

Exercise 7.2

Q1. Identify the following statements as True or False.

i) $|x| = 0$ has only one solution.
(True)

ii) All absolute value equations have two solutions. (False)

iii) The equation $|x| = 2$ is equivalent to $x = 2$ or $x = -2$. (True)

iv) The equation $|x-4|=-4$ has no solution. (True)

v) The equation $|2x-3|=5$ is equivalent to $2x-3=5$ or $2x+3=5$ (False.)

Q2. Solve for 'x'.

i) $|3x-5|=4$

$$\Rightarrow +(3x-5)=4 \text{ or } -(3x-5)=4$$

$$3x-5=4 \text{ or } 3x-5=-4$$

$$3x=4+5 \text{ or } 3x=-4+5$$

$$3x=9 \text{ or } 3x=1$$

$$x=3 \text{ or } x=\frac{1}{3}$$

Check:

Substituting $x=3$ in given equation

$$|3(3)-5|=4$$

$$|9-5|=4$$

$$|4|=4$$

$$4=4 \text{ which is true}$$

Putting $x=\frac{1}{3}$ in given equation

$$\left|3\left(\frac{1}{3}\right)-5\right|=4$$

$$|1-5|=4$$

$$|-4|=4$$

$$4=4 \text{ which is true, so}$$

$$\text{Solution Set} = \left\{3, \frac{1}{3}\right\}$$

ii) $\frac{1}{2}|3x+2|-4=11$

$$\frac{1}{2}|3x+2|=11+4$$

$$\frac{1}{2}|3x+2|=15$$

$$|3x+2|=15 \times 2$$

$$|3x+2|=30$$

$$+(3x+2)=30 \text{ or } -(3x+2)=30$$

$$3x+2=30 \text{ or } 3x+2=-30$$

$$3x=30-2 \text{ or } 3x=-30-2$$

$$3x=28 \text{ or } 3x=-32$$

$$x=\frac{28}{3} \text{ or } x=\frac{-32}{3}$$

Check:

Putting $x=\frac{28}{3}$ in the given equation

$$\frac{1}{2}\left|\cancel{x}\left(\frac{28}{\cancel{x}}\right)+2\right|-4=11$$

$$\frac{1}{2}|28+2|-4=11$$

$$\frac{1}{2}|30|-4=11$$

$$\frac{1}{2}(30)-4=11$$

$$15-4=11$$

$$11=11 \text{ which is true}$$

Now putting $x=-\frac{32}{3}$ in the given equation.

$$\frac{1}{2}\left|\cancel{x}\left(-\frac{32}{\cancel{x}}\right)+2\right|-4=11$$

$$\frac{1}{2}|-32+2|-4=11$$

$$\frac{1}{2}|-30|-4=11$$

$$\frac{1}{2}(30)-4=11$$

$$15 - 4 = 11$$

$11 = 11$ which is true, so

$$\text{Hence Solution Set} = \left\{ \frac{28}{3}, -\frac{32}{3} \right\}$$

$$\text{iii) } |2x + 5| = 11$$

$$+(2x + 5) = 11 \quad \text{or} \quad -(2x + 5) = 11$$

$$2x + 5 = 11 \quad \text{or} \quad 2x + 5 = -11$$

$$2x = 11 - 5 \quad \text{or} \quad 2x = -11 - 5$$

$$2x = 6 \quad \text{or} \quad 2x = -16$$

$$x = \frac{6}{2} \quad \text{or} \quad x = \frac{-16}{2}$$

$$x = 3 \quad \text{or} \quad x = -8$$

Check:

Putting $x = 3$ in the given equation.

$$|2(3) + 5| = 11$$

$$|6 + 5| = 11$$

$$|11| = 11$$

$11 = 11$ which is true

Now putting $x = -8$ in the given equation.

$$|2(-8) + 5| = 11$$

$$|-16 + 5| = 11$$

$$|-11| = 11$$

$11 = 11$ which is true, so

$$\text{Solution Set} = \{3, -8\}$$

$$\text{iv) } |3 + 2x| = |6x - 7|$$

$$\frac{|3 + 2x|}{|6x - 7|} = 1$$

$$\frac{|3 + 2x|}{|6x - 7|} = 1$$

$$+\left(\frac{3 + 2x}{6x - 7}\right) = 1 \quad \text{or} \quad -\left(\frac{3 + 2x}{6x - 7}\right) = 1$$

$$\frac{3 + 2x}{6x - 7} = 1 \quad \text{or} \quad \frac{3 + 2x}{6x - 7} = -1$$

$$3 + 2x = 6x - 7 \quad \text{or} \quad 3 + 2x = -6x + 7$$

$$3 + 7 = 6x - 2x \quad \text{or} \quad 2x + 6x = 7 - 3$$

$$10 = 4x \quad \text{or} \quad 8x = 4$$

$$\Rightarrow x = \frac{10}{4} \quad \text{or} \quad x = \frac{4}{8}$$

$$x = \frac{5}{2} \quad \text{or} \quad x = \frac{1}{2}$$

Check:

Putting $x = \frac{5}{2}$ in the given equation

$$\left| 3 + \cancel{2}\left(\frac{5}{\cancel{2}}\right) \right| = \left| \cancel{6}\left(\frac{5}{\cancel{2}}\right) - 7 \right|$$

$$|3 + 5| = |15 - 7|$$

$$|8| = |8|$$

$8 = 8$ which is true

Now putting $x = \frac{1}{2}$ in the given equation

$$\left| 3 + \cancel{2}\left(\frac{1}{\cancel{2}}\right) \right| = \left| \cancel{6}\left(\frac{1}{\cancel{2}}\right) - 7 \right|$$

$$|3 + 1| = |3 - 7|$$

$$|4| = |-4|$$

$4 = 4$ which is true, so

$$\text{Solution Set} = \left\{ \frac{5}{2}, \frac{1}{2} \right\}$$

$$\text{v) } |x + 2| - 3 = 5 - |x - 2|$$

$$|x + 2| + |x + 2| = 5 + 3$$

$$2|x + 2| = 8$$

$$|x + 2| = \frac{8}{2}$$

$$|x + 2| = 4$$

$$\begin{aligned}
 +(x+2) &= 4 & \text{or} & & -(x+2) &= 4 \\
 x+2 &= 4 & \text{or} & & x+2 &= -4 \\
 x &= 4-2 & \text{or} & & x &= -4-2 \\
 x &= 2 & \text{or} & & x &= -6
 \end{aligned}$$

Check:

Putting $x = 2$ in the give equation

$$|2+2|-3=5-|2+2|$$

$$|4|-3=5-|4|$$

$$4-3=5-4$$

$$1=1 \quad \text{which is true}$$

Now putting $x = -6$ in the given equation.

$$|-6+2|-3=5-|-6+2|$$

$$|-4|-3=5-|-4|$$

$$4-3=5-4$$

$$1=1 \quad \text{which is true, so}$$

$$\text{Solution Set} = \{2, -6\}$$

$$\text{vi) } \frac{1}{2}|x+3|+21=9$$

$$\frac{1}{2}|x+3|=9-21$$

$$\frac{1}{2}|x+3|=-12$$

$$|x+3|=-24$$

As the value of absolute cannot be negative, so Solution Set = $\{ \}$

$$\text{vii) } \left| \frac{3-5x}{4} \right| - \frac{1}{3} = \frac{2}{3}$$

$$\left| \frac{3-5x}{4} \right| = \frac{2}{3} + \frac{1}{3}$$

$$\left| \frac{3-5x}{4} \right| = \frac{\cancel{2}}{\cancel{3}}$$

$$\left| \frac{3-5x}{4} \right| = 1$$

$$+\left(\frac{3-5x}{4}\right)=1 \quad \text{or} \quad -\left(\frac{3-5x}{4}\right)=1$$

$$\frac{3-5x}{4}=1 \quad \text{or} \quad \frac{3-5x}{4}=-1$$

$$3-5x=4 \quad \text{or} \quad 3-5x=-4$$

$$3-4=5x \quad \text{or} \quad 3+4=5x$$

$$-1=5x \quad \text{or} \quad 7=5x$$

$$x=-\frac{1}{5} \quad \text{or} \quad x=\frac{7}{5}$$

Check:

Putting $x = -\frac{1}{5}$ in the given equation

$$\left| \frac{3-5\left(-\frac{1}{5}\right)}{4} \right| - \frac{1}{3} = \frac{2}{3}$$

$$\left| \frac{3+1}{4} \right| - \frac{1}{3} = \frac{2}{3}$$

$$\left| \frac{4}{4} \right| - \frac{1}{3} = \frac{2}{3}$$

$$|1| - \frac{1}{3} = \frac{2}{3}$$

$$1 - \frac{1}{3} = \frac{2}{3}$$

$$\frac{3-1}{3} = \frac{2}{3}$$

$$\frac{2}{3} = \frac{2}{3}$$

which is true,

Now putting $x = \frac{7}{5}$ in the given equation

$$\left| \frac{3-\cancel{5}\left(\frac{\cancel{7}}{\cancel{5}}\right)}{4} \right| - \frac{1}{3} = \frac{2}{3}$$

$$\left| \frac{3-7}{4} \right| - \frac{1}{3} = \frac{2}{3}$$

$$\left| -\frac{4}{4} \right| - \frac{1}{3} = \frac{2}{3}$$

$$|-1| - \frac{1}{3} = \frac{2}{3}$$

$$1 - \frac{1}{3} = \frac{2}{3}$$

$$\frac{2}{3} = \frac{2}{3} \text{ which is true}$$

$$\text{So, solution set} = \left\{ -\frac{1}{5}, \frac{7}{5} \right\}$$

$$\text{viii)} \quad \left| \frac{x+5}{2-x} \right| = 6$$

$$+\left(\frac{x+5}{2-x} \right) = 6 \quad \text{or} \quad -\left(\frac{x+5}{2-x} \right) = 6$$

$$\frac{x+5}{2-x} = 6 \quad \text{or} \quad \frac{x+5}{2-x} = -6$$

$$x+5 = 12-6x \quad \text{or} \quad x+5 = 12+6x$$

$$x+6x = 12-5 \quad \text{or} \quad 5+12 = 6x-x$$

$$7x = 7 \quad \text{or} \quad 17 = 5x$$

$$x = 1 \quad \text{or} \quad x = \frac{17}{5}$$

Check:

Putting $x=1$ in the given equation.

$$\left| \frac{1+5}{2-1} \right| = 6$$

$$\left| \frac{6}{1} \right| = 6$$

$$|6| = 6$$

$$6 = 6$$

Now putting $x = \frac{17}{5}$ in the given equation

$$\left| \frac{\frac{17}{5} + 5}{2 - \frac{17}{5}} \right| = 6$$

$$\left| \frac{17+25}{10-17} \right| = 6$$

$$\left| \frac{42}{-7} \right| = 6$$

$$|-6| = 6$$

$$6 = 6 \text{ which is true}$$

$$\text{So, solution set} = \left\{ 1, \frac{17}{5} \right\}$$

Definition of inequality

Let a, b be real numbers, then a is greater than b if the difference $a - b$ is positive and we denote this order relation by the inequality $a > b$. An equivalent statement is that b is less than a , symbolized by $b < a$. Similarly, if $a - b$ is negative, then a is less than b and expressed in symbols as $a < b$.

Properties of Inequalities

1. Law of Trichotomy

For any $a, b \in \mathbb{R}$, one and only one of the following statements is true.

$$a < b \quad \text{or} \quad a = b, \quad \text{or} \quad a > b$$

An important special case of this property is the case for $b = 0$, namely, $a < 0$ or $a = 0$ or $a > 0$ for any $a \in \mathbb{R}$

2. Transitive Property

Let $a, b, c \in \mathbb{R}$.

(i) If $a > b$ and $b > c$, then $a > c$

(ii) If $a < b$ and $b < c$, then $a < c$

3. Additive Closure Property

For $a, b, c \in \mathbb{R}$,

(i) If $a > b$, then $a + c > b + c$

If $a < b$, then $a + c > b + c$

(ii) If $a > 0$ and $b > 0$, then $a + b > 0$

If $a < 0$ and $b < 0$, then $a + b < 0$

4. Multiplicative Property

Let $a, b, c, d \in \mathbb{R}$,

- (i) If $a > 0$ and $b > 0$, then $ab > 0$,
whereas $a < 0$ and $b < 0 \Rightarrow ab > 0$
- (ii) If $a > b$ and $c > 0$, then $ac > bc$
Or if $a < b$ and $c > 0$, then $ac < bc$
- (iii) If $a > b$ and $c < 0$, then $ac < bc$
Or if $a < b$ and $c < 0$, then $ac > bc$
- The above property (iii) states that the sign of inequality is reversed if each side is multiplied by a negative real number.
- (iv) If $a > b$ and $c > d$, then $ac > bd$

Example

Solve $9 - 7x > 19 - 2x$, where $x \in \mathbb{R}$.

Solution

$$\begin{aligned} 9 - 7x &> 19 - 2x \\ 9 - 5x &> 19 \\ -5x &> 10 \\ x &< -2 \end{aligned}$$

Hence the solution set = $\{x \mid x < -2\}$

Example

Solve $\frac{1}{2}x - \frac{2}{3} \leq x + \frac{1}{3}$, where $x \in \mathbb{R}$.

Solution

$$\frac{1}{2}x - \frac{2}{3} \leq x + \frac{1}{3}$$

To clear fractions we multiply each side by 6, the L.C.M of 2 and 3 and get

$$\begin{aligned} 6 \left[\frac{1}{2}x - \frac{2}{3} \right] &\leq 6 \left[x + \frac{1}{3} \right] \\ 6 \times \frac{1}{2}x - \frac{6 \times 2}{3} &\leq 6x + 6 \times \frac{1}{3} \\ \text{or } 3x - 4 &\leq 6x + 2 \\ \text{or } -4 - 2 &\leq 6x - 3x \end{aligned}$$

$$\text{or } -6 \leq 3x$$

$$\text{or } -\frac{6}{3} \leq x$$

$$-2 \leq x \Rightarrow x \geq -2$$

Hence the solution set

$$= \{x \mid x \geq -2\}$$

Example

Solve the double inequality

$$-2 < \frac{1-2x}{3} < 1, \text{ where } x \in \mathbb{R}.$$

Solution

The given inequality is a double inequality and represents two separate inequalities

$$-2 < \frac{1-2x}{3} \quad \text{and} \quad \frac{1-2x}{3} < 1$$

$$-2 < \frac{1-2x}{3} < 1$$

$$\text{or } -6 < 1-2x < 3$$

$$\text{or } -7 < -2x < 2$$

$$\text{or } \frac{7}{2} > x > -1$$

$$\text{i.e., } -1 < x < 3.5$$

$$\text{Hence S.S} = \{x \mid -1 < x < 3.5\}$$

Example

Solve the inequality

$$4x - 1 \leq 3 \leq 7 + 2x, \text{ where } x \in \mathbb{R}.$$

Solution

The given inequality holds if and only if both the separate inequalities $4x - 1 \leq 3$ and $3 \leq 7 + 2x$ hold. We solve each of these inequalities separately.

The first inequality $4x - 1 \leq 3$

gives

$$4x \leq 4 \quad \text{i.e., } x \leq 1 \quad \dots(i)$$

$$3 \leq 7 + 2x \Rightarrow -4 \leq 2x$$

i.e.

$$-2 \leq x \Rightarrow x \geq -2$$

....(ii)

Combining (i) and (ii) we have

$$-2 \leq x \leq 1$$

Thus the solution set = $\{x \mid -2 \leq x \leq 1\}$.