + y^2 + z^2 = 64$ find value of $xy+yz+zx$.

Solution:

$$x + y + z = 12$$

Squaring both sides

$$(x + y + z)^2 = (12)^2$$

$$x^2+y^2+z^2+2xy+2yz+2zx = 144$$

$$x^2 + y^2 + z^2 + 2(xy+yz+zx) = 144$$

$$64 + 2(xy+yz+zx) = 144$$

$$2(xy + yz + zx) = 144 - 64$$

$$2(xy + yz + zx) = 80$$

$$xy + yz + zx = \frac{80}{2} = 40.$$

6. If $x + y = 7$ and $xy = 12$ then find value of $x^3 + y^3$.

Solution:

$$x + y = 7$$

$$(x + y)^3 = (7)^3$$

$$x^3 + y^3 + 3xy(x+y) = 343$$

$$x^3 + y^3 + 3(12)(7) = 343$$

$$x^3 + y^3 + 252 = 343$$

$$x^3 + y^3 = 343 - 252$$

$$x^3 + y^3 = 91$$

7. If $3x + 4y = 11$ and $xy = 12$ then find value of $27x^3 + 64y^3$.

Solution:

$$3x + 4y = 11$$

$$(3x + 4y)^3 = (11)^3$$

$$(3x)^3 + (4y)^3 + 3(3x)(4y)(3x+4y) = 1331$$

$$27x^3 + 64y^3 + 36xy(3x+4y) = 1331$$

$$27x^3 + 64y^3 + 36(12)(11) = 1331$$

$$27x^3 + 64y^3 + 4752 = 1331$$

$$27x^3 + 64y^3 = 1331 - 4752 = -3421$$

8. If $x - y = 4$ and $xy = 21$ then find value of $x^3 - y^3$.

Solution:

$$x - y = 4$$

$$(x-y)^3 = (4)^3$$

$$x^3 - y^3 - 3xy(x-y) = 64$$

$$x^3 - y^3 - 3(21)(4) = 64$$

$$x^3 - y^3 - 252 = 64$$

$$x^3 - y^3 = 64 + 252$$

$$x^3 - y^3 = 316$$

9. If $5x - 6y = 13$ and $xy = 6$ then find value of $125x^3 - 216y^3$.

Solution:

$$5x - 6y = 13$$

$$\Rightarrow (5x - 6y)^3 = (13)^3$$

$$\Rightarrow (5x)^3 - (6y)^3 - 3(5x)(6y)(5x - 6y) = 2197$$

$$125x^3 - 216y^3 - 90xy(5x - 6y) = 2197$$

$$125x^3 - 216y^3 - 90(6)(13) = 2197$$

$$125x^3 - 216y^3 - 7020 = 2197$$

$$125x^3 - 216y^3 = 2197 + 7020$$

$$125x^3 - 216y^3 = 9217$$

10. If $x + \frac{1}{x} = 3$ then find $x^3 + \frac{1}{x^3}$.

$$x + \frac{1}{x} = 3 \text{ Cubing both sides}$$

$$\left(x + \frac{1}{x}\right)^3 = (3)^3$$

$$x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right) = 27$$

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 27$$

$$x^3 + \frac{1}{x^3} + 3(3) = 27$$

$$x^3 + \frac{1}{x^3} = 27 - 9$$

$$x^3 + \frac{1}{x^3} = 18$$

11. If $x - \frac{1}{x} = 7$, then find value of

$$x^3 - \frac{1}{x^3}$$

$$x - \frac{1}{x} = 7 \text{ Taking cube of both sides}$$

$$\left(x - \frac{1}{x}\right)^3 = (7)^3$$

$$x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right) = 343$$

$$x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 343$$

$$x^3 - \frac{1}{x^3} - 3(7) = 343$$

$$x^3 - \frac{1}{x^3} - 21 = 343$$

$$x^3 - \frac{1}{x^3} = 343 + 21$$

$$x^3 - \frac{1}{x^3} = 364$$

12. If $3x + \frac{1}{3x} = 5$, then find value of

$$27x^3 + \frac{1}{27x^3}$$

$$\left(3x + \frac{1}{3x}\right)^3 = (5)^3$$

$$(3x)^3 + \left(\frac{1}{3x}\right)^3 + 3(3x)\left(\frac{1}{3x}\right)\left(3x + \frac{1}{3x}\right) = 125$$

$$27x^3 + \frac{1}{27x^3} + 3\left(3x + \frac{1}{3x}\right) = 125$$

$$27x^3 + \frac{1}{27x^3} + 3(5) = 125$$

$$27x^3 + \frac{1}{27x^3} + 15 = 125$$

$$27x^3 + \frac{1}{27x^3} = 125 - 15$$

$$27x^3 + \frac{1}{27x^3} = 110$$

13. If $\left(5x - \frac{1}{5x}\right) = 6$, then find value of

$$125x^3 - \frac{1}{25x^3}$$

$$\left(5x - \frac{1}{5x}\right) = 6$$

Taking cube of both sides

$$\left(5x - \frac{1}{5x}\right)^3 = (6)^3$$

$$(5x)^3 - \left(\frac{1}{5x}\right)^3 - 3\left(5x\right)\left(\frac{1}{5x}\right)\left(5x - \frac{1}{5x}\right) = 216$$

$$125x^3 - \frac{1}{125x^3} - 3\left(5x - \frac{1}{5x}\right) = 216$$

$$125x^3 - \frac{1}{125x^3} - 3(6) = 216$$

$$125x^3 - \frac{1}{25x^3} - 18 = 216$$

$$125x^3 - \frac{1}{125x^3} = 216 + 18$$

$$125x^3 - \frac{1}{125x^3} = 234$$

14. Factorize (i) $x^3 - y^3 - x + y$

$$(i) \quad x^3 - y^3 - x + y$$

$$= (x - y)(x^2 + xy + y^2) - 1(x - y)$$

$$= (x - y)[x^2 + xy + y^2 - 1]$$

$$(ii) \quad 8x^3 - \frac{1}{27y^3}$$

$$= (2x)^3 - \left(\frac{1}{3y}\right)^3$$

$$= \left(2x - \frac{1}{3y}\right)\left((2x)^2 + (2x)\left(\frac{1}{3y}\right) + \left(\frac{1}{3y}\right)^2\right)$$

$$= \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$$

15. Find products, using formulae

$$(i) \quad (x^2 + y^2)(x^4 - x^2y^2 + y^4)$$

$$= (x^2)^3 + (y^2)^3$$

$$\text{Ref} = (a + b)(a^2 - ab + b^2) = a^3 + b^3$$

$$= x^6 + y^6$$

$$(ii) \quad (x^3 - y^3)(x^6 + x^3y^3 + y^6)$$

$$= (x^3)^3 - (y^3)^3$$

$$\text{Ref.} (a - b)(a^2 + ab + b^2) = a^3 - b^3$$

$$= x^9 - y^9$$

$$(iii) \quad (x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)$$

$$(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$$

$$= (x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)$$

$$(x^2 + y^2)(x^4 - x^2y^2 + y^4)$$

$$= (x^3 - y^3)(x^3 + y^3)[(x^2)^3 + (y^2)^3]$$

$$= [(x^3)^2 - (y^3)^2](x^6 + y^6)$$

$$= (x^6 - y^6)(x^6 + y^6)$$

$$= (x^6)^2 - (y^6)^2$$

$$= x^{12} - y^{12}$$

$$16. \quad (2x^2 - 1)(2x^2 + 1)(4x^4 + 2x^2 + 1)$$

$$(4x^4 - 2x^2 + 1)$$

$$= (2x^2 - 1)(4x^4 + 2x^2 + 1)(2x^2 + 1)$$

$$(4x^4 - 2x^2 + 1)$$

$$= ((2x^2)^3 - (1)^3)((2x^2)^3 + (1)^3)$$

$$= (8x^6 - 1)(8x^6 + 1)$$

$$= (8x^6)^2 - (1)^2$$

$$= 64x^{12} - 1$$

Define Surd

An irrational radical with rational radicand is called a surd.

Hence the radical $\sqrt[n]{a}$ is a surd if

- (i) a is rational
- (ii) the result $\sqrt[n]{a}$ is irrational.

e.g., $\sqrt{3}, \sqrt{2/5}, \sqrt[3]{7}, \sqrt[4]{10}$ are surds.

But $\sqrt{\pi}$ is not surd because π is not rational.

Note: Every surd is an irrational number but every irrational number is not surd

Example

Simplify by combining similar terms.

$$(i) \quad 4\sqrt{3} - 3\sqrt{27} + 2\sqrt{75}$$

$$(ii) \quad \sqrt[3]{128} - \sqrt[3]{250} + \sqrt[3]{432}$$

Solution

$$(i) \quad 4\sqrt{3} - 3\sqrt{27} + 2\sqrt{75}$$

$$= 4\sqrt{3} - 3\sqrt{9 \times 3} + 2\sqrt{25 \times 3} = 4\sqrt{3} - 3\sqrt{9} \sqrt{3} + 2\sqrt{25} \times \sqrt{3}$$

$$= 4\sqrt{3} - 9\sqrt{3} + 10\sqrt{3} = (4 - 9 + 10)\sqrt{3} = 5\sqrt{3}$$

$$(ii) \quad \sqrt[3]{128} - \sqrt[3]{250} + \sqrt[3]{432}$$

$$= \sqrt[3]{64 \times 2} - \sqrt[3]{125 \times 2} + \sqrt[3]{216 \times 2}$$

$$= \sqrt[3]{(4)^3 \times 2} - \sqrt[3]{(5)^3 \times 2} + \sqrt[3]{(6)^3 \times 2}$$

$$= \sqrt[3]{(4)^3} \sqrt[3]{2} - \sqrt[3]{(5)^3} \sqrt[3]{2} + \sqrt[3]{(6)^3} \sqrt[3]{2}$$

$$= 4\sqrt[3]{2} - 5\sqrt[3]{2} + 6\sqrt[3]{2} = (4 - 5 + 6)\sqrt[3]{2} = 5\sqrt[3]{2}$$

Example

Simplify and express the answer in the simplest form.

$$(i) \quad \sqrt{14}\sqrt{35}$$

$$(ii) \quad \frac{\sqrt[6]{12}}{\sqrt{3}\sqrt[3]{2}}$$

Solution

$$(i) \quad \sqrt{14}\sqrt{35} = \sqrt{14 \times 35} = \sqrt{7 \times 2 \times 7 \times 5} = \sqrt{(7)^2 \times 2 \times 5}$$

$$= \sqrt{(7)^2 \times 10} = \sqrt{(7)^2} \times \sqrt{10} = 7\sqrt{10}$$

(ii) We have $\frac{\sqrt[6]{12}}{\sqrt{3} \sqrt[3]{2}}$

For $\sqrt{3}$ $\sqrt[3]{2}$ the L.C.M of orders 2 and 3 is 6.

$$\text{Thus } \sqrt{3} = (3)^{1/2} = (3)^{3/6} = \sqrt[6]{3^3} = \sqrt[6]{27}$$

$$\text{and } \sqrt[3]{2} = (2)^{1/3} = (2)^{2/6} = \sqrt[6]{(2)^2} = \sqrt[6]{4}$$

$$\text{Hence } \frac{\sqrt[6]{12}}{\sqrt{3} \sqrt[3]{2}} = \frac{\sqrt[6]{12}}{\sqrt[6]{27} \sqrt[6]{4}} = \frac{\sqrt[6]{12}}{\sqrt[6]{108}} = \sqrt[6]{\frac{12}{108}} = \sqrt[6]{\frac{1}{9}}$$

Its simplest form is

$$\sqrt[6]{\left(\frac{1}{3}\right)^2} = \left(\frac{1}{3}\right)^{2/6} = \left(\frac{1}{3}\right)^{1/3} = \sqrt[3]{\frac{1}{3}}$$