

ALGEBRAIC MANIPULATION

Highest Common Factor (H.C.F.)

If two or more algebraic expressions are given then their common factor of highest power is called the H.C.F of the expressions.

Least Common Multiple (L.C.M)

If an algebraic expression $p(x)$ is exactly divisible by two or more expressions, then $p(x)$ is called the Common Multiple of the given expressions. The Least Common Multiple (L.C.M) is the product of common factors together with non-common factors of the given expressions.

Finding H.C.F

We can find H.C.F of given expressions by the following two methods.

(i) By Factorization

(ii) By division

H.C.F. by Factorization

Example

Find the H.C.F of the following polynomials.

$$x^2 - 4, x^2 + 4x + 4, 2x^2 + x - 6$$

Solution

By factorization,

$$x^2 - 4 = (x + 2)(x - 2)$$

$$x^2 + 4x + 4 = (x + 2)^2 = (x + 2)(x + 2)$$

$$2x^2 + x - 6 = 2x^2 + 4x - 3x - 6 = 2x(x + 2) - 3(x + 2) \\ = (x + 2)(2x - 3)$$

Common factors = $x + 2$

$$\text{H.C.F} = x + 2$$

H.C.F. by Division

Example

Use division method to find the H.C.F. of the polynomials

$$p(x) = x^3 - 7x^2 + 14x - 8 \text{ and}$$

$$q(x) = x^3 - 7x + 6$$

Solution

$$\begin{array}{r} x^3 - 7x + 6 \overline{) x^3 - 7x^2 + 14x - 8} \\ \underline{+ x^3 \quad \quad - 7x + 6} \\ -7x^2 + 21x - 14 \end{array}$$

Here the remainder can be factorized as

$$-7x^2 + 21x - 14 = -7(x^2 - 3x + 2)$$

We ignore -7 because it is not common to both the given polynomials and consider $x^2 - 3x + 2$.

$$\begin{array}{r} x^2 - 3x + 2 \overline{) x^3 + 0x^2 - 7x + 6} \\ \underline{+ x^3 - 3x^2 + 2x} \\ 3x^2 - 9x + 6 \\ \underline{3x^2 - 9x + 6} \\ 0 \end{array}$$

Hence H.C. F of $p(x)$ and $q(x)$ is $x^2 - 3x + 2$

Example

Find the L.C.M of $p(x)=12(x^3-y^3)$ and $q(x)=8(x^3-xy^2)$

Solution

By prime factorization of the given expressions, we have

$$p(x)=12(x^3-y^3)=2^2 \times 3 \times (x-y)(x^2+xy+y^2) \text{ and}$$

$$q(x)=8(x^3-xy^2)=8x(x^2-y^2)=2^3 x(x+y)(x-y) \text{ Hence L.C.M. of } p(x) \text{ and } q(x),$$

$$2^3 \times 3 \times x(x+y)(x-y)(x^2+xy+y^2)=24x(x+y)(x^3-y^3)$$

Relation between H.C.F and L.C.M**Example**

By factorization, find (i) H.C.F (ii) L.C.M of $p(x)=12(x^5-x^4)$ and $q(x)=8(x^4-3x^3+3x^2)$. Establish a relation between $p(x)$, $q(x)$ and H.C.F and L.C.M of the expressions $p(x)$ and $q(x)$.

Solution

Firstly, let us factorize completely the given expressions $p(x)$ and $q(x)$ into irreducible factors. We have

$$p(x)=12(x^5-x^4)=12x^4(x-1)=2^2 \times 3 \times x^4(x-1) \text{ and}$$

$$q(x) = 8(x^4-3x^3+2x^2)=8x^2(x^2-3x+2)=2^3 x^2(x-1)(x-2)$$

$$\text{H.C.F. of } p(x) \text{ and } q(x) = 2^2 x^2(x-1)=4x^2(x-1)$$

$$\text{L.C.M of } p(x) \text{ and } q(x) = 2^3 \times 3 \times x^4(x-1)(x-2)$$

$$\begin{aligned} \text{Now } p(x) \times q(x) &= 12x^4(x-1) \times 8x^2(x-1)(x-2) \\ &= 96x^6(x-1)^2(x-2) \dots\dots\dots(i) \end{aligned}$$

and (L.C.M) (H.C.F)

$$= [2^3 \times 3 \times x^4(x-1)(x-2)] [4x^2(x-1)]$$

$$= [24x^4(x-1)(x-2)] [4x^2(x-1)]$$

$$= 96x^6(x-1)^2(x-2) \dots\dots\dots(ii)$$

From (i) and (ii)

$$\text{L.C.M} \times \text{H.C.F} = P(x) \times q(x)$$

Note

$$(1) \quad \text{L.C.M} = \frac{p(x) \times q(x)}{\text{H.C.F}} \quad \text{or}$$

$$\text{H.C.F} = \frac{p(x) \times q(x)}{\text{L.C.M}}$$

(2) If L.C.M, H.C.F and one of $p(x)$ or $q(x)$ are known, then

$$p(x) = \frac{\text{L.C.M} \times \text{H.C.F}}{q(x)}$$

$$q(x) = \frac{\text{L.C.M} \times \text{H.C.F}}{p(x)}$$

Example

Find H.C.F of the polynomials,

$$p(x) = 20(2x^3 + 3x^2 - 2x)$$

$$q(x) = 9(5x^4 + 40x)$$

Then using the above formula (I) find the L.C.M of $p(x)$ and $q(x)$.

Solution

We have

$$p(x) = 20(2x^3 + 3x^2 - 2x) = 20x(2x^2 + 3x - 2)$$

$$= 20x(2x^2 + 4x - x - 2) = 20x[2x(x+2) - (x+2)] = 20x(x+2)(2x-1) = 2^2 \times 5 \times x(x+2)(2x-1)$$

$$q(x) = 9(5x^4 + 40x) = 45x(x^3 + 8) = 45x\left[(x^3) + (2)^3\right]$$

$$= 45x(x+2)(x^2 - 2x + 4) = 5 \times 3^2 \times x(x+2)(x^2 - 2x + 4) \text{ Thus H.C.F of } p(x) \text{ and } q(x) \text{ is:}$$

$$= 5x(x+2)$$

$$\text{Now, using the formula} \quad \text{L.C.M.} = \frac{p(x) \times q(x)}{\text{H.C.F}}$$

We obtain

$$\begin{aligned} \text{L.C.M.} &= \frac{2^2 \times 5 \times x(x+2)(2x-1) \times 5 \times 3^2 \times x(x+2)(x^2 - 2x + 4)}{5x(x+2)} \\ &= 4 \times 5 \times 9 \times x(x+2)(2x-1)(x^2 - 2x + 4) \\ &= 180x(x+2)(2x-1)(x^2 - 2x + 4) \end{aligned}$$

Example

Find the L.C.M of

$$p(x) = 6x^3 - 7x^2 - 27x + 8 \quad \text{and}$$

$$q(x) = 6x^3 + 17x^2 + 9x - 4$$

Solution

We have, by long division,

$$\begin{array}{r} 1 \\ 6x^3 - 7x^2 - 27x + 8 \overline{) 6x^3 + 17x^2 + 9x - 4} \\ \underline{6x^3 - 7x^2 - 27x + 8} \\ 24x^2 + 36x - 12 \end{array}$$

But the remainder $24x^2 + 36x - 12$

$$= 12(2x^2 + 3x - 1)$$

Thus, ignoring 12, we have

$$\begin{array}{r} 2x^2 + 3x - 1 \overline{) 6x^3 - 7x^2 - 27x + 8} \\ \underline{6x^3 + 9x^2 - 3x} \\ -16x^2 - 24x + 8 \\ \underline{-16x^2 - 24x + 8} \\ 0 \end{array}$$

Hence H.C.F of $p(x)$ and $q(x)$ is

$$= 2x^2 + 3x - 1$$

$$x^2 + 6x - 27 = x^2 - 3x + 9x - 27$$

$$= x(x-3) + 9(x-3)$$

$$= (x-3)(x+9) \quad \dots\dots(ii)$$

$$2x^2 - 18 = 2(x^2 - 9)$$

$$= 2[(x)^2 - (3)^2]$$

$$= 2(x+3)(x-3) \quad \dots\dots(iii)$$

From (i), (ii) and (iii)

Common factors = $(x-3)$

$$HCF = x-3$$

$$iii) x^3 - 2x^2 + x, x^2 + 2x - 3, x^2 + 3x - 4$$

Sol: By factorization

$$x^3 - 2x^2 + x = x(x^2 - 2x + 1)$$

$$= x(x^2 - x - x + 1)$$

$$= x[x(x-1) - 1(x-1)]$$

$$= x(x-1)(x-1) \quad \dots\dots(i)$$

$$x^2 + 2x - 3 = x^2 - x + 3x - 3$$

$$= x(x-1) + 3(x-1)$$

$$= (x-1)(x+3) \quad \dots\dots(ii)$$

$$x^2 + 3x - 4 = x^2 - x + 4x - 4$$

$$= x(x-1) + 4(x-1)$$

$$= (x-1)(x+4) \quad \dots\dots(iii)$$

From (i), (ii) and (iii)

Common factors: $x-1$

$$HCF = x-1$$

$$iv) 18(x^3 + 9x^2 + 8x), 24(x^2 - 3x + 2)$$

Sol: By factorization

$$18(x^3 + 9x^2 + 8x) = 18x(x^2 + 9x + 8)$$

$$= 18x(x^2 - x - 8x + 8)$$

$$= 18x[x(x-1) - 8(x-1)]$$

$$= 2 \times 3 \times 3 \times x(x-1)(x-8) \quad \dots\dots(i)$$

$$24(x^2 - 3x + 2) =$$

$$24(x^2 - x - 2x + 2)$$

$$= 2 \times 2 \times 2 \times 3[x(x-1) - 2(x-1)]$$

$$= 2 \times 2 \times 2 \times 3(x-1)(x-2) \quad \dots(ii)$$

From (i) and (ii)

$$HCF = 2 \times 3(x-1)$$

$$= 6(x-1)$$

$$v) 36(3x^4 + 5x^3 - 2x^2), 54(27x^4 - x)$$

Sol: By factorization

$$36(3x^4 + 5x^3 - 2x^2) = 36x^2(3x^2 + 5x - 2)$$

$$= 36x^2(3x^2 + 6x - x - 2)$$

$$= 36x^2[3x(x+2) - 1(x+2)]$$

$$= 2 \times 2 \times 3 \times 3 \times x \times x(x+2)(3x-1) \quad \dots(i)$$

$$54(27x^4 - x) = 54x(27x^3 - 1)$$

$$= 54x[(3x)^3 - (1)^3]$$

$$= 54x(3x-1)[(3x)^2 + (3x)(1) + (1)^2]$$

$$= 2 \times 3 \times 3 \times 3 \times x(3x-1)(9x^2 + 3x + 1) \quad \dots(ii)$$

From (i) and (ii)

$$\text{Common factors} = 2, 3, 3, x, (3x-1)$$

$$HCF = 2 \times 3 \times 3 \times x(3x-1)$$

$$= 18x(3x-1)$$

Q3. Find the H.C.F of the following by division methal.

$$i) p(x) = x^3 + 3x^2 - 16x + 12, q(x) = x^3 + x^2 - 10x + 8$$

$$\begin{array}{r} \text{Sol: } x^3 + x^2 - 10x + 8 \overline{) x^3 + 3x^2 - 16x + 12} \\ \underline{-x^3 + x^2 + 10x + 8} \\ 2x^2 - 6x + 4 \end{array}$$

Dividing remainder by 2

$$\begin{array}{r}
 x^2 - 3x + 2 \\
 x^2 - 3x + 2 \overline{) \begin{array}{l} x^4 + x^2 - 10x + 8 \\ -x^4 + 3x^2 + 2x \\ \hline 4x^2 - 12x + 8 \\ -4x^2 - 12x + 8 \\ \hline 0 \end{array}}
 \end{array}$$

Hence HCF = $x^2 - 3x + 2$

ii) $P(x) = x^4 + x^3 - 2x^2 + x - 3,$

$q(x) = 5x^3 + 3x^2 - 17x + 6$

$$\begin{array}{r}
 5x^3 + 3x^2 - 17x + 6 \overline{) \begin{array}{l} x^4 + x^3 - 2x^2 + x - 3 \\ \times 5 \\ \hline 5x^4 + 5x^3 - 10x^2 + 5x - 15 \\ -5x^4 + 3x^3 + 17x^2 + 6x \\ \hline 2x^3 + 7x^2 - x - 15 \\ \times 5 \\ \hline 10x^3 + 35x^2 - 5x - 75 \\ -10x^3 + 6x^2 + 34x + 12 \\ \hline 29x^2 + 29x - 87 \end{array}}
 \end{array}$$

(Multiplying by 5)

(Multiplying by 5)

Divided by 29

$x^2 + x - 3$

$$\begin{array}{r}
 5x - 2 \\
 x^2 + x - 3 \overline{) \begin{array}{l} 5x^3 + 3x^2 - 17x + 6 \\ -5x^3 + 5x^2 + 15x \\ \hline -2x^2 - 2x + 6 \\ -2x^2 + 2x + 6 \\ \hline 0 \end{array}}
 \end{array}$$

Hence H.C.F = $x^2 + x - 3$

iii) $p(x) = 2x^5 - 4x^4 - 6x,$

$q(x) = x^5 + x^4 - 3x^3 - 3x^2$

$$\begin{array}{r}
 2 \\
 x^5 + x^4 - 3x^3 - 3x^2 \overline{) \begin{array}{l} 2x^5 - 4x^4 - 6x \\ -2x^5 + 2x^4 + 6x^3 + 6x^2 \\ \hline -6x^4 + 6x^3 + 6x^2 - 6x \end{array}}
 \end{array}$$

Dividing by -6

$$\begin{array}{r}
 x^4 - x^3 - x^2 + x \\
 x^4 - x^3 - x^2 + x \overline{) \begin{array}{l} x^6 + x^4 - 3x^3 - 3x^2 \\ -x^6 + x^4 + x^3 + x^2 \\ \hline 2x^4 - 2x^3 - 4x^2 \\ -2x^4 + 2x^3 + 2x^2 + 2x \\ \hline -2x^2 - 2x \end{array}}
 \end{array}$$

Dividing by -2

$x^2 + x$

$$\begin{array}{r}
 x^2 - 2x + 1 \\
 x^2 + x \overline{) \begin{array}{l} x^4 - x^3 - x^2 + x \\ -x^4 + x^3 \\ \hline -2x^3 - x^2 + x \\ -2x^3 + 2x^2 \\ \hline x^2 + x \\ -x^2 - x \\ \hline 0 \end{array}}
 \end{array}$$

Hence H.C.F = $x^2 + x = x(x+1)$

Q4. Find the L.C.M of the following expressions:

i) $39x^7y^3z$ and $91x^5y^6z^7$

Sol: By factorization

$39x^7y^3z = 13 \times 3 \times x \times x \times x \times x \times x \times y \times y \times y \times z$

$91x^5y^6z^7 = 13 \times 7 \times x \times x \times x \times x \times y \times y \times y \times y \times y \times z \times z \times z \times z \times z \times z$

Hence L.C.M =

$13 \times 3 \times 7 \times x \times x \times x \times x \times x \times y \times y \times y \times y \times y \times z \times z \times z \times z \times z \times z$

$= 273x^7y^6z^7$

ii) $102xy^2z, 85x^2yz$ and $187xyz^2$

Sol: By factorization

$102xy^2z = 2 \times 3 \times 17 \times x \times y \times y \times z$

$85x^2yz = 5 \times 17 \times x \times x \times y \times z$

$187xyz^2 = 11 \times 17 \times x \times y \times z \times z$

$$\begin{aligned}\text{Hence L.C.M} &= 17 \times 11 \times 5 \times 3 \times 2 \times x \times x \times y \times y \times z \times z \\ &= 5610x^2y^2z^2\end{aligned}$$

Q5. Find the L.C.M of the following expressions by factorization:

i) $x^2 - 25x + 100$ and $x^2 - x - 20$

Sol: By factorization

$$\begin{aligned}x^2 - 25x + 100 &= x^2 - 5x - 20x + 100 \\ &= x(x-5) - 20(x-5) \\ &= (x-5)(x-20) \dots\dots\dots(i) \\ x^2 - x - 20 &= x^2 - 5x + 4x - 20 \\ &= x(x-5) + 4(x-5) \\ &= (x-5)(x+4) \dots\dots\dots(ii)\end{aligned}$$

From (i) and (ii)

$$\text{L.C.M} = (x-5)(x-20)(x+4)$$

ii) $x^2 + 4x + 4$, $x^2 - 4$, $2x^2 + x - 6$

Sol: By factorization

$$\begin{aligned}x^2 + 4x + 4 &= x^2 + 2x + 2x + 4 \\ &= x(x+2) + 2(x+2) \\ &= (x+2)(x+2) \dots\dots\dots(i) \\ x^2 - 4 &= (x)^2 - (2)^2 \\ &= (x+2)(x-2) \dots\dots\dots(ii) \\ 2x^2 + x - 6 &= 2x^2 + 4x - 3x - 6 \\ &= 2x(x+2) - 3(x+2) \\ &= (x+2)(2x-3) \dots\dots\dots(iii)\end{aligned}$$

From (i), (ii) and (iii)

$$\begin{aligned}\text{LCM} &= (x+2)(x+2)(x-2)(2x-3) \\ &= (x+2)^2(x-2)(2x-3)\end{aligned}$$

iii) $2(x^4 - y^4)$, $3(x^3 + 2x^2y - xy^2 - 2y^3)$

Sol: By factorization

$$2(x^4 - y^4) = 2[(x^2)^2 - (y^2)^2]$$

$$\begin{aligned}&= 2(x^2 + y^2)(x^2 - y^2) \\ &= 2(x^2 + y^2)(x+y)(x-y) \dots\dots\dots(i)\end{aligned}$$

$$\begin{aligned}3(x^3 + 2x^2y - xy^2 - 2y^3) &= 3[x^2(x+2y) - y^2(x+2y)] \\ &= 3(x+2y)(x^2 - y^2) \\ &= 3(x+2y)(x+y)(x-y) \dots\dots\dots(ii)\end{aligned}$$

From (i) & (ii)

L.C.M =

$$\begin{aligned}&2 \times 3(x+y)(x-y)(x^2 + y^2)(x+2y) \\ &= 6(x^4 - y^4)(x+2y)\end{aligned}$$

iv) $4(x^4 - 1)$, $6(x^3 - x^2 - x + 1)$

Sol: By factorization

$$\begin{aligned}4(x^4 - 1) &= 4[(x^2)^2 - (1)^2] \\ &= 4(x^2 + 1)(x^2 - 1) \\ &= 2 \times 2(x^2 + 1)[(x)^2 - (1)^2] \\ &= 2 \times 2(x^2 + 1)(x+1)(x-1) \dots\dots\dots(i) \\ 6(x^3 - x^2 - x + 1) &= 6[x^2(x-1) - 1(x-1)] \\ &= 6(x-1)(x^2 - 1) = 2 \times 3(x-1)[(x)^2 - (1)^2] \\ &= 2 \times 3(x-1)(x-1)(x+1) \dots\dots(ii)\end{aligned}$$

From (i) & (ii)

$$\begin{aligned}\text{LCM} &= 2 \times 2 \times 3(x+1)(x-1)(x^2 + 1)(x-1) \\ &= 12(x^4 - 1)(x-1)\end{aligned}$$

Q6. For what value of k is $(x+4)$, the H.C.F of $x^2 + x - (2k+2)$ and $2x^2 + kx - 12$?

Sol: $k = ?$

$$p(x) = x^2 + x - (2k+2) \text{ and}$$

$$q(x) = 2x^2 + kx - 12$$

As given that $x+4$ is HCF, so $p(x)$ and $q(x)$ will be exactly divisible by $(x+4)$

$$\begin{array}{r}
 x-3 \\
 x+4 \overline{) x^2 + x - (2k+2)} \\
 \underline{\cancel{x^2} + 4x} \\
 -3x - (2k+2) \\
 \underline{ 3x + 12} \\
 12 - (2k+2)
 \end{array}$$

$$= 12 - 2k - 2$$

$$= 10 - 2k$$

As $p(x)$ is exactly divisible by $x+4$, so,

$$10 - 2k = 0$$

$$10 = 2k$$

$$\frac{10}{2} = k$$

$$k = 5$$

Q7. If $(x+3)(x-2)$ is the H.C.F of

$$p(x) = (x+3)(2x^2 - 3x + k) \text{ and}$$

$$q(x) = (x-2)(3x^2 + 7x - l), \text{ find } k \text{ and } l.$$

Sol: $k = ?$ and $l = ?$

As $(x+3)(x-2)$ is the H.C.F, so $p(x)$ and $q(x)$ will be exactly divisible by

$(x+3)(x-2)$ i.e., $\frac{p(x)}{HCF}$ has remainder zero.

$$\frac{(x+3)(2x^2 - 3x + k)}{(x+3)(x-2)} = \frac{2x^2 - 3x + k}{x-2}$$

$$\begin{array}{r}
 2x+1 \\
 x-2 \overline{) 2x^2 - 3x + k} \\
 \underline{\pm 2x^2 \mp 4x} \\
 x + k \\
 \underline{ \pm x \mp 2} \\
 k + 2
 \end{array}$$

As remainder = 0, then

$$k + 2 = 0$$

$$\boxed{k = -2}$$

and $\frac{q(x)}{HCF}$ has zero remainder

$$\frac{(x-2)(3x^2 + 7x - l)}{(x+3)(x-2)} = \frac{3x^2 + 7x - l}{x+3}$$

$$\begin{array}{r}
 3x-2 \\
 x+3 \overline{) 3x^2 + 7x - l} \\
 \underline{\pm 3x^2 \pm 9x} \\
 -2x - l \\
 \underline{ 2x \mp 6} \\
 -l + 6
 \end{array}$$

As remainder = 0

$$-l + 6 = 0$$

$$-l = -6$$

$$\Rightarrow \boxed{l = 6}$$

Q8. The LCM and HCF of two polynomials $p(x)$ and $q(x)$ are $2(x^4 - 1)$ and $(x+1)(x^2 + 1)$ respectively. If $p(x) = x^3 + x + 1$, find $q(x)$.

$$\text{Sol: LCM} = 2(x^4 - 1),$$

$$\text{HCF} = (x+1)(x^2 + 1)$$

$$p(x) = x^3 + x^2 + x + 1, \quad q(x) = ?$$

$$\text{As } p(x) \times q(x) = (\text{LCM}) \times (\text{HCF})$$

$$q(x) = \frac{(\text{LCM}) \times (\text{HCF})}{p(x)}$$

$$= \frac{2(x^4 - 1) \times (x+1)(x^2 + 1)}{x^3 + x^2 + x + 1}$$

$$= \frac{2(x^4 - 1)(x^3 + x^2 + x + 1)}{x^3 + x^2 + x + 1}$$

$$q(x) = 2(x^4 - 1)$$

Q9. Let $p(x) = 10(x^2 - 9)(x^2 - 3x + 2)$ and $q(x) = 10x(x+3)(x-1)^2$. If the H.C.F. of $p(x), q(x)$ is $10(x+3)(x-1)$, find their L.C.M.

Sol: $p(x) = 10(x^2 - 9)(x^2 - 3x + 2)$,

$$q(x) = 10x(x+3)(x-1)^2$$

$$\text{H.C.F.} = 10(x+3)(x-1), \text{ L.C.M.} = ?$$

As $(\text{L.C.M.}) \times (\text{H.C.F.}) = p(x) \times q(x)$

$$\text{L.C.M.} = \frac{p(x) \times q(x)}{\text{H.C.F.}}$$

$$= \frac{10(x^2 - 9)(x^2 - 3x + 2) \times 10x(x+3)(x-1)^2}{10(x+3)(x-1)}$$

$$= \frac{(x^2 - 9)(x^2 - 3x + 2) \times 10x \cancel{(x+3)} \cancel{(x-1)} (x-1)}{\cancel{(x+3)} \cancel{(x-1)}}$$

$$= 10x(x-1)(x^2 - 9)(x^2 - 3x + 2)$$

$$= 10x(x-1)(x^2 - 9)(x^2 - x - 2x + 2)$$

$$= 10x(x-1)(x^2 - 9)[x(x-1) - 2(x-1)]$$

$$= 10x(x-1)(x^2 - 9)(x-1)(x-2)$$

$$= 10x(x-1)^2(x^2 - 9)(x-2)$$

Q10. Let the product of L.C.M and H.C.F of two polynomials be $(x+3)^2(x-2)(x+5)$. If one polynomial is $(x+3)(x-2)$ and the second polynomial is $x^2 + kx + 15$, find the value of k .

Sol: $k = ?$

Product of L.C.M. & H.C.F is

$$\text{LCM} \times \text{HCF} = (x+3)^2(x-2)(x+5)$$

$$p(x) = (x+3)(x-2)$$

$$q(x) = x^2 + kx + 15$$

As $p(x) \times q(x) = \text{LCM} \times \text{HCF}$

$$(x+3)(x-2)(x^2 + kx + 15)$$

$$= (x+3)^2(x-2)(x+5)$$

$$x^2 + kx + 15 = \frac{(x+3)\cancel{(x+3)}\cancel{(x-2)}(x+5)}{\cancel{(x+3)}\cancel{(x-2)}}$$

$$x^2 + kx + 15 = (x+3)(x+5)$$

$$x^2 + kx + 15 = x^2 + 3x + 5x + 15$$

$$x^2 + kx + 15 = x^2 + 8x + 15$$

Comparing co-efficient of 'x'

$$\Rightarrow kx = 8x$$

$$\boxed{k = 8}$$

Q11. Waqas wishes to distribute 128 bananas and also 176 apples equally among a certain number of children. Find the highest number of the Children. Who can get the fruit in this way?

Sol: No. of bananas = 128

No. of apples = 176

Highest no. of children who get the fruit in this way is H.C.F.

So No. of bananas =

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

No. of apples =

$$2 \times 2 \times 2 \times 2 \times 11$$

Hence required no. of children =

$$2 \times 2 \times 2 \times 2 = 16$$

Example

Simplify

$$\frac{x+3}{x^2-3x+2} + \frac{x+2}{x^2-4x+3} + \frac{x+1}{x^2-5x+6}, x \neq 1, 2, 3$$

Solution

$$\frac{x+3}{x^2-3x+2} + \frac{x+2}{x^2-4x+3} + \frac{x+1}{x^2-5x+6}$$

$$= \frac{x+3}{x^2-2x-x+2} + \frac{x+2}{x^2-3x-x+3} + \frac{x+1}{x^2-3x-2x+6}$$

$$= \frac{x+3}{x(x-2)-1(x-2)} + \frac{x+2}{x(x-3)-1(x-3)} + \frac{x+1}{x(x-3)-2(x-3)}$$

$$\begin{aligned}
&= \frac{x+3}{(x-2)(x-1)} + \frac{x+2}{(x-3)(x-1)} + \frac{x+1}{(x-3)(x-2)} \\
&= \frac{(x+3)(x-3) + (x+2)(x-2) + (x+1)(x-1)}{(x-1)(x-2)(x-3)} \\
&= \frac{x^2-9+x^2-4+x^2-1}{(x-1)(x-2)(x-3)} \\
&= \frac{3x^2-14}{(x-1)(x-2)(x-3)}
\end{aligned}$$

Example

Express the product $\frac{x^3-8}{x^2-4} \times \frac{x^2+6x+8}{x^2-2x+1}$

as an algebraic expression reduced lowest forms $x \neq 2, -2, 1$

Solution

By factorizing completely, we have

$$\begin{aligned}
&\frac{x^3-8}{x^2-4} \times \frac{x^2+6x+8}{x^2-2x+1} \\
&= \frac{(x-2)(x^2+2x+4) \times (x+2)(x+4)}{(x-2)(x+2) \times (x-1)^2} \dots (i)
\end{aligned}$$

Now the factors of numerator are $(x-2), (x^2+2x+4), (x+2)$ and $(x+4)$ and the factors of denominator are

$(x-2), (x+2)$ and $(x-1)^2$.

Therefore, their H.C.F. is $(x-2) \times (x+2)$

By cancelling H.C.F. i.e., $(x-2) \times (x+2)$ from (i), we get the simplified form of given product as the fraction $\frac{(x^2+2x+4)(x+4)}{(x-1)^2}$

Example

Divide $\frac{x^2+x+1}{x^2-9}$ by $\frac{x^3-1}{x^2-4x+3}$

and simplify by reducing to lowest forms.

Solution

$$\begin{aligned}
&\text{We have } \frac{x^2+x+1}{x^2-9} \div \frac{x^3-1}{x^2-4x+3} \\
&= \frac{(x^2+x+1)}{(x^2-9)} \times \frac{(x^2-4x+3)}{(x^3-1)} \\
&= \frac{(x^2+x+1)(x^2-x-3x+3)}{(x^2-9)(x^3-1)} \\
&= \frac{(x^2+x+1)[x(x-1)-3(x-1)]}{(x+3)(x-3)(x-1)(x^2+x+1)} \\
&= \frac{(x^2+x+1)(x-3)(x-1)}{(x+3)(x-3)(x-1)(x^2+x+1)} = \frac{1}{x+3}, x \neq -3
\end{aligned}$$