

Exercise 5.4

Factorize each of the following cubic polynomials by factor theorem.

Q.1 $x^3 - 2x^2 - x + 2$

Let $P(x) = x^3 - 2x^2 - x + 2$

Put $x = 1$

$$P(1) = (1)^3 - 2(1)^2 - (1) + 2$$

$$= 1 - 2 - 1 + 2$$

$$= -3 + 3 = 0$$

As, $R = 0$,

So $(x - 1)$ is a factor

Put $x = -1$

$$P(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$$

$$= -1 - 2 + 1 + 2$$

As $R = 0$,

So $(x + 1)$ is the second factor of $p(x)$.

Put $x = 2$

$$P(2) = (2)^3 - 2(2)^2 - (2) + 2$$

$$= 8 - 8 - 2 + 2$$

$$= 10 - 10$$

$$= 0$$

As $R = 0$,

So $(x - 2)$ is the third factor

Hence $P(x) = x^3 - 2x^2 - x + 2$

$$= (x - 1)(x + 1)(x - 2)$$

Q.2 $x^3 - x^2 - 22x + 40$

Sol:

Let $P(x) = x^3 - x^2 - 22x + 40$

Put $x = 1$

$$P(1) = (1)^3 - (1)^2 - 22(1) + 40$$

$$= 1 - 1 - 22 + 40$$

$$=18 \neq 0$$

Hence $x-1$ is not a zero of $P(x)$

Put $x=-1$

$$\begin{aligned} P(-1) &= (-1)^3 - (-1)^2 - 22(-1) + 40 \\ &= -1 - 1 + 22 + 40 \\ &= 60 \neq 0 \end{aligned}$$

Hence $x=-1$ is not a zero of $P(x)$

Put $x=2$

$$\begin{aligned} P(2) &= (2)^3 - (2)^2 - 22(2) + 40 \\ &= 8 - 4 - 44 + 40 = 0 \end{aligned}$$

Hence $x-2$ is a zero of $P(x)$

So $(x-2)$ is a factor

Put $x=-2$

$$\begin{aligned} P(-2) &= (-2)^3 - (-2)^2 - 22(-2) + 40 \\ &= -8 - 4 + 44 + 40 = 72 \end{aligned}$$

Hence $x=-2$ is not a zero of $P(x)$

Put $x=3$

$$\begin{aligned} P(3) &= (3)^3 - (3)^2 - 22(3) + 40 \\ &= 27 - 9 - 66 + 40 \\ &= 67 - 75 \\ &= -8 \neq 0 \end{aligned}$$

Hence $x=3$ is not a zero of $P(x)$

Put $x=-3$

$$\begin{aligned} P(-3) &= (-3)^3 - (-3)^2 - 22(-3) + 40 \\ &= -27 - 9 + 66 + 40 \\ &= 106 - 36 \\ &= 70 \neq 0 \end{aligned}$$

Hence $x=-3$ is not a zero of $P(x)$

Put $x=4$

$$\begin{aligned} P(4) &= (4)^3 - (4)^2 - 22(4) + 40 \\ &= 64 - 16 - 88 + 40 \end{aligned}$$

$$= 104 - 104$$

$$= 0$$

Hence $x=4$ is a zero of $P(x)$

So $(x-4)$ is second factor

Put $x=-4$

$$\begin{aligned} P(-4) &= (-4)^3 - (-4)^2 - 22(-4) + 40 \\ &= -64 - 16 + 88 + 40 \\ &= -80 + 128 \\ &= 48 \neq 0 \end{aligned}$$

So, $x=-4$ is not a zero of $P(x)$

Put $x=5$

$$\begin{aligned} P(5) &= (5)^3 - (5)^2 - 22(5) + 40 \\ &= 125 - 25 - 110 + 40 \\ &= 165 - 135 \\ &= 30 \neq 0 \end{aligned}$$

So, $x=5$ is not a zero of $P(x)$

Put $x=-5$

$$\begin{aligned} P(-5) &= (-5)^3 - (-5)^2 - 22(-5) + 40 \\ &= -125 - 25 + 110 + 40 \\ &= -150 + 150 \\ &= 0 \end{aligned}$$

So, $x=-5$ is a zero of $P(x)$

Hence $x+5$ is third factor of $P(x)$

$$\begin{aligned} \text{Hence } P(x) &= x^3 - x^2 - 22x + 40 \\ &= (x-2)(x-4)(x+5) \end{aligned}$$

$$\text{Q.3 } x^3 - 6x^2 + 3x + 10$$

Sol:

$$\text{Let } P(x) = x^3 - 6x^2 - 6x^2 + 3x + 10$$

Put $x=1$

$$\begin{aligned} P(1) &= (1)^3 - 6(1)^2 + 3(1) + 10 \\ &= 1 - 6 + 3 + 10 \end{aligned}$$

$$=14-6$$

$$=8 \neq 0$$

So, $x = 1$ is not a zero of $P(x)$

Put $x = -1$

$$P(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10$$

$$= -1 - 6 - 3 + 10$$

$$= -10 + 10$$

$$= 0$$

So, $x = -1$ is a zero of $P(x)$.

Hence $(x + 1)$ is a factor of $P(x)$

Put $x = 2$

$$P(2) = (2)^3 - 6(2)^2 + 3(2) + 10$$

$$= 8 - 24 + 6 + 10$$

$$= 24 - 24$$

$$= 0$$

So, $x = 2$ is a zero of $P(x)$.

Hence $(x - 2)$ is second factor of $P(x)$

Put $x = -2$

$$P(-2) = (-2)^3 - 6(-2)^2 + 3(-2) + 10$$

$$= -8 - 24 - 6 + 10$$

$$= -28 \neq 0$$

So, $x = -2$ is not a zero of $P(x)$

Put $x = 3$

$$P(3) = (3)^3 - 6(3)^2 + 3(3) + 10$$

$$= 27 - 6(9) + 9 + 10$$

$$= 46 - 54$$

$$= -8 \neq 0$$

So, $x = 3$ is not a zero of $P(x)$

Put $x = -3$

$$P(-3) = (-3)^3 - 6(-3)^2 + 3(-3) + 10$$

$$= -27 - 6(9) - 9 + 10$$

$$= -90 + 10$$

$$= -80 \neq 0$$

So, $x = -3$ is not a zero of $P(x)$

Put $x = 4$

$$P(4) = (4)^3 - 6(4)^2 + 3(4) + 10$$

$$= 64 - 6(16) + 12 + 10$$

$$= 86 - 96$$

$$= -10 \neq 0$$

So, $x = 4$ is not a zero of $P(x)$

Put $x = -4$

$$P(-4) = (-4)^3 - 6(-4)^2 + 3(-4) + 10$$

$$= -64 - 6(16) - 12 + 10$$

$$= -64 - 96 - 12 + 10$$

$$= -172 + 10$$

$$= -162$$

$$= -162 \neq 0$$

Put $x = 5$

$$P(5) = (5)^3 - 6(5)^2 + 3(5) + 10$$

$$= 125 - 150 + 15 + 10$$

$$= 150 - 150$$

$$= 0$$

So, $x = 5$ is a zero of $P(x)$

Hence $(x - 5)$ is third factor of $P(x)$

$$\text{Hence } P(x) = x^3 - 6x^2 + 3x + 10$$

$$= (x + 1)(x - 2)(x - 5)$$

$$\text{Q.4 } x^3 + x^2 - 10x + 8$$

Sol:

$$\text{Let } P(x) = x^3 + x^2 - 10x + 8$$

Put $x = 1$

$$P(1) = (1)^3 + (1)^2 - 10(1) + 8$$

$$= 1 + 1 - 10 + 8$$

$$= 0$$

So, $x = 1$ is a zero of $P(x)$

Hence $(x-1)$ is a factor of $P(x)$

Put $x=-1$

$$\begin{aligned}P(-1) &= (-1)^3 + (-1)^2 - 10(-1) + 8 \\&= -1 + 1 + 10 + 8 \\&= 18 \neq 0\end{aligned}$$

So, $x=-1$ is not a zero of $P(x)$

Put $x=2$

$$\begin{aligned}P(2) &= (2)^3 + (2)^2 - 10(2) + 8 \\&= 8 + 4 - 20 + 8 \\&= 20 - 20 \\&= 0\end{aligned}$$

So, $x=2$ is a zero of $P(x)$

Hence $x-2$ is second factor of $P(x)$

Put $x=-2$

$$\begin{aligned}P(-2) &= (-2)^3 + (-2)^2 - 10(-2) + 8 \\&= -8 + 4 + 20 + 8 \\&= 24 \neq 0\end{aligned}$$

So, $x=-2$ is not a zero of $P(x)$

Put $x=3$

$$\begin{aligned}P(3) &= (3)^3 + (3)^2 - 10(3) + 8 \\&= 27 + 9 - 30 + 8 \\&= 44 - 30 \\&= 14 \neq 0\end{aligned}$$

Put $x=-3$

$$\begin{aligned}P(-3) &= (-3)^3 + (-3)^2 - 10(-3) + 8 \\&= -27 + 9 + 30 + 8 \\&= -27 + 47 \\&= 20 \neq 0\end{aligned}$$

So, $x=-3$ is not a zero of $P(x)$

Put $x=4$

$$P(4) = (4)^3 + (4)^2 - 10(4) + 8$$

$$= 64 + 16 - 40 + 8$$

$$= 88 - 40$$

$$= 48 \neq 0$$

So, $x=4$ is not a zero of $P(x)$

Put $x=-4$

$$\begin{aligned}P(-4) &= (-4)^3 + (-4)^2 - 10(-4) + 8 \\&= -64 + 16 + 40 + 8 \\&= -64 + 64 \\&= 0\end{aligned}$$

So, $x=-4$ is a zero of $P(x)$

Hence $x+4$ is third factor of $P(x)$

$$\begin{aligned}\text{Hence } P(x) &= x^3 + x^2 - 10x + 8 \\&= (x-1)(x-2)(x+4)\end{aligned}$$

$$\text{Q.5 } x^3 - 2x^2 - 5x + 6$$

Sol:

$$P(x) = x^3 - 2x^2 - 5x + 6$$

Put $x=1$

$$\begin{aligned}P(1) &= (1)^3 - 2(1)^2 - 5(1) + 6 \\&= 1 - 2 - 5 + 6 \\&= 7 - 7 \\&= 0\end{aligned}$$

So, $x=1$ is a zero of $P(1)$

Hence $x-1$ is a factor of $P(x)$

Put $x=-1$

$$\begin{aligned}P(-1) &= (-1)^3 - 2(-1)^2 - 5(-1) + 6 \\&= -1 - 2 + 5 + 6 \\&= -3 + 11 \\&= 8 \neq 0\end{aligned}$$

So, $x=-1$ is not a zero of $P(x)$

Put $x=2$

$$P(2) = (2)^3 - 2(2)^2 - 5(2) + 6$$

$$=8-8-10+6$$

$$=-4 \neq 0$$

So, $x=2$ is not a zero of $P(x)$

Put $x=-2$

$$P(-2)=(-2)^3-2(-2)^2-5(-2)$$

$$=-8-8+10+6$$

$$=0$$

So, $x=-2$ is a zero of $P(x)$

Hence $(x+2)$ is second factor of $P(x)$

Put $x=3$

$$P(3)=(3)^3-2(3)^2-5(3)+6$$

$$=27-18-15+6$$

$$=33-33$$

$$=0$$

So, $x=3$ is a zero of $P(x)$

Hence $(x-3)$ is third factor of $P(x)$

$$\text{Hence } P(x)=x^3-2x^2-5x+6$$

$$=(x-1)(x+2)(x-3)$$

$$\text{Q.6 } x^3+5x^2-2x-24$$

Sol:

$$\text{Let } P(x)=x^3+5x^2-2x-24$$

Put $x=1$

$$P(1)=(1)^3+5(1)^2-2(1)-24$$

$$=1+5-2-24$$

$$=6-26$$

$$=-20 \neq 0$$

So, $x=1$ is not a zero of $P(x)$

Put $x=-1$

$$P(-1)=(-1)^3+5(-1)^2-2(-1)-24$$

$$=-1+5+2-24$$

$$=7-25$$

$$=-18 \neq 0$$

So, $x=-1$ is not a zero of $P(x)$

Put $x=2$

$$P(2)=(2)^3+5(2)^2-2(2)-24$$

$$=8+20-4-24$$

$$=28-28$$

$$=0$$

So, $x=2$ is a zero of $P(x)$

Hence $(x-2)$ is a factor of $P(x)$

Put $x=-2$

$$P(-2)=(-2)^3+5(-2)^2-2(-2)-24$$

$$=-8+5(4)+4-24$$

$$=-32+24$$

$$=-8 \neq 0$$

So, $x=-2$ is not a zero of $P(x)$

Put $x=3$

$$P(3)=(3)^3+5(3)^2-2(3)-24$$

$$=27+5(9)-6-24$$

$$=72-30$$

$$=42 \neq 0$$

So, $x=3$ is not a zero of $P(x)$

Put $x=-3$

$$P(-3)=(-3)^3+5(-3)^2-2(-3)-24$$

$$=-27+45+6-24$$

$$=51-51$$

$$=0$$

So, $x=-3$ is a zero of $P(x)$

Hence $(x+3)$ is second factor of $P(x)$

Put $x=4$

$$P(4)=(4)^3+5(4)^2-2(4)-24$$

$$=64+5(16)-8-24$$

$$=144-32$$

$$=112 \neq 0$$

So, $x=4$ is not a zero of $P(x)$

Put $x=-4$

$$\begin{aligned} P(-4) &= (-4)^3 + 5(-4)^2 - 2(-4) - 24 \\ &= -64 + 80 + 8 - 24 \\ &= 0 \end{aligned}$$

So, $x = -4$ is a zero of $P(x)$

Hence $(x+4)$ is third factor of $P(x)$

$$\begin{aligned} \text{Hence } P(x) &= x^3 + 5x^2 - 2x - 24 \\ &= (x-2)(x+3)(x+4) \end{aligned}$$

Q. 7 $3x^3 - x^2 - 12x + 4$

Sol: $P(x) = 3x^3 - x^2 - 12x + 4$

Put $x=1$

$$\begin{aligned} P(1) &= 3(1)^3 - (1)^2 - 12(1) + 4 \\ &= 3 - 1 - 12 + 4 \\ &= 7 - 13 \\ &= -6 \neq 0 \end{aligned}$$

So, $x=1$ is not a zero of $P(x)$

Put $x=-1$

$$\begin{aligned} P(-1) &= 3(-1)^3 - (-1)^2 - 12(-1) + 4 \\ &= -3 - 1 + 12 + 4 \\ &= -4 + 16 \\ &= 12 \neq 0 \end{aligned}$$

So, $x=-1$ is not a zero of $P(x)$

Put $x=2$

$$\begin{aligned} P(2) &= 3(2)^3 - (2)^2 - 12(2) + 4 \\ &= 24 - 4 - 24 + 4 \\ &= 28 - 28 \\ &= 0 \end{aligned}$$

So, $x=2$ is a zero of $P(x)$

Hence $(x-2)$ is a factor of $P(x)$

Put $x=-2$

$$\begin{aligned} P(-2) &= 3(-2)^3 - (-2)^2 - 12(-2) + 4 \\ &= -24 - 4 + 24 + 4 \\ &= -28 + 28 \\ &= 0 \end{aligned}$$

So, $x=-2$ is a zero of $P(x)$

Hence $(x+2)$ is second factor of $P(x)$

Put $3x=1$

$$x = \frac{1}{3}$$

$$\begin{aligned} P\left(\frac{1}{3}\right) &= 3\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - 12\left(\frac{1}{3}\right) + 4 \\ &= \cancel{3}\left(\frac{1}{27}\right) - \frac{1}{9} - 12\left(\frac{1}{3}\right) + 4 \\ &= \frac{1}{9} - \frac{1}{9} - 4 + 4 \\ &= 0 \end{aligned}$$

So, $x = \frac{1}{3}$ is a zero of $P(x)$

Hence $(3x-1)$ is third factor of $P(x)$

$$\begin{aligned} \text{Hence } P(x) &= 3x^3 - x^2 - 12x + 4 \\ &= (x-2)(x+2)(3x-1) \end{aligned}$$

Q.8 $2x^3 + x^2 - 2x - 1$

Let $P(x) = 2x^3 + x^2 - 2x - 1$

Put $x=1$

$$\begin{aligned} P(1) &= 2(1)^3 + (1)^2 - 2(1) - 1 \\ &= 2 + 1 - 2 - 1 \\ &= 3 - 3 \\ &= 0 \end{aligned}$$

So, $x=1$ is a zero of $P(x)$

Hence $(x-1)$ is a factor of $P(x)$

Put $x=-1$

$$P(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$$

$$= -2 + 1 + 2 - 1$$

$$= -1 + 1$$

$$= 0$$

So, $x = -1$ is a zero of $P(x)$

Hence $(x + 1)$ is second factor of $P(x)$

Put $2x = 1$

$$x = \frac{1}{2}$$

$$P\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) - 1$$

$$= 2\left(\frac{1}{8}\right) + \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) - 1$$

$$= \frac{1}{4} + \frac{1}{4} - 1 - 1$$

$$= -\frac{3}{2} \neq 0$$

So, $x - 2$ is not a zero of $P(x)$

$$\text{Put } x = \frac{-1}{2}$$

$$P\left(\frac{-1}{2}\right) = 2\left(\frac{-1}{2}\right)^3 + \left(\frac{-1}{2}\right)^2 - 2\left(\frac{-1}{2}\right) - 1$$

$$= 2\left(-\frac{1}{8}\right) + \frac{1}{4} + 1 - 1$$

$$= -\frac{1}{4} + \frac{1}{4} + 1 - 1$$

$$= 0$$

So, $x = \frac{-1}{2}$ is a zero of $P(x)$

Hence $2x + 1$ is third factor of $P(x)$

$$\text{Hence } P(x) = 2x^3 + x^2 - 2x - 1$$

$$= (x - 1)(x + 1)(2x + 1)$$

Objective

1. The factor of $x^2 - 5x + 6$ are: _____

(a) $x + 1, x - 6$ (b) $x - 2, x - 3$

(c) $x + 6, x - 1$ (d) $x + 2, x + 3$

2. Factors of $8x^3 + 27y^3$ are: _____

(a) $(2x + 3y)(4x^2 - 9y^2)$

(b) $(2x - 3y)(4x^2 - 9y^2)$

(c) $(2x + 3y)(4x^2 - 6xy + 9y^2)$

(d) $(2x - 3y)(4x^2 + 6xy + 9y^2)$

3. Factors of $3x^2 - x - 2$ are:

(a) $(x + 1)(3x - 2)$ (b) $(x + 1)(3x + 2)$

(c) $(x - 1)(3x - 2)$ (d) $(x - 1)(3x + 2)$

4. Factors of $a^4 - 4b^4$ are: _____

(a) $(a - b)(a + b)(a^2 + 4b^2)$

(b) $(a^2 - 2b^2)(a^2 + 2b^2)$

(c) $(a - b)(a + b)(a^2 - 4b^2)$

(d) $(a - 2b)(a^2 + 2b^2)$

5. What will be added to complete the square of $9a^2 - 12ab$? _____

(a) $-16b^2$

(b) $16b^2$

(c) $4b^2$

(d) $-4b^2$

6. Find m so that $x^2 + 4x + m$ is a complete square:

(a) 8

(b) -8

(c) 4

(d) 16

7. Factors of $5x^2 - 17xy - 12y^2$ are _____

(a) $(x + 4y)(5x + 3y)$

(b) $(x - 4y)(5x - 3y)$

(c) $(x - 4y)(5x + 3y)$

(d) $(5x - 4y)(x + 3y)$

8. Factors of $27x^3 - \frac{1}{x^3}$ are ____

- (a) $\left(3x - \frac{1}{x}\right)\left(9x^2 + 3 + \frac{1}{x^2}\right)$
 (b) $\left(3x + \frac{1}{x}\right)\left(9x^2 + 3 + \frac{1}{x^2}\right)$
 (c) $\left(3x - \frac{1}{x}\right)\left(9x^2 - 3 + \frac{1}{x^2}\right)$
 (d) $\left(3x + \frac{1}{x}\right)\left(9x^2 - 3 + \frac{1}{x^2}\right)$

9. If $x - 2$ is a factor of $p(x) = x^2 + 2kx + 8$, then $K =$ ____

- (a) -3 (b) 3
 (c) 4 (d) 5

10. $4a^2 + 4ab + (\dots)$ is a complete square

- (a) b^2 (b) $2b$
 (c) a^2 (d) $4b^2$

11. $\frac{x^2}{y^2} - 2 + \frac{y^2}{x^2} = \dots\dots\dots$

- (a) $\left(\frac{x}{y} - \frac{y}{x}\right)^2$ (b) $\left(\frac{x}{y} + \frac{y}{x}\right)^2$
 (c) $\left(\frac{x}{y} - \frac{y}{x}\right)^3$ (d) $\left(\frac{x}{y} + \frac{y}{x}\right)^3$

12. $(x+y)(x^2 - xy + y^2) =$ ____

- (a) $x^3 - y^3$ (b) $x^3 + y^3$
 (c) $(x+y)^3$ (d) $(x-y)^3$

13. Factors of $x^4 - 16$ is ____

- (a) $(x-2)^2$
 (b) $(x-2)(x+2)(x^2+4)$
 (c) $(x-2)(x+2)$
 (d) $(x+2)^2$

14. Factors of $3x - 3a + xy - ay$.

- (a) $(3+y)(x-a)$
 (b) $(3-y)(x+a)$
 (c) $(3-y)(x-a)$
 (d) $(3+y)(x+a)$

15. Factors of $pqr + qr^2 - pr^2 - r^3$ is:

- (a) $r(p+r)(q-r)$ (b) $r(p-r)(q+r)$
 (c) $r(p-r)(q-r)$ (d) $r(p+r)(q+r)$

Answer Key

1.	b	2.	c	3.	d	4.	b	5.	c
6.	c	7.	c	8.	a	9.	a	10.	a
11.	a	12.	b	13.	b	14.	a	15.	a