

Exercise 6.2

Simplify each of the following as a rational expression.

$$\begin{aligned}\text{Q1. } & \frac{x^2 - x - 6}{x^2 - 9} + \frac{x^2 + 2x - 24}{x^2 - x - 12} \\ &= \frac{x^2 - 3x + 2x - 6}{(x)^2 - (3)^2} + \frac{x^2 + 6x - 4x - 24}{x^2 + 3x - 4x - 12}\end{aligned}$$

$$\begin{aligned}&= \frac{x(x-3) + 2(x-3)}{(x+3)(x-3)} + \frac{x(x+6) - 4(x+6)}{x(x+3) - 4(x+3)} \\ &= \frac{(x-3)(x+2)}{(x+3)(x-3)} + \frac{(x+6)(x-4)}{(x+3)(x-4)} \\ &= \frac{x+2}{x+3} + \frac{x+6}{x+3} = \frac{x+2+x+6}{x+3}\end{aligned}$$

$$= \frac{2x+8}{x+3}$$

$$= \frac{2(x+4)}{x+3}$$

Q2. $\left[\frac{x+1}{x-1} - \frac{x-1}{x+1} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$

$$= \left[\frac{(x+1)^2 - (x-1)^2}{(x-1)(x+1)} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$$

$$= \left[\frac{(x^2+2x+1) - (x^2-2x+1)}{(x)^2 - (1)^2} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$$

$$= \left[\frac{x^2+2x+1-x^2+2x-1}{x^2-1} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$$

$$= \left[\frac{4x}{x^2-1} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$$

$$= \left[\frac{4x(x^2+1) - 4x(x^2-1)}{(x^2-1)(x^2+1)} \right] + \frac{4x}{x^4-1}$$

$$= \frac{4x^3+4x-4x^3+4x}{(x^2)^2 - (1)^2} + \frac{4x}{x^4-1}$$

$$= \frac{8x}{x^4-1} + \frac{4x}{x^4-1}$$

$$= \frac{8x+4x}{x^4-1}$$

$$= \frac{12x}{x^4-1}$$

Q3. $\frac{1}{x^2-8x+15} + \frac{1}{x^2-4x+3} - \frac{2}{x^2-6x+5}$

$$= \frac{1}{x^2-3x-5x+15} + \frac{1}{x^2-3x-x+3} - \frac{2}{x^2-5x-x+5}$$

$$= \frac{1}{x(x-3)-5(x-3)} + \frac{1}{x(x-3)-1(x-3)} - \frac{2}{x(x-5)-1(x-5)}$$

$$= \frac{1}{(x-3)(x-5)} + \frac{1}{(x-3)(x-1)} - \frac{2}{(x-5)(x-1)}$$

$$= \frac{x-1+x-5-2(x-3)}{(x-1)(x-3)(x-5)}$$

$$= \frac{x-1+x-5-2x+6}{(x-1)(x-3)(x-5)}$$

$$= \frac{2x-6-2x+6}{(x-1)(x-3)(x-5)}$$

$$= \frac{0}{(x-1)(x-3)(x-5)}$$

$$= 0$$

Q4. $\frac{(x+2)(x+3)}{x^2-9} + \frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

$$= \frac{(x+2)(x+3)}{(x)^2 - (3)^2} + \frac{(x+2) \cdot 2(x^2-16)}{(x-4)(x^2+2x-3x-6)}$$

$$= \frac{(x+2)(x+3)}{(x-3)(x+3)} + \frac{2(x+2)[(x)^2 - (4)^2]}{(x-4)(x^2+2x-3x-6)}$$

$$= \frac{(x+2)}{x-3} + \frac{2(x+2)(x+4)(x-4)}{(x-4)(x+2)(x-3)}$$

$$= \frac{x+2}{x-3} + \frac{2x+8}{x-3}$$

$$= \frac{x+2+2x+8}{x-3}$$

$$= \frac{3x+10}{x-3}$$

Q5. $\frac{x+3}{2x^2+9x+9} + \frac{1}{2(2x-3)} - \frac{4x}{4x^2-9}$

$$= \frac{x+3}{2x^2+6x+3x+9} + \frac{1}{2(2x-3)} - \frac{4x}{(2x)^2 - (3)^2}$$

$$= \frac{x+3}{2x(x+3)+3(x+3)} + \frac{1}{2(2x-3)} - \frac{4x}{(2x+3)(2x-3)}$$

$$\begin{aligned}
&= \frac{\cancel{x+3}}{(\cancel{x+3})(2x+3)} + \frac{1}{2(2x-3)} - \frac{4x}{(2x+3)(2x-3)} \\
&= \frac{1}{2x+3} + \frac{1}{2(2x-3)} - \frac{4x}{(2x+3)(2x-3)} \\
&= \frac{2(2x-3) + 2x+3 - 2(4x)}{2(2x+3)(2x-3)} \\
&= \frac{4x-6+2x+3-8x}{2(2x+3)(2x-3)} \\
&= \frac{-2x-3}{2(2x+3)(2x-3)} \\
&= \frac{-1(\cancel{2x+3})}{2(\cancel{2x+3})(2x-3)} \\
&= \frac{-1}{2(2x-3)} \\
&= \frac{1}{2(3-2x)}
\end{aligned}$$

Q6. $A - \frac{1}{A}$, where $A = \frac{a+1}{a-1}$

so $\frac{1}{A} = \frac{a-1}{a+1}$

Now $A - \frac{1}{A} = \frac{a+1}{a-1} - \frac{a-1}{a+1}$

$$\begin{aligned}
&= \frac{(a+1)^2 - (a-1)^2}{(a-1)(a+1)} \\
&= \frac{(a^2 + 2a + 1) - (a^2 - 2a + 1)}{(a)^2 - (1)^2} \\
&= \frac{\cancel{a^2} + 2a + \cancel{1} - \cancel{a^2} + 2a - \cancel{1}}{a^2 - 1} \\
&= \frac{4a}{a^2 - 1}
\end{aligned}$$

Q7. $\left[\frac{x-1}{x-2} + \frac{2}{2-x} \right] - \left[\frac{x+1}{x+2} + \frac{4}{4-x^2} \right]$

$$\begin{aligned}
&= \left[\frac{-(x-1)}{2-x} + \frac{2}{2-x} \right] - \left[\frac{x+1}{x+2} + \frac{4}{(2)^2 - (x)^2} \right] \\
&= \left[-\frac{(x-1)}{2-x} + \frac{2}{2-x} \right] - \left[\frac{x+1}{x+2} + \frac{4}{(2+x)(2-x)} \right] \\
&= \left[\frac{-x+1+2}{2-x} \right] - \left[\frac{(x+1)(2-x)+4}{(2+x)(2-x)} \right] \\
&= \frac{3-x}{2-x} - \left[\frac{2x-x^2+2-x+4}{(2+x)(2-x)} \right] \\
&= \frac{3-x}{2-x} - \left[\frac{6+x-x^2}{(2+x)(2-x)} \right] \\
&= \frac{3-x}{2-x} - \left[\frac{6+3x-2x-x^2}{(2+x)(2-x)} \right] \\
&= \frac{3-x}{2-x} - \left[\frac{3(2+x)-x(2+x)}{(2+x)(2-x)} \right] \\
&= \frac{3-x}{2-x} - \left[\frac{(\cancel{2+x})(3-x)}{(\cancel{2+x})(2-x)} \right] \\
&= \frac{3-x}{2-x} - \frac{3-x}{2-x} \\
&= \frac{3-x-3+x}{2-x} \\
&= \frac{0}{2-x} \\
&= 0
\end{aligned}$$

Q8. What rational expression should

be subtracted from $\frac{2x^2+2x-7}{x^2+x-6}$ to get

$$\frac{x-1}{x-2} = ?$$

Sol: Let the required expression be A,

$$\text{then } \frac{2x^2 + 2x - 7}{x^2 + x - 6} - A = \frac{x-1}{x-2}$$

$$\text{or } \frac{2x^2 + 2x - 7}{x^2 + x - 6} - \frac{x-1}{x-2} = A$$

$$\begin{aligned} \text{So } A &= \frac{2x^2 + 2x - 7}{x^2 + 3x - 2x - 6} - \frac{x-1}{x-2} \\ &= \frac{2x^2 + 2x - 7}{x(x+3) - 2(x+3)} - \frac{x-1}{x-2} \\ &= \frac{2x^2 + 2x - 7}{(x+3)(x-2)} - \frac{x-1}{x-2} \\ &= \frac{2x^2 + 2x - 7 - (x-1)(x+3)}{(x+3)(x-2)} \\ &= \frac{2x^2 + 2x - 7 - (x^2 - x + 3x - 3)}{(x+3)(x-2)} \\ &= \frac{(2x^2 + 2x - 7) - (x^2 + 2x - 3)}{(x+3)(x-2)} \\ &= \frac{2x^2 + 2x - 7 - x^2 - 2x + 3}{(x+3)(x-2)} \\ &= \frac{x^2 - 4}{(x+3)(x-2)} \\ &= \frac{(x)^2 - (2)^2}{(x+3)(x-2)} \\ &= \frac{(x+2)(x-2)}{(x+3)(x-2)} \\ &= \frac{x+2}{x+3} \end{aligned}$$

Perform the indicated operations and simplify to the lowest forms.

$$\text{Q9. } \frac{x^2 + x - 6}{x^2 - x - 6} \times \frac{x^2 - 4}{x^2 - 9}$$

$$\begin{aligned} &= \frac{x^2 + 3x - 2x - 6}{x^2 - 3x + 2x - 6} \times \frac{(x)^2 - (2)^2}{(x)^2 - (3)^2} \\ &= \frac{x(x+3) - 2(x+3)}{x(x-3) + 2(x-3)} \times \frac{(x+2)(x-2)}{(x+3)(x-3)} \\ &= \frac{(x+3)(x-2)}{(x-3)(x+2)} \times \frac{(x+2)(x-2)}{(x+3)(x-3)} \\ &= \frac{(x-2)^2}{(x-3)^2} \end{aligned}$$

$$\begin{aligned} \text{Q10. } &\frac{x^3 - 8}{x^2 - 4} \times \frac{x^2 + 6x + 8}{x^2 - 2x + 1} \\ &= \frac{(x)^3 - (2)^3}{(x)^2 - (2)^2} \times \frac{x^2 + 2x + 4x + 8}{x^2 - x - x + 1} \\ &= \frac{(x-2)[(x)^2 + (x)(2) + (2)^2]}{(x-2)(x+2)} \times \frac{x(x+2) + 4(x+2)}{x(x-1) - 1(x-1)} \\ &= \frac{x^2 + 2x + 4}{x+2} \times \frac{(x+2)(x+4)}{(x-1)(x-1)} \\ &= \frac{(x^2 + 2x + 4)(x+4)}{(x-1)^2} \end{aligned}$$

$$\begin{aligned} \text{Q11. } &\frac{x^4 - 8x}{2x^2 + 5x - 3} \times \frac{2x-1}{x^2 + 2x + 4} \times \frac{x+3}{x^2 - 2x} \\ &= \frac{x(x^3 - 8)}{2x^2 + 6x - x - 3} \times \frac{2x-1}{x^2 + 2x + 4} \times \frac{x+3}{x(x-2)} \\ &= \frac{x[(x)^3 - (2)^3]}{2x(x+3) - 1(x+3)} \times \frac{2x-1}{x^2 + 2x + 4} \times \frac{x+3}{x(x-2)} \\ &= \frac{x(x-2)(x^2 + 2x + 4)}{(x+3)(2x-1)} \times \frac{2x-1}{x^2 + 2x + 4} \times \frac{x+3}{x(x-2)} \\ &= 1 \end{aligned}$$

$$\text{Q12. } \frac{2y^2 + 7y - 4}{3y^2 - 13y + 4} \div \frac{4y^2 - 1}{6y^2 + y - 1}$$

$$\begin{aligned}
&= \frac{2y^2 + 8y - y - 4}{3y^2 - y - 12y + 4} \div \frac{(2y)^2 - (1)^2}{6y^2 + 3y - 2y - 1} \\
&= \frac{2y(y+4) - 1(y+4)}{y(3y-1) - 4(3y-1)} \div \frac{(2y+1)(2y-1)}{3y(2y+1) - 1(2y+1)} \\
&= \frac{(y+4)\cancel{(2y-1)}}{\cancel{(3y-1)}(y-4)} \div \frac{\cancel{(2y+1)}(2y-1)}{\cancel{(2y+1)}\cancel{(3y-1)}} \\
&= \frac{(y+4)\cancel{(2y-1)}}{\cancel{(3y-1)}(y-4)} \times \frac{\cancel{(2y+1)}(3y-1)}{\cancel{(2y+1)}\cancel{(2y-1)}} \\
&= \frac{y+4}{y-4}
\end{aligned}$$

Q13. $\left[\frac{x^2 + y^2}{x^2 - y^2} - \frac{x^2 - y^2}{x^2 + y^2} \right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y} \right]$

$$\begin{aligned}
&= \left[\frac{(x^2 + y^2)^2 - (x^2 - y^2)^2}{(x^2 - y^2)(x^2 + y^2)} \right] \div \left[\frac{(x+y)^2 - (x-y)^2}{(x-y)(x+y)} \right] \\
&= \frac{x^4 + y^4 + 2x^2y^2 - (x^4 + y^4 - 2x^2y^2)^2}{(x^2 - y^2)(x^2 + y^2)} \\
&\quad + \frac{x^2 + y^2 + 2xy - x^2 - y^2 + 2xy}{x^2 - y^2} \\
&= \frac{\cancel{x^4} + \cancel{y^4} + 2x^2y^2 - \cancel{x^4} - \cancel{y^4} + 2x^2y^2}{(x^2 - y^2)(x^2 + y^2)} \\
&\quad + \frac{\cancel{x^2} + \cancel{y^2} + 2xy - \cancel{x^2} - \cancel{y^2} + 2xy}{x^2 - y^2} \\
&= \frac{4x^2y^2}{(x^2 - y^2)(x^2 + y^2)} \div \frac{4xy}{x^2 - y^2} \\
&= \frac{\cancel{4x^2}\cancel{y^2}}{(\cancel{x^2} - \cancel{y^2})(x^2 + y^2)} \times \frac{\cancel{x^2} - \cancel{y^2}}{\cancel{4xy}} \\
&= \frac{xy}{x^2 + y^2}
\end{aligned}$$

Square Root of Algebraic Expression

The square root of a given expression $p(x)$ as another expression $q(x)$ such that $q(x) \cdot q(x) = p(x)$.

As $5 \times 5 = 25$, so square root of 25 is 5

It means we can find square root of the expression $p(x)$ if it can be expressed as a perfect square.

Example

Use factorization to find the square root of the expression

$$4x^2 - 12x + 9$$

Solution

$$\begin{aligned}
&\text{We have, } 4x^2 - 12x + 9 \\
&= 4x^2 - 6x - 6x + 9 = 2x(2x - 3) - 3(2x - 3) \\
&= (2x - 3)(2x - 3) = (2x - 3)^2
\end{aligned}$$

$$\begin{aligned}
&\text{Hence } \sqrt{4x^2 - 12x + 9} \\
&= \pm(2x - 3)
\end{aligned}$$

Example

Find the square root of $x^2 + \frac{1}{x^2} + 12\left(x + \frac{1}{x}\right) + 38, x \neq 0$

Solution

$$\begin{aligned}
&\text{We have } x^2 + \frac{1}{x^2} + 12\left(x + \frac{1}{x}\right) + 38 \\
&= x^2 + \frac{1}{x^2} + 2 + 12\left(x + \frac{1}{x}\right) + 36, \\
&\quad (\text{adding and subtracting } 2)
\end{aligned}$$

$$= \left(x + \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right)(6) + (6)^2$$

$$= \left[\pm \left(x + \frac{1}{x} + 6\right) \right]^2;$$

$$\text{since } a^2 + 2ab + b^2 = (a+b)^2$$

Hence the required square root is

$$\pm \left(x + \frac{1}{x} + 6\right)$$

Example

Find the square root of $4x^4 + 12x^3 + x^2 - 12x + 4$

Solution

$$\begin{array}{r}
 2x^2 + 3x - 2 \\
 \hline
 4x^4 + 12x^3 + x^2 - 12x + 4 \\
 \underline{4x^4} \\
 12x^3 + x^2 - 12x + 4 \\
 \underline{12x^3 + 9x^2} \\
 -8x^2 - 12x + 4 \\
 \underline{-8x^2 + 12x + 4} \\
 0
 \end{array}$$

Thus square root of given expression is $\pm(2x^2 + 3x - 2)$

Example 2

Find the square root of the expression

$$4\frac{x^2}{y^2} + 8\frac{x}{y} + 16 + 12\frac{y}{x} + 9\frac{y^2}{x^2}$$

Solution

We note that the given expression is in descending powers of x .

$$\begin{array}{r}
 2\frac{x}{y} + 2 + 3\frac{y}{x} \\
 \hline
 4\frac{x^2}{y^2} + 8\frac{x}{y} + 16 + 12\frac{y}{x} + 9\frac{y^2}{x^2} \\
 \underline{4\frac{x^2}{y^2}} \phantom{+ 8\frac{x}{y}} \phantom{+ 12\frac{y}{x}} \phantom{+ 9\frac{y^2}{x^2}} \\
 8\frac{x}{y} + 16 \\
 \underline{\pm 8\frac{x}{y} + 4} \\
 12 + 12\frac{y}{x} + 9\frac{y^2}{x^2} \\
 \underline{\pm 12 \pm 12\frac{y}{x} \pm 9\frac{y^2}{x^2}} \\
 0
 \end{array}$$

Hence the square root of given expression is $\pm \left(2\frac{x}{y} + 2 + 3\frac{y}{x}\right)$

Example

To make the expression $x^4 - 10x^3 + 33x^2 - 42x + 20$ a perfect square,

- What should be added to it?
- What should be subtracted from it?
- What should be the value of x ?

$$\begin{array}{r}
 x^2 - 5x + 4 \\
 \hline
 x^4 - 10x^3 + 33x^2 - 42x + 20 \\
 \underline{\pm x^4} \\
 -10x^3 + 33x^2 \\
 \underline{-10x^3 + 25x^2} \\
 8x^2 - 42x + 20 \\
 \underline{-8x^2 - 40x + 16} \\
 -2x + 4
 \end{array}$$

For making the given expression a perfect square the remainder must be zero.

Hence

(i) We should add $(2x-4)$ to the given expression

(ii) We should subtract $(-2x+4)$ from the given expression

(iii) We should take $-2x+4=0$ to find the value of x . This gives the required value of x i.e., $x=2$.