

LINEAR EQUATIONS AND INEQUALITIES

Define Linear Equations

A linear equation in one unknown variable x is an equation of the form

$$ax + b = 0, \text{ where } a, b \in R \text{ and } a \neq 0.$$

A solution to a linear equation is any replacement or substitution for the variable x that makes the statement true. Two linear equations are said to be equivalent if they have exactly the same solution.

Example

$$\text{Solve the equation } \frac{3x}{2} - \frac{x-2}{3} = \frac{25}{6}$$

Solution

Multiplying each side of the given equation by 6

$$\begin{aligned} 9x - 2(x-2) &= 25 \\ \Rightarrow 9x - 2x + 4 &= 25 \\ \Rightarrow 7x &= 21 \\ \Rightarrow x &= 3 \end{aligned}$$

Check

Substituting $x = 3$ in original equation,

$$\begin{aligned} \frac{3}{2}(3) - \frac{3-2}{3} &= \frac{25}{6} \\ \frac{9}{2} - \frac{1}{3} &= \frac{25}{6} \\ \frac{25}{6} &= \frac{25}{6} \end{aligned}$$

Since $x = 3$ makes the original statement true, therefore the solution is correct.

Note

Some fractional equations may have no solution.

Example

$$\text{Solve } \frac{3}{y-1} - 2 = \frac{3y}{y-1}, \quad y \neq 1$$

Solution

Multiplying both sides by $y - 1$, we get

$$\begin{aligned} 3 - 2(y-1) &= 3y \\ \Rightarrow 3 - 2y + 2 &= 3y \\ \Rightarrow -5y &= -5 \\ \Rightarrow y &= 1 \end{aligned}$$

Check

Substituting $y = 1$ in the given equation, we have

$$\begin{aligned} \frac{3}{1-1} - 2 &= \frac{3(1)}{1-1} \\ \frac{3}{0} - 2 &= \frac{3}{0} \end{aligned}$$

$$\text{But } \frac{3}{0} \text{ is undefined. So } y=1$$

cannot be a solution.

Thus the given equation has no solution.

Example

$$\text{Solve } \frac{3x-1}{3} - \frac{2x}{x-1} = x, \quad x \neq 1$$

Solution

Multiplying each side by $3(x-1)$

$$\begin{aligned}
(x-1)(3x-1)-6x &= 3x(x-1) \\
\Rightarrow 3x^2-4x+1-6x &= 3x^2-3x \\
\Rightarrow -10x+1 &= -3x \\
\Rightarrow -7x &= -1 \\
\Rightarrow x &= \frac{1}{7}
\end{aligned}$$

Check

On substituting $x = \frac{1}{7}$ the original equation is verified a true statement. That means the restriction $x \neq 1$ has no effect on the solution because $\frac{1}{7} \neq 1$.

Hence our solution $x = \frac{1}{7}$ is correct.

Define Radical equation

When the variable in an equation occurs under a radical, the equation is called a radical equation.

Example

Solve the equations

$$\begin{aligned}
\text{(a)} \quad \sqrt{2x-3}-7 &= 0 \\
\text{(b)} \quad \sqrt[3]{3x+5} &= \sqrt[3]{3x+5} = \sqrt[3]{x-1}
\end{aligned}$$

Solution

(a) To isolate the radical, we can rewrite the given equation as

$$\begin{aligned}
\sqrt{2x-3} &= 7 \\
\Rightarrow 2x-3 &= 49 \dots\dots \\
\Rightarrow 2x &= 52 \Rightarrow x = 26
\end{aligned}$$

Check

Let us substitute $x=26$ in the original equation. Then

$$\begin{aligned}
\sqrt{2(26)-3}-7 &= 0 \\
\sqrt{52-3}-7 &= 0
\end{aligned}$$

$$\begin{aligned}
\sqrt{49}-7 &= 0 \\
0 &= 0
\end{aligned}$$

Hence the solution set is $\{26\}$.

(b) We have

$$\sqrt[3]{3x+5} = \sqrt[3]{x-1}$$

Taking cube of each side

$$\begin{aligned}
\Rightarrow 3x+5 &= x-1, \\
\Rightarrow 2x &= -06 \Rightarrow x = -3
\end{aligned}$$

Check

We substitute $x = -3$ in the original equation. Then

$$\begin{aligned}
\sqrt[3]{3(-3)+5} &= \sqrt[3]{-3-1} \\
\sqrt[3]{-9+5} &= \sqrt[3]{-4} \\
\Rightarrow \sqrt[3]{-4} &= \sqrt[3]{-4}
\end{aligned}$$

Thus $x = -3$ satisfies the original equation.

Here $\sqrt[3]{-4}$ is a real number because we raised each side of the equation to an odd power.

Thus the solution set = $\{-3\}$

Example

Solve and check: $\sqrt{5x-7}-$

$$\sqrt{x+10} = 0$$

Solution

When two terms of a radical equation contain variables in the radicand, we express the equation such that one of these terms is on each side. So we rewrite the equation in this form to get

$$\sqrt{5x-7} - \sqrt{x+10} = 0$$

Squaring each side

$$5x-7 = x+10,$$

$$5x-x = 10+7$$

$$4x = 17 \Rightarrow x = \frac{17}{4}$$

Check

Substituting $x = \frac{17}{4}$ in original equation

$$\sqrt{5x-7} - \sqrt{x+10} = 0$$

$$\sqrt{5\left(\frac{17}{4}\right) - 7} - \sqrt{\frac{17}{4} + 10} = 0$$

$$\sqrt{\frac{57}{4}} - \sqrt{\frac{57}{4}} = 0$$

$$0 = 0$$

i.e., $x = \frac{17}{4}$ makes the given equation a true statement.

$$\text{Thus solution set} = \left\{ \frac{17}{4} \right\}.$$

Example

$$\text{Solve } \sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$$

Solution

$$\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$$

Squaring both sides we get

$$x+7+x+2+2\sqrt{(x+7)(x+2)} = 6x+13$$

$$\Rightarrow 2\sqrt{x^2+9x+14} = 4x+4$$

$$\Rightarrow \sqrt{x^2+9x+14} = 2x+2$$

Squaring again

$$x^2+9x+14 = 4x^2+8x+4$$

$$\Rightarrow 3x^2 - x - 10 = 0$$

$$\Rightarrow 3x^2 - 6x + 5x - 10 = 0$$

$$\Rightarrow 3x(x-2) + 5(x-2) = 0$$

$$\Rightarrow (x-2)(3x+5) = 0$$

$$\Rightarrow x = 2, \frac{-5}{3}$$

On checking, we see that $x=2$ satisfies the equation, but $x = \frac{-5}{3}$ does not satisfy the equation. So solution set is $\{2\}$ and $x = \frac{-5}{3}$ is an extraneous root.

Exercise 7.1

Q1. Solve the following equations.

i) $\frac{2}{3}x - \frac{1}{2}x = x + \frac{1}{6}$

Sol: Multiplying both sides by 6

$$2\cancel{6}\left(\frac{2}{\cancel{3}}x\right) - \cancel{6}\left(\frac{1}{\cancel{2}}x\right) = 6(x) + \cancel{6}\left(\frac{1}{\cancel{6}}\right)$$

$$4x - 3x = 6x + 1$$

$$x = 6x + 1$$

$$-1 = 6x - x$$

$$-1 = 5x$$

$$\Rightarrow \boxed{x = -\frac{1}{5}}$$

Check:

Substituting $x = -\frac{1}{5}$ in the given equation

$$\frac{2}{3}\left(-\frac{1}{5}\right) - \frac{1}{2}\left(-\frac{1}{5}\right) = -\frac{1}{5} + \frac{1}{6}$$

$$-\frac{2}{15} + \frac{1}{10} = -\frac{1}{5} + \frac{1}{6}$$

$$\frac{-4+3}{30} = \frac{-6+5}{30}$$

$$-\frac{1}{30} = -\frac{1}{30} \text{ which is true}$$

Hence solution set = $\left\{-\frac{1}{5}\right\}$

ii) $\frac{x-3}{3} - \frac{x-2}{2} = -1$

Multiplying both sides by 6

$$2\cancel{6}\left(\frac{x-3}{\cancel{3}}\right) - \cancel{3}\cancel{6}\left(\frac{x-2}{\cancel{2}}\right) = 6(-1)$$

$$2x - \cancel{6} - 3x + \cancel{6} = -6$$

$$-x = -6$$

$$\boxed{x = 6}$$

Check:

Substituting $x = 6$ in the given equation

$$\frac{6-3}{3} - \frac{6-2}{2} = -1$$

$$\frac{3}{3} - \frac{4}{2} = -1$$

$$1 - 2 = -1$$

$-1 = -1$ which is true, so solution set = $\{6\}$

iii) $\frac{1}{2}\left(x - \frac{1}{6}\right) + \frac{2}{3} = \frac{5}{6} + \frac{1}{3}\left(\frac{1}{2} - 3x\right)$

$$\frac{1}{2}x - \frac{1}{12} + \frac{2}{3} = \frac{5}{6} + \frac{1}{6} - \frac{1}{3}(\cancel{3}x)$$

Multiplying both sides by 12

$$\cancel{12}\left(\frac{1}{\cancel{2}}x\right) - \cancel{12}\left(\frac{1}{\cancel{12}}\right) + \cancel{12}\left(\frac{2}{\cancel{3}}\right) = \cancel{12}\left(\frac{5}{\cancel{6}}\right) + \cancel{12}\left(\frac{1}{\cancel{6}}\right) - \cancel{12}(x)$$

$$6x - 1 + 8 = 10 + 2 - 12x$$

$$6x + 7 = 12 - 12x$$

$$6x + 12x = 12 - 7$$

$$18x = 5$$

$$\boxed{x = \frac{5}{18}}$$

Check:

Substituting $x = \frac{5}{18}$ in the given equation

$$\frac{1}{2}\left(\frac{5}{18} - \frac{1}{6}\right) + \frac{2}{3} = \frac{5}{6} + \frac{1}{3}\left(\frac{1}{2} - \cancel{3} \times \frac{5}{\cancel{6} + 8}\right)$$

$$\frac{1}{2}\left(\frac{5-3}{18}\right) + \frac{2}{3} = \frac{5}{6} + \frac{1}{3}\left(\frac{3-5}{6}\right)$$

$$\frac{1}{2}\left(\frac{2}{18}\right) + \frac{2}{3} = \frac{5}{6} - \frac{2}{18}$$

$$\frac{1+12}{18} = \frac{15-2}{18}$$

$$\frac{13}{18} = \frac{13}{18} \text{ which is true, so}$$

$$\text{Solution set} = \left\{\frac{5}{18}\right\}$$

(iv) $x + \frac{1}{3} = 2\left(x - \frac{2}{3}\right) - 6x$

$$x + \frac{1}{3} = 2x - \frac{4}{3} - 6x$$

Multiplying both sides by 3

$$3x + \cancel{3} \times \frac{1}{\cancel{3}} = 3(2x) - \cancel{3}\left(\frac{4}{\cancel{3}}\right) - 3(6x)$$

$$3x + 1 = 6x - 4 - 18x$$

$$3x + 1 = -12x - 4$$

$$15x = -5$$

$$x = -\frac{5}{15}$$

$$\boxed{x = -\frac{1}{3}}$$

Check:

Substituting $x = -\frac{1}{3}$ in the given equation

$$-\frac{1}{3} + \frac{1}{3} = 2\left(-\frac{1}{3} - \frac{2}{3}\right) - \cancel{6}\left(-\frac{1}{\cancel{3}}\right)$$

$$0 = 2\left(-\frac{x}{3}\right) + 2$$

$$0 = -2 + 2$$

$$0 = 0 \text{ which is true, so}$$

$$\text{Solution set} = \left\{-\frac{1}{3}\right\}$$

$$\text{v)} \quad \frac{5(x-3)}{6} - x = 1 - \frac{x}{9}$$

Multiplying both sides by 18

$$18 \times \frac{5(x-3)}{6} - 18x = 18 - 2x$$

$$15(x-3) - 18x = 18 - 2x$$

$$15x - 45 - 18x = 18 - 2x$$

$$15x - 18x + 2x = 18 + 45$$

$$-x = 63$$

$$\Rightarrow x = -63$$

Check:

Substituting $x = -63$ in the given equation

$$\frac{5(-63-3)}{6} - (-63) = 1 - \frac{(-63)}{9}$$

$$5\left(\frac{-66}{6}\right) + 63 = 1 + \frac{63}{9}$$

$$-55 + 63 = 1 + 7$$

$$8 = 8 \text{ which is true, so}$$

$$\text{Solution set} = \{-63\}$$

$$\text{vi)} \quad \frac{x}{3x-6} = 2 - \frac{2x}{x-2}$$

$$\frac{x}{3(x-2)} = 2 - \frac{2x}{x-2}$$

Multiplying both sides by $3(x-2)$

$$x(x-2) \times \frac{x}{3(x-2)} = 2 \times 3(x-2) - \frac{2x}{x-2} \times 3(x-2)$$

$$x = 6x - 12 - 6x$$

$$\boxed{x = -12}$$

Check:

Substituting $x = -12$ in the given equation

$$\frac{-12}{3(-12)-6} = 2 - \frac{2(-12)}{-12-2}$$

$$\frac{-12}{-36-6} = 2 - \frac{(-24)}{-14}$$

$$\frac{-12}{-42} = 2 - \frac{12}{7}$$

$$\frac{2}{7} = \frac{14-12}{7}$$

$$\frac{2}{7} = \frac{2}{7}$$

which is true, so

$$\text{Solution Set} = \{-12\}$$

$$\text{vii)} \quad \frac{2x}{2x+5} = \frac{2}{3} - \frac{5}{4x+10}, \quad x \neq -\frac{5}{2}$$

$$\frac{2x}{2x+5} = \frac{2}{3} - \frac{5}{2(2x+5)}$$

Multiplying both sides by $6(2x+5)$

$$6(2x+5) \times \frac{2x}{2x+5} = \frac{2}{3} \times 6(2x+5) - \frac{5}{2(2x+5)} \times 6(2x+5)$$

$$12x = 8x + 20 - 15$$

$$12x - 8x = 5$$

$$4x = 5$$

$$4x = 5$$

$$x = \frac{5}{4}$$

Check:

Substituting $x = \frac{5}{4}$ in the given equation

$$\frac{\cancel{2}\left(\frac{5}{\cancel{4}}\right)}{\cancel{2}\left(\frac{5}{\cancel{4}}\right)+5} = \frac{2}{3} - \frac{5}{\cancel{4}\left(\frac{5}{\cancel{4}}\right)+10}$$

$$\frac{\cancel{2}}{\cancel{5+10}} = \frac{2}{3} - \frac{\cancel{5}}{\cancel{15}}$$

$$\frac{\cancel{5}}{\cancel{15}} = \frac{2}{3} - \frac{1}{3}$$

$$\frac{1}{3} = \frac{1}{3} \text{ which is true, so}$$

$$\text{Solution set} = \left\{ \frac{5}{4} \right\}$$

$$\text{viii) } \frac{2x}{x-1} + \frac{1}{3} = \frac{5}{6} + \frac{2}{x-1}, x \neq 1$$

Multiplying both sides by $6(x-1)$

$$6(\cancel{x-1}) \times \frac{2x}{\cancel{x-1}} + \cancel{6}(x-1) \times \frac{1}{\cancel{6}} \\ = \cancel{6}(x-1) \times \frac{5}{\cancel{6}} + 6(\cancel{x-1}) \times \frac{2}{\cancel{x-1}}$$

$$12x + 2x - 2 = 5x - 5 + 12$$

$$12x + 2x - 5x = 2 - 5 + 12$$

$$9x = 9$$

$$x = \frac{9}{9}$$

$$\boxed{x=1}$$

Check:

Substituting $x=1$ in the given equation

$$\frac{2(1)}{1-1} + \frac{1}{3} = \frac{5}{6} + \frac{2}{1-1}$$

$$\frac{2}{0} + \frac{1}{3} = \frac{5}{6} + \frac{2}{0}$$

As $\frac{2}{0}$ is undefined, so $x=1$ cannot be a solution thus the given equation has no solution.

$$\text{ix) } \frac{2}{x^2-1} - \frac{1}{x+1} = \frac{1}{x+1}, x \neq \pm 1$$

$$\frac{2}{(x+1)(x-1)} - \frac{1}{x+1} = \frac{1}{x+1}$$

Multiplying both sides by $(x+1)(x-1)$

$$\cancel{(x+1)}(\cancel{x-1}) \times \frac{2}{\cancel{(x+1)}(\cancel{x-1})}$$

$$-\cancel{(x+1)}(\cancel{x-1}) \times \frac{1}{\cancel{x+1}} = \frac{1}{\cancel{x+1}} \times \cancel{(x+1)}(\cancel{x-1})$$

$$2 - x + 1 = x - 1$$

$$2 + 1 + 1 = x + x$$

$$2x = 4$$

$$x = \frac{4}{2}$$

$$x = 2$$

Check:

Substituting $x=2$ in the given equation

$$\frac{2}{(2)^2-1} - \frac{1}{2+1} = \frac{1}{2+1}$$

$$\frac{2}{4-1} - \frac{1}{3} = \frac{1}{3}$$

$$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$$\frac{1}{3} = \frac{1}{3} \text{ which is true, so}$$

$$\text{Solution Set} = \{2\}$$

$$\text{x) } \frac{2}{3x+6} = \frac{1}{6} - \frac{1}{2x+4}, x \neq -2$$

$$\frac{2}{3(x+2)} = \frac{1}{6} - \frac{1}{2(x+2)}$$

Multiplying both sides by $6(x+2)$

$$\cancel{6}(\cancel{x+2}) \times \frac{2}{\cancel{6}(\cancel{x+2})} =$$

$$\frac{1}{\cancel{6}} \times \cancel{6}(x+2) - \frac{1}{\cancel{2}(\cancel{x+2})} \times \cancel{3}\cancel{6}(\cancel{x+2})$$

$$4 = x + 2 - 3$$

$$4 = x - 1$$

$$4 + 1 = x$$

$$\boxed{x = 5}$$

Check:

Substituting $x = 5$ in the given equation

$$\frac{2}{3(5)+6} = \frac{1}{6} - \frac{1}{2(5)+4}$$

$$\frac{2}{15+6} = \frac{1}{6} - \frac{1}{10+4}$$

$$\frac{2}{21} = \frac{1}{6} - \frac{1}{14}$$

$$\frac{2}{21} = \frac{7-3}{42}$$

$$\frac{2}{21} = \frac{4}{42}$$

$$\frac{2}{21} = \frac{2}{21}$$

which is true, so

$$\text{Solution Set} = \{5\}$$

Q2. Solve each question and check for extraneous solution, if any.

i) $\sqrt{3x+4} = 2$

Squaring both sides

$$(\sqrt{3x+4})^2 = (2)^2$$

$$3x+4=4$$

$$3x=4-4$$

$$3x=0$$

$$x = \frac{0}{3}$$

$$\boxed{x = 0}$$

Check:

Substituting $x = 0$ in the given equation

$$\sqrt{3x+4} = 2$$

$$\sqrt{3(0)+4} = 2$$

$$\sqrt{0+4} = 2$$

$$\sqrt{4} = 2$$

$$2 = 2 \text{ which is true, so}$$

$$\text{Solution Set} = \{0\}$$

ii) $\sqrt[3]{2x-4} - 2 = 0$

$$\sqrt[3]{2x-4} = 2$$

Taking cube of both sides

$$(\sqrt[3]{2x-4})^3 = (2)^3$$

$$2x-4=8$$

$$2x=8+4$$

$$2x=12$$

$$x = \frac{12}{2}$$

$$\boxed{x = 6}$$

Check:

Putting $x = 6$ in the given equation.

$$\sqrt[3]{2x-4} - 2 = 0$$

$$\sqrt[3]{2(6)-4} - 2 = 0$$

$$\sqrt[3]{12-4} - 2 = 0$$

$$\sqrt[3]{8} - 2 = 0$$

$$\sqrt[3]{2^3} - 2 = 0$$

$$2 - 2 = 0$$

$$0 = 0 \text{ which is true, so}$$

$$\text{Solution Set} = \{6\}$$

iii) $\sqrt{x-3} - 7 = 0$

$$\text{or } \sqrt{x-3} = 7$$

Squaring both sides

$$(\sqrt{x-3})^2 = (7)^2$$

$$x-3=49$$

$$x=49+3$$

$$x=52$$

Check:

Putting $x = 52$ in the given equation

$$\sqrt{x-3} - 7 = 0$$

$$\sqrt{52-3} - 7 = 0$$

$$\sqrt{49} - 7 = 0$$

$$7 - 7 = 0$$

$$0 = 0 \text{ which true, so}$$

$$\text{Solution Set} = \{52\}$$

$$\text{iv) } 2\sqrt{t+4} = 5$$

$$\sqrt{t+4} = \frac{5}{2}$$

Squaring both sides

$$(\sqrt{t+4})^2 = \left(\frac{5}{2}\right)^2$$

$$t+4 = \frac{25}{4}$$

$$t = \frac{25}{4} - 4$$

$$= \frac{25-16}{4}$$

$$\boxed{t = \frac{9}{4}}$$

Check:

Putting $t = \frac{9}{4}$ in the given equation.

$$2\sqrt{t+4} = 5$$

$$2\sqrt{\frac{9}{4}+4} = 5$$

$$2\sqrt{\frac{9+16}{4}} = 5$$

$$2\sqrt{\frac{25}{4}} = 5$$

$$\cancel{2}\left(\frac{5}{\cancel{2}}\right) = 5$$

$$5 = 5 \text{ which is true, so}$$

$$\text{Solution Set} = \left\{\frac{9}{4}\right\}$$

$$\text{v) } \sqrt[3]{2x+3} = \sqrt[3]{x-2}$$

Taking cube of both sides

$$(\sqrt[3]{2x+3})^3 = (\sqrt[3]{x-2})^3$$

$$2x+3 = x-2$$

$$2x - x = -2 - 3$$

$$\boxed{x = -5}$$

Check:

Putting $x = -5$ in the given equation.

$$\sqrt[3]{2x+3} = \sqrt[3]{x-2}$$

$$\sqrt[3]{2(-5)+3} = \sqrt[3]{-5-2}$$

$$\sqrt[3]{-10+3} = \sqrt[3]{-7}$$

$$\sqrt[3]{-7} = \sqrt[3]{-7} \text{ which is true, so}$$

$$\text{Solution Set} = \{-5\}$$

$$\text{vi) } \sqrt[3]{2-t} = \sqrt[3]{2t-28}$$

Taking cube of both sides

$$(\sqrt[3]{2-t})^3 = (\sqrt[3]{2t-28})^3$$

$$2-t = 2t-28$$

$$2+28 = 2t+t$$

$$3t = 30$$

$$t = \frac{30}{\cancel{3}}$$

$$\boxed{t = 10}$$

Check:

Putting $t = 3$ in the given equation

$$\sqrt[3]{2-t} = \sqrt[3]{2t-28}$$

$$\sqrt[3]{2-10} = \sqrt[3]{2 \times 10 - 28}$$

$$\sqrt[3]{-8} = \sqrt[3]{20-28}$$

$$\sqrt[3]{-8} = \sqrt[3]{-8} \text{ which is true, so}$$

$$\text{Solution Set} = \{10\}$$

$$\text{vii) } \sqrt{2t+6} - \sqrt{2t-5} = 0 \text{ or}$$

$$\sqrt{2t+6} = \sqrt{2t-5}$$

Squaring both sides

$$(\sqrt{2t+6})^2 = (\sqrt{2t-5})^2$$

$$2t+6 = 2t-5$$

$$\cancel{2t} - \cancel{2t} + 6 = -5$$

$$6 = -5 \text{ which is not possible, so}$$

$$\text{Solution Set} = \{ \}$$

$$\text{viii) } \sqrt{\frac{x+1}{2x+5}} = 2, \quad x \neq -\frac{5}{2}$$

Squaring both sides

$$\left(\sqrt{\frac{x+1}{2x+5}}\right)^2 = (2)^2$$

$$\frac{x+1}{2x+5} = 4$$

$$x+1 = 4(2x+5)$$

$$x+1 = 8x+20$$

$$1-20 = 8x-x$$

Definition

The absolute value of a real number 'a' denoted by |a|, is defined as

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

e.g., |6| = 6, |0| = 0 and |-6| = -(-6) = 6

Some properties of Absolute Value

If $a, b \in \mathbb{R}$, then

$$(i) \quad |a| \geq 0$$

$$-19 = 7x$$

$$\Rightarrow \boxed{x = -\frac{19}{7}}$$

Check:

Putting $x = -\frac{19}{7}$ in the given equation

$$\sqrt{\frac{x+1}{2x+5}} = 2$$

$$\sqrt{\frac{\frac{-19}{7}+1}{2\left(\frac{-19}{7}\right)+5}} = 2$$

$$\sqrt{\frac{\cancel{-19}+7}{\cancel{-38}+35}} = 2$$

$$\sqrt{\frac{\cancel{-12}}{\cancel{-3}}} = 2$$

$$\sqrt{4} = 2$$

$$2 = 2 \text{ which is true, so}$$

$$\text{Solution Set} = \left\{ \frac{-19}{7} \right\}$$

$$(ii) \quad |-a| = |a|$$

$$(iii) \quad |ab| = |a| \cdot |b|$$

$$(iv) \quad \left| \frac{a}{b} \right| = \frac{|a|}{|b|}, \quad b \neq 0$$

Example

Solve and check, $|2x+3| = 11$

Solution

By definition, depending on whether $(2x+3)$ is positive or negative the given equation is equivalent to

$$+(2x+3) = 11 \text{ or } -(2x+3) = 11$$

In practice, these two equations are usually written as

$$2x+3 = +11 \text{ or } 2x+3 = -11$$

$$2x = 8 \text{ or } 2x = -14$$

$$x = 4 \text{ or } x = -7$$

Check

Substituting $x = 4$, in the original equation, we get

$$|2(4) + 3| = 11$$

$$\text{i.e., } 11 = 11, \text{ true}$$

Now substituting $x = -7$, we have

$$|2(-7) + 3| = 11$$

$$|-11| = 11$$

$$11 = 11, \text{ true}$$

Hence $x = 4, -7$ are the solutions to the given equation.

Or Solution set = $\{-7, 4\}$

Example

$$\text{Solve } |8x - 3| = |4x + 5|$$

Solution

Since two numbers having the same absolute value are either equal or differ in sign, therefore, the given equation is equivalent to

$$8x - 3 = 4x + 5 \text{ or } 8x - 3 = -(4x + 5)$$

$$8x - 3 = 4x + 5 \text{ or } 8x - 3 = -4x - 5$$

$$8x - 4x = 5 + 3 \text{ or } 8x + 4x = -5 + 3$$

$$4x = 8 \text{ or } 12x = -2$$

$$x = 2 \text{ or } x = -1/6$$

On checking we find that

$x = 2, x = -\frac{1}{6}$ both satisfy the original equation.

Hence the solution set $\left\{-\frac{1}{6}, 2\right\}$.

Sometimes it may happen that the solution(s) obtained do not satisfy the original equation. Such solution(s) (called extraneous) must be rejected. Therefore, it is always advisable to check the solutions in the original equation.

Example 3

$$\text{Solve and check } |3x + 10| = 5x + 6$$

Solution

The given equation is equivalent to

$$\pm(3x+10) = 5x+6$$

$$\text{i.e., } 3x+10 = 5x+6 \text{ or } 3x+10 = -(5x+6)$$

$$3x+10 = 5x+6 \text{ or } 3x+10 = -5x-6$$

$$3x-5x = 6-10 \text{ or } 3x+5x = -6-10$$

$$-2x = -4 \text{ or } 8x = -16$$

$$x = 2 \text{ or } x = -2$$

On checking in the original equation we see that $x = -2$ does not satisfy it.

Hence the only solution is $x = 2$.