

Exercise 4.4

1. Rationalize the denominator

$$(i) \quad \frac{3}{4\sqrt{3}} = \frac{3}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{4\sqrt{3} \times \sqrt{3}}$$

$$= \frac{3\sqrt{3}}{4(3)} = \frac{\sqrt{3}}{4}$$

$$(ii) \quad \frac{14}{\sqrt{98}} = \frac{14}{7\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\cancel{14}\sqrt{2}}{\cancel{14}} = \sqrt{2}$$

$$(iii) \quad \frac{6}{\sqrt{8} \cdot \sqrt{27}} = \frac{6}{2\sqrt{2} \cdot 3\sqrt{3}}$$

$$= \frac{\cancel{6}}{\cancel{6}\sqrt{6}}$$

$$= \frac{1}{\sqrt{6}}$$

$$= \frac{1}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{\sqrt{6}}{6}$$

$$(iv) \quad \frac{1}{3+2\sqrt{5}} = \frac{1}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$= \frac{3-2\sqrt{5}}{(3)^2 - (2\sqrt{5})^2} = \frac{3-2\sqrt{5}}{9-20}$$

$$= \frac{3-2\sqrt{5}}{-11}$$

$$(v) \quad \frac{15}{\sqrt{31}-4}$$

$$= \frac{15}{\sqrt{31}-4} \times \frac{\sqrt{31}+4}{\sqrt{31}+4}$$

$$= \frac{15(\sqrt{31}+4)}{(\sqrt{31})^2 - (4)^2}$$

$$= \frac{15(\sqrt{31}+4)}{31-16}$$

$$= \frac{\cancel{15}(\sqrt{31}+4)}{\cancel{15}}$$

$$= \sqrt{31}+4$$

$$(vi) \quad \frac{2}{\sqrt{5}-\sqrt{3}} = \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2}$$

$$= \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$$

$$= \frac{\cancel{2}(\sqrt{5}+\sqrt{3})}{\cancel{2}}$$

$$= \sqrt{5}+\sqrt{3}$$

$$(vii) \quad \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{(\sqrt{3}-1)^2}{3-1}$$

$$= \frac{(\sqrt{3})^2 + 1^2 - 2(1)\sqrt{3}}{2}$$

$$= \frac{3+1-2\sqrt{3}}{2}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= \frac{2(2-\sqrt{3})}{2}$$

$$= 2-\sqrt{3}$$

$$(viii) \quad \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2 - (\sqrt{3})^2}$$

$$= \frac{(\sqrt{5}+\sqrt{3})^2}{5-3}$$

$$= \frac{(\sqrt{5})^2 + (\sqrt{3})^2 + 2(\sqrt{5})(\sqrt{3})}{2}$$

$$= \frac{5+3+2\sqrt{15}}{2}$$

$$= \frac{8+2\sqrt{15}}{2}$$

$$= \frac{2(4+\sqrt{15})}{2}$$

$$= 4+\sqrt{15}$$

(2) Find conjugate of $x+\sqrt{y}$:

(i) $3+\sqrt{7}$

Conjugate of $3+\sqrt{7}$ is $3-\sqrt{7}$

(ii) $4-\sqrt{5}$

Conjugate of $4-\sqrt{5}$ is $4+\sqrt{5}$

(iii) $2+\sqrt{3}$

Conjugate of $2+\sqrt{3}$ is $2-\sqrt{3}$

(iv) $2+\sqrt{5}$

Conjugate of $2+\sqrt{5}$ is $2-\sqrt{5}$

(v) $5+\sqrt{7}$

Conjugate of $5+\sqrt{7}$ is $5-\sqrt{7}$

(vi) $4-\sqrt{15}$

Conjugate of $4-\sqrt{15}$ is $4+\sqrt{15}$

(vii) $7-\sqrt{6}$

Conjugate of $7-\sqrt{6}$ is $7+\sqrt{6}$

(viii) $9+\sqrt{2}$

Conjugate of $9+\sqrt{2}$ is $9-\sqrt{2}$

Q.3 If $x=2-\sqrt{3}$ find $\frac{1}{x}$

(i) $x = 2-\sqrt{3}$

$$\frac{1}{x} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$\frac{1}{x} = \frac{2+\sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$\frac{1}{x} = \frac{2+\sqrt{3}}{4-3}$$

$$\frac{1}{x} = 2+\sqrt{3}$$

(ii) $x=4-\sqrt{17}$ find $\frac{1}{x}$

$$\frac{1}{x} = \frac{1}{4-\sqrt{17}} \times \frac{4+\sqrt{17}}{4+\sqrt{17}}$$

$$\frac{1}{x} = \frac{4+\sqrt{17}}{(4)^2 - (\sqrt{17})^2}$$

$$= \frac{4+\sqrt{17}}{16-17}$$

$$= \frac{4+\sqrt{17}}{-1}$$

$$= -(4+\sqrt{17})$$

$$= -4-\sqrt{17}$$

(iii) If $x = \sqrt{3} + 2$, find $x + \frac{1}{x}$

$$x = \sqrt{3} + 2$$

$$\frac{1}{x} = \frac{1}{\sqrt{3} + 2} \times \frac{\sqrt{3} - 2}{\sqrt{3} - 2}$$

$$\frac{1}{x} = \frac{\sqrt{3} - 2}{(\sqrt{3})^2 - (2)^2}$$

$$\frac{1}{x} = \frac{\sqrt{3} - 2}{3 - 4}$$

$$\frac{1}{x} = \frac{\sqrt{3} - 2}{-1}$$

$$\frac{1}{x} = -\sqrt{3} + 2 = 2 - \sqrt{3}$$

$$x + \frac{1}{x} = \cancel{\sqrt{3}} + 2 - \cancel{\sqrt{3}} + 2$$

$$x + \frac{1}{x} = 4$$

Q4. Simplify

(i) $\frac{1 + \sqrt{2}}{\sqrt{5} + \sqrt{3}} + \frac{1 - \sqrt{2}}{\sqrt{5} - \sqrt{3}}$

$$\frac{1 + \sqrt{2}}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}} + \frac{1 - \sqrt{2}}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}$$

$$= \frac{(1 + \sqrt{2})(\sqrt{5} - \sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2} + \frac{(1 - \sqrt{2})(\sqrt{5} + \sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2}$$

$$= \frac{(1 + \sqrt{2})(\sqrt{5} - \sqrt{3})}{5 - 3} + \frac{(1 - \sqrt{2})(\sqrt{5} + \sqrt{3})}{5 - 3}$$

$$= \frac{\sqrt{5} - \sqrt{3} + \sqrt{2}\sqrt{5} - \sqrt{2}\sqrt{3}}{2} + \frac{\sqrt{5} + \sqrt{3} - \sqrt{2}\sqrt{5} - \sqrt{2}\sqrt{3}}{2}$$

$$= \frac{\sqrt{5} - \sqrt{3} + \sqrt{10} - \sqrt{6}}{2} + \frac{\sqrt{5} + \sqrt{3} - \sqrt{10} - \sqrt{6}}{2}$$

$$= \frac{\sqrt{5} - \cancel{\sqrt{3}} + \cancel{\sqrt{10}} - \sqrt{6} + \sqrt{5} + \cancel{\sqrt{3}} - \cancel{\sqrt{10}} - \sqrt{6}}{2}$$

$$= \frac{2\sqrt{5} - 2\sqrt{6}}{2}$$

$$= \cancel{2}(\sqrt{5} - \sqrt{6})$$

$$= \sqrt{5} - \sqrt{6}$$

(ii) $\frac{1}{2 + \sqrt{3}} + \frac{2}{\sqrt{5} - \sqrt{3}} + \frac{1}{2 + \sqrt{5}}$

$$= \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} + \frac{2}{\sqrt{5} - \sqrt{3}}$$

$$\times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} + \frac{1}{2 + \sqrt{5}} \times \frac{2 - \sqrt{5}}{2 - \sqrt{5}}$$

$$= \frac{2 - \sqrt{3}}{(2)^2 - (\sqrt{3})^2} + \frac{2(\sqrt{5} + \sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2} + \frac{2 - \sqrt{5}}{(2)^2 - (\sqrt{5})^2}$$

$$= \frac{2 - \sqrt{3}}{4 - 3} + \frac{2(\sqrt{5} + \sqrt{3})}{5 - 3} + \frac{2 - \sqrt{5}}{4 - 5}$$

$$= 2 - \sqrt{3} + \frac{2(\sqrt{5} + \sqrt{3})}{2} + \frac{2 - \sqrt{5}}{-1}$$

$$= \cancel{2} - \cancel{\sqrt{3}} + \sqrt{5} + \cancel{\sqrt{3}} - \cancel{2} + \sqrt{5} = 2\sqrt{5}$$

(iii) $\frac{2}{\sqrt{5} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{2}} - \frac{3}{\sqrt{5} + \sqrt{2}}$

$$= \frac{2}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{2}}$$

$$\times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}} - \frac{3}{\sqrt{5} + \sqrt{2}} \times \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}}$$

$$= \frac{2(\sqrt{5} - \sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2} + \frac{\sqrt{3} - \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} - \frac{3(\sqrt{5} - \sqrt{2})}{(\sqrt{5})^2 - (\sqrt{2})^2}$$

$$= \frac{2(\sqrt{5} - \sqrt{3})}{5 - 3} + \frac{\sqrt{3} - \sqrt{2}}{3 - 2} - \frac{3(\sqrt{5} - \sqrt{2})}{5 - 2}$$

$$\begin{aligned}
 &= \frac{\cancel{2}(\sqrt{5}-\sqrt{3})}{\cancel{2}} + \frac{\sqrt{3}-\sqrt{2}}{1} - \frac{\cancel{2}(\sqrt{5}-\sqrt{2})}{\cancel{2}} \\
 &= \cancel{\sqrt{5}} - \cancel{\sqrt{3}} + \sqrt{3} - \sqrt{2} - \cancel{\sqrt{5}} + \cancel{\sqrt{2}} \\
 &= 0
 \end{aligned}$$

Q5(i) If $x = 2 + \sqrt{3}$, find value of $x - \frac{1}{x}$

$$\text{and } \left(x - \frac{1}{x}\right)^2$$

$$x = 2 + \sqrt{3}$$

$$\frac{1}{x} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$\frac{1}{x} = \frac{2 - \sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$\frac{1}{x} = 2 - \sqrt{3}$$

$$\begin{aligned}
 x - \frac{1}{x} &= 2 + \sqrt{3} - (2 - \sqrt{3}) \\
 &= \cancel{2} + \sqrt{3} - \cancel{2} + \sqrt{3} \\
 &= 2\sqrt{3}
 \end{aligned}$$

$$\left(x - \frac{1}{x}\right)^2 = (2\sqrt{3})^2$$

$$\left(x - \frac{1}{x}\right)^2 = 12$$

(ii) If $x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ find the value of

$$x + \frac{1}{x}, x^2 + \frac{1}{x} \text{ and } x^3 + \frac{1}{x^3}$$

$$x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$x = \frac{(\sqrt{5}-\sqrt{2})^2}{(\sqrt{5})^2 - (\sqrt{2})^2}$$

$$x = \frac{(\sqrt{5})^2 + (\sqrt{2})^2 - 2(\sqrt{5})(\sqrt{2})}{5 - 2}$$

$$x = \frac{5 + 2 - 2\sqrt{10}}{3}$$

$$x = \frac{7 - 2\sqrt{10}}{3}$$

$$\frac{1}{x} = \frac{3}{7 - 2\sqrt{10}} \times \frac{7 + 2\sqrt{10}}{7 + 2\sqrt{10}}$$

$$\frac{1}{x} = \frac{3(7 + 2\sqrt{10})}{(7)^2 - (2\sqrt{10})^2}$$

$$\frac{1}{x} = \frac{3(7 + 2\sqrt{10})}{49 - 40}$$

$$\frac{1}{x} = \frac{3(7 + 2\sqrt{10})}{9}$$

$$\frac{1}{x} = \frac{7 + 2\sqrt{10}}{3}$$

$$\begin{aligned}
 x + \frac{1}{x} &= \frac{7 - 2\sqrt{10}}{3} + \frac{7 + 2\sqrt{10}}{3} \\
 &= \frac{7 - 2\sqrt{10} + 7 + 2\sqrt{10}}{3} = \frac{14}{3}
 \end{aligned}$$

Now

$$x + \frac{1}{x} = \frac{14}{3}$$

Squaring

$$\left(x + \frac{1}{x}\right)^2 = \left(\frac{14}{3}\right)^2$$

$$x^2 + \frac{1}{x^2} + 2 = \frac{196}{9}$$

$$x^2 + \frac{1}{x^2} = \frac{196}{9} - 2$$

$$x^2 + \frac{1}{x^2} = \frac{196 - 18}{9} = \frac{178}{9}$$

Also

$$x^3 + \frac{1}{x^3} = ?$$

$$x + \frac{1}{x} = \frac{14}{3}$$

$$\left(x + \frac{1}{x}\right)^3 = \left(\frac{14}{3}\right)^3$$

$$x^3 + \frac{1}{x^3} + 3\left(x\right)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right) = \frac{2744}{27}$$

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \frac{2744}{27}$$

$$x^3 + \frac{1}{x^3} + 3\left(\frac{14}{3}\right) = \frac{2744}{27}$$

$$x^3 + \frac{1}{x^3} = \frac{2744}{27} - 14$$

$$= \frac{2366}{27}$$

Q6. Determine the rational numbers a and b. If

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} = a + b\sqrt{3}$$

Given

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} = a + b\sqrt{3}$$

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = a + b\sqrt{3}$$

$$\frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2 - (1)^2} + \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2 - (1)^2} = a + b\sqrt{3}$$

$$\frac{(\sqrt{3})^2 + (1)^2 - 2(\sqrt{3})(1)}{3-1} + \frac{(\sqrt{3})^2 + (1)^2 + 2\sqrt{3}}{3-1} = a + b\sqrt{3}$$

$$\frac{3+1-2\sqrt{3}}{2} + \frac{3+1+2\sqrt{3}}{2} = a + b\sqrt{3}$$

$$\frac{4-2\sqrt{3}}{2} + \frac{4+2\sqrt{3}}{2} = a + b\sqrt{3}$$

$$\frac{\cancel{2}(2-\sqrt{3})}{\cancel{2}} + \frac{\cancel{2}(2+\sqrt{3})}{\cancel{2}} = a + b\sqrt{3}$$

$$2 - \cancel{\sqrt{3}} + 2 + \cancel{\sqrt{3}} = a + b\sqrt{3}$$

$$4 = a + b\sqrt{3}$$

$$\Rightarrow a + b\sqrt{3} = 4$$

Hence on comparing the two sides, we get

$$\Rightarrow a = 4 \text{ and } b = 0$$

Exercise

Q1. If $x + \frac{1}{x} = 3$ find

(i) $x^2 + \frac{1}{x^2}$

(ii) $x^4 + \frac{1}{x^4}$

(i) $x + \frac{1}{x} = 3$

$$\left(x + \frac{1}{x}\right)^2 = (3)^2$$

$$x^2 + \frac{1}{x^2} + 2 = 9$$

$$x^2 + \frac{1}{x^2} = 9 - 2$$

$$x^2 + \frac{1}{x^2} = 7$$

(ii) $x^4 + \frac{1}{x^4}$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (7)^2$$

$$x^4 + \frac{1}{x^4} + 2 = 49$$

$$x^4 + \frac{1}{x^4} = 49 - 2$$

$$x^4 + \frac{1}{x^4} = 47$$

Q2. If $x - \frac{1}{x} = 2$ find

(i) $x^2 + \frac{1}{x^2}$

(ii) $x^4 + \frac{1}{x^4}$

(i) $x - \frac{1}{x} = 2$

Squaring

$$\left(x - \frac{1}{x}\right)^2 = (2)^2$$

$$x^2 + \frac{1}{x^2} - 2 = 4$$

$$x^2 + \frac{1}{x^2} = 4 + 2$$

$$x^2 + \frac{1}{x^2} = 6$$

(ii) $\left(x^2 + \frac{1}{x^2}\right) = (6)^2$

$$x^4 + \frac{1}{x^4} + 2 = 36$$

$$x^4 + \frac{1}{x^4} = 36 - 2$$

$$x^4 + \frac{1}{x^4} = 34$$

Q3. Find value of $x^3 + y^3$ and xy if

$$x + y = 5 \text{ and } x - y = 3$$

$$4xy = (x + y)^2 - (x - y)^2$$

$$4xy = (5)^2 - (3)^2$$

Now

$$4xy = 25 - 9 = 16$$

$$xy = \frac{16}{4} = 4$$

$$x + y = 5$$

taking cube both sides

$$(x + y)^3 = (5)^3$$

$$x^3 + y^3 + 3xy(x + y) = 125$$

$$x^3 + y^3 + 3(4)(5) = 125$$

$$x^3 + y^3 + 60 = 125$$

$$x^3 + y^3 = 125 - 60$$

$$x^3 + y^3 = 65$$

Q4. If $P = 2 + \sqrt{3}$ find (i) $P + \frac{1}{P}$

(ii) $P - \frac{1}{P}$ (iii) $P^2 + \frac{1}{P^2}$ (iv) $P^2 - \frac{1}{P^2}$

$$P = 2 + \sqrt{3}$$

$$\frac{1}{P} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$\frac{1}{P} = \frac{2 - \sqrt{3}}{(2)^2 - (\sqrt{3})^2} = \frac{2 - \sqrt{3}}{4 - 3} = 2 - \sqrt{3}$$

i) $P + \frac{1}{P} = 2 + \cancel{\sqrt{3}} + 2 - \cancel{\sqrt{3}} = 4$

ii) $P - \frac{1}{P} = \cancel{2} + \sqrt{3} - \cancel{2} + \sqrt{3} = 2\sqrt{3}$

iii) $P^2 + \frac{1}{P^2} = ?$

$$\left(P + \frac{1}{P}\right)^2 = (4)^2$$

$$P^2 + \frac{1}{P^2} + 2 = 16$$

$$P^2 + \frac{1}{P^2} = 16 - 2$$

$$P^2 + \frac{1}{P^2} = 14$$

iv) $P^2 - \frac{1}{P^2} = ?$

$$\begin{aligned}
 P^2 - \frac{1}{P^2} &= \left(P + \frac{1}{P}\right) \left(P - \frac{1}{P}\right) \\
 &= (4)(\sqrt{3}) \\
 &= 8\sqrt{3}
 \end{aligned}$$

Q5. If $q = \sqrt{5} + 2$ Find (i) $q + \frac{1}{q}$

(ii) $q - \frac{1}{q}$ (iii) $q^2 + \frac{1}{q^2}$ (iv) $q^2 - \frac{1}{q^2}$

Solution: $q = \sqrt{5} + 2$

$$\frac{1}{q} = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$\frac{1}{q} = \frac{\sqrt{5}-2}{(\sqrt{5})^2 - (2)^2}$$

$$\frac{1}{q} = \frac{\sqrt{5}-2}{1} = \sqrt{5}-2$$

(i) $q + \frac{1}{q} = \sqrt{5} + 2 + \sqrt{5} - 2$
 $= 2\sqrt{5}$

(ii) $q - \frac{1}{q} = \sqrt{5} + 2 - \sqrt{5} + 2$
 $= 4$

(iii) $q^2 + \frac{1}{q^2}$

$$\left(q + \frac{1}{q}\right)^2 = (2\sqrt{5})^2$$

$$q^2 + \frac{1}{q^2} + 2 = 20$$

$$q^2 + \frac{1}{q^2} = 20 - 2$$

$$q^2 + \frac{1}{q^2} = 18$$

(iv) $q^2 - \frac{1}{q^2} = \left(q + \frac{1}{q}\right) \left(q - \frac{1}{q}\right)$

$$\begin{aligned}
 &= (2\sqrt{5})(4) \\
 &= 8\sqrt{5}
 \end{aligned}$$

Q6. Simplify

i) $\frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} - \sqrt{a^2-2}}$

$$\begin{aligned}
 &= \frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} - \sqrt{a^2-2}} \times \frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} + \sqrt{a^2-2}} \\
 &= \frac{(\sqrt{a^2+2} + \sqrt{a^2-2})^2}{(\sqrt{a^2+2})^2 - (\sqrt{a^2-2})^2} \\
 &= \frac{(\sqrt{a^2+2})^2 + (\sqrt{a^2-2})^2 + 2(\sqrt{a^2+2})(\sqrt{a^2-2})}{a^2 + 2 - a^2 + 2} \\
 &= \frac{a^2 + 2 + a^2 - 2 + 2\sqrt{a^4-4}}{4} \\
 &= \frac{2a^2 + 2\sqrt{a^4-4}}{4} \\
 &= \frac{\cancel{2}(a^2 + \sqrt{a^4-4})}{\cancel{2}} \\
 &= \frac{a^2 + \sqrt{a^4-4}}{2}
 \end{aligned}$$

(ii) $\frac{1}{a - \sqrt{a^2 - x^2}} - \frac{1}{a + \sqrt{a^2 - x^2}}$

$$\begin{aligned}
 &= \frac{1}{a - \sqrt{a^2 - x^2}} \times \frac{a + \sqrt{a^2 - x^2}}{a + \sqrt{a^2 - x^2}} \\
 &\quad - \frac{1}{a + \sqrt{a^2 - x^2}} \times \frac{a - \sqrt{a^2 - x^2}}{a - \sqrt{a^2 - x^2}} \\
 &= \frac{a + \sqrt{a^2 - x^2}}{(a)^2 - (\sqrt{a^2 - x^2})^2} - \frac{a - \sqrt{a^2 - x^2}}{(a)^2 - (\sqrt{a^2 - x^2})^2}
 \end{aligned}$$

$$= \frac{a + \sqrt{a^2 - x^2}}{\cancel{a^2} - \cancel{a^2} + x^2} - \frac{a - \sqrt{a^2 - x^2}}{\cancel{a^2} - \cancel{a^2} + x^2}$$

$$= \frac{a + \sqrt{a^2 - x^2}}{x^2} - \frac{a - \sqrt{a^2 - x^2}}{x^2}$$

$$= \frac{\cancel{a} + \sqrt{a^2 - x^2} - \cancel{a} + \sqrt{a^2 - x^2}}{x^2}$$

$$= \frac{2\sqrt{a^2 - x^2}}{x^2}$$

Objective

- $4x + 3y - 2$ is an algebraic ____
 (a) Expression
 (b) Sentence
 (c) Equation
 (d) In equation
- The degree of polynomial $4x^4 + 2x^2y$ is ____
 (a) 1 (b) 2
 (c) 3 (d) 4
- $a^3 + b^3$ is equal to ____
 (a) $(a-b)(a^2 + ab + b^2)$
 (b) $(a+b)(a^2 - ab + b^2)$
 (c) $(a-b)(a^2 - ab + b^2)$
 (d) $(a-b)(a^2 + ab - b^2)$
- $(3 + \sqrt{2})(3 - \sqrt{2})$ is equal to: ____
 (a) 7 (b) -7
 (c) -1 (d) 1
- Conjugate of Surd $a + \sqrt{b}$ is ____
 (a) $-a + \sqrt{b}$ (b) $a - \sqrt{b}$
 (c) $\sqrt{a} + \sqrt{b}$ (d) $\sqrt{a} - \sqrt{b}$
- $\frac{1}{a-b} - \frac{1}{a+b}$ is equal to
 (a) $\frac{2a}{a^2 - b^2}$ (b) $\frac{2b}{a^2 - b^2}$
 (c) $\frac{-2a}{a^2 - b^2}$ (d) $\frac{-2b}{a^2 - b^2}$
- $\frac{a^2 - b^2}{a + b}$ is equal to:
 (a) $(a-b)^2$ (b) $(a+b)^2$
 (c) $a+b$ (d) $a-b$
- $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$ is equal to: ____
 (a) $a^2 + b^2$ (b) $a^2 - b^2$
 (c) $a - b$ (d) $a + b$
- The degree of the polynomial $x^2y^2 + 3xy + y^3$ is ____
 (a) 4 (b) 5
 (c) 6 (d) 2
- $x^2 - 4 =$ ____
 (a) $(x-2)(x+2)$ (b) $(x-2)(x-2)$
 (c) $(x+2)(x+2)$ (d) None
- $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)(\dots\dots\dots)$
 (a) $x^2 - 1 + \frac{1}{x^2}$ (b) $x^2 + 1 + \frac{1}{x^2}$
 (c) $x^2 + 1 - \frac{1}{x^2}$ (d) $x^2 - 1 - \frac{1}{x^2}$
- $2(a^2 + b^2) =$ ____
 (a) $(a+b)^2 + (a-b)^2$
 (b) $(a+b)^2 - (a-b)^2$
 (c) $(a+b)^2$ (d) $4ab$
- Order of surd $\sqrt[3]{x}$ is ____
 (a) 3 (b) $\frac{1}{3}$
 (c) 0 (d) 1

14. $\frac{1}{2-\sqrt{3}} = \underline{\hspace{2cm}}$

- (a) $2+\sqrt{3}$ (b) $2-\sqrt{3}$
 (d) $-2+\sqrt{3}$ (d) $-2-\sqrt{3}$

15. $(a+b)^2 - (a-b)^2 = \underline{\hspace{2cm}}$

- (a) $2(a^2 + b^2)$ (b) $4ab$
 (c) $2ab$ (d) $3ab$

16. $\sqrt{14} \cdot \sqrt{35} = \underline{\hspace{2cm}}$

- (a) $\sqrt[4]{10}$ (b) $\sqrt[5]{10}$
 (c) $7\sqrt{10}$ (d) $8\sqrt{10}$

17. A surd which contains a single term is called surd.

- (a) Monomial
 (b) Binomial
 (c) Trinomial
 (d) None

ANSWER KEY

1.	a	2.	d	3.	b	4.	a	5.	b
6.	b	7.	d	8.	c	9.	a	10.	a
11.	a	12.	a	13.	a	14.	a	15.	b
16.	c	17.	a						