

Exercise 7.3

Q1. Solve the following in equalities.

i) $3x+1 < 5x-4$

$$1+4 < 5x-3x$$

$$5 < 2x$$

$$\frac{5}{2} < x$$

or $x > \frac{5}{2}$

$$\text{Solution Set} = \left\{ x \mid x > \frac{5}{2} \right\}$$

ii) $4x-10.3 \leq 21x-1.8$

$$4x-21x \leq 10.3-1.8$$

$$-17x \leq 8.5$$

$$17x \geq -8.5$$

$$x \geq -\frac{8.5}{17}$$

$$x \geq -0.5$$

$$\text{Solution Set} = \left\{ x \mid x \geq -0.5 \right\}$$

iii) $4 - \frac{1}{2}x \geq -7 + \frac{1}{4}x$

$$4+7 \geq \frac{1}{4}x + \frac{1}{2}x$$

$$11 \geq \frac{x+2x}{4}$$

$$11 \geq \frac{3}{4}x$$

$$\frac{11 \times 4}{3} \geq x$$

$$\frac{44}{3} \geq x$$

or $x \leq \frac{44}{3}$

$$\text{Solution Set} = \left\{ x \mid x \leq \frac{44}{3} \right\}$$

iv) $x-2(5-2x) \geq 6x-3\frac{1}{2}$

$$x-2(5-2x) \geq 6x-\frac{7}{2}$$

Multiplying both sides by 2

$$2x-4(5-2x) \geq 12x-7$$

$$2x-20+8x \geq 12x-7$$

$$2x+8x-12x \geq 20-7$$

$$-2x \geq 13$$

$$2x \leq -13$$

$$x \leq -\frac{13}{2}$$

$$\text{Solution Set} = \left\{ x \mid x \leq -\frac{13}{2} \right\}$$

v) $\frac{3x+2}{9} - \frac{2x+1}{3} > -1$

Multiplying both sides by 9

$$3x+2-3(2x+1) > -9$$

$$3x+2-6x-3 > -9$$

$$-3x-1 > -9$$

$$-3x > 1-9$$

$$-3x > -8$$

$$x < \frac{-8}{-3}$$

$$x < \frac{8}{3}$$

$$\text{Solution Set} = \left\{ x \mid x < \frac{8}{3} \right\}$$

$$\text{vi) } 3(2x+1) - 2(2x+5) < 5(3x-2)$$

$$6x+3-4x-10 < 15x-10$$

$$2x-7 < 15x-10$$

$$10-7 < 15x-2x$$

$$3 < 13x$$

$$\frac{3}{13} < x$$

$$\text{or } x > \frac{3}{13}$$

$$\text{Solution Set} = \left\{ x \mid x > \frac{3}{13} \right\}$$

$$\text{vii) } 3(x-1) - (x-2) > -2(x+4)$$

$$3x-3-x+2 > -2x-8$$

$$2x-1 > -2x-8$$

$$2x+2x > 1-8$$

$$4x > -7$$

$$x > -\frac{7}{4}$$

$$\text{Solution Set} = \left\{ x \mid x > -\frac{7}{4} \right\}$$

$$\text{viii) } 2\frac{2}{3}x + \frac{2}{3}(5x-4) > -\frac{1}{3}(8x+7)$$

$$\frac{8}{3}x + \frac{2}{3}(5x-4) > -\frac{1}{3}(8x+7)$$

Multiplying both sides by 3

$$8x+2(5x-4) > -(8x+7)$$

$$8x+10x-8 > -8x-7$$

$$18x-8 > -8x-7$$

$$18x+8x > 8-7$$

$$26x > 1$$

$$x > \frac{1}{26}$$

$$\text{Solution Set} = \left\{ x \mid x > \frac{1}{26} \right\}$$

Q2. Solve the following inequalities.

$$\text{i) } -4 < 3x+5 < 8$$

$$-4 < 3x+5 \quad \text{and} \quad 3x+5 < 8$$

$$-4-5 < 3x \quad \text{and} \quad 3x < 8-5$$

$$-9 < 3x \quad \text{and} \quad 3x < 3$$

$$-\frac{9}{3} < x \quad \text{and} \quad x < \frac{3}{3}$$

$$-3 < x \quad \text{and} \quad x < 1$$

$$-3 < x < 1$$

$$\text{Solution Set} = \{x \mid -3 < x < 1\}$$

$$\text{ii) } -5 \leq \frac{4-3x}{2} < 1$$

$$-5 \leq \frac{4-3x}{2} \quad \text{and} \quad \frac{4-3x}{2} < 1$$

$$-10x \leq 4-3x \quad \text{and} \quad 4-3x < 2$$

$$-10-4 \leq -3x \quad \text{and} \quad -3x < 2-4$$

$$-14 \leq -3x \quad \text{and} \quad -3x < -2$$

$$14 \geq 3x \quad \text{and} \quad 3x > 2$$

$$\frac{14}{3} \geq x \quad \text{and} \quad x > \frac{2}{3}$$

$$\frac{14}{3} \geq x > \frac{2}{3}$$

$$\text{Solution Set} = \left\{ x \mid \frac{14}{3} \geq x > \frac{2}{3} \right\}$$

$$\text{iii) } -6 < \frac{x-2}{4} < 6$$

$$-6 < \frac{x-2}{4} \quad \text{and} \quad \frac{x-2}{4} < 6$$

$$-24 < x-2 \quad \text{and} \quad x-2 < 24$$

$$-24+2 < x \quad \text{and} \quad x < 24+2$$

$$-22 < x \quad \text{and} \quad x < 26$$

$$\text{Solution Set} = \{x \mid -22 < x < 26\}$$

$$\text{iv) } 3 \geq \frac{7-x}{2} \geq 1$$

$$\begin{aligned}
 3 &\geq \frac{7-x}{2} & \text{and} & \quad \frac{7-x}{2} \geq 1 \\
 6 &\geq 7-x & \text{and} & \quad 7-x \geq 2 \\
 6-7 &\geq -x & \text{and} & \quad -x \geq 2-7 \\
 -1 &\geq -x & \text{and} & \quad -x \geq -5 \\
 1 &\leq x & \text{and} & \quad x \leq 5 \\
 1 &\leq x \leq 5
 \end{aligned}$$

$$\text{Solution Set} = \{x \mid 1 \leq x \leq 5\}$$

$$\begin{aligned}
 \text{v)} \quad 3x-10 &\leq 5 < x+3 \\
 3x-10 &\leq 5 & \text{and} & \quad 5 < x+3 \\
 -5-10 &\leq -3x & \text{and} & \quad -x < 3-5 \\
 -15 &\leq -3x & \text{and} & \quad -x < -2 \\
 15 &\geq 3x & \text{and} & \quad x > 2 \\
 5 &\geq x & \text{and} & \quad x > 2 \\
 5 &\geq x > 2
 \end{aligned}$$

$$\text{Solution Set} = \{x \mid 5 \geq x > 2\}$$

$$\begin{aligned}
 \text{vi)} \quad -3 &\leq \frac{x-4}{-5} < 4 \\
 -3 &\leq \frac{x-4}{-5} & \text{and} & \quad \frac{x-4}{-5} < 4
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \quad 3 &\geq \frac{x-4}{5} & \text{and} & \quad \frac{x-4}{5} > -4 \\
 15 &\geq x-4 & \text{and} & \quad x-4 > 20 \\
 15+4 &\geq x & \text{and} & \quad x > 4-20 \\
 19 &\geq x & \text{and} & \quad x > -16 \\
 19 &\geq x \geq -16
 \end{aligned}$$

$$\text{Solution Set} = \{x \mid 19 \geq x > -16\}$$

$$\begin{aligned}
 \text{vii)} \quad 1-2x &< 5-x \leq 25-6x \\
 1-2x &< 5-x & \text{and} & \quad 5-x \leq 25-6x \\
 1-5 &\leq 2x-x & \text{and} & \quad 6x-x \leq 25-5 \\
 -4 &< x & \text{and} & \quad 5x \leq 20 \\
 -4 &< x & \text{and} & \quad x \leq 4 \\
 -4 &< x \leq 4
 \end{aligned}$$

$$\text{Solution Set} = \{x \mid -4 < x \leq 4\}$$

$$\begin{aligned}
 \text{viii)} \quad 3x-2 &< 2x+1 < 4x+17 \\
 3x-2 &< 2x+1 & \text{and} & \quad 2x+1 < 4x+17 \\
 -2-1 &< 2x-3x & \text{and} & \quad 2x-4x < 17-1 \\
 -3 &< -x & \text{and} & \quad -2x < 16 \\
 3 &> x & \text{and} & \quad 2x > -16 \\
 3 &> x & \text{and} & \quad x > -8 \quad 3 > x > -8
 \end{aligned}$$

$$\text{Solution Set} = \{x \mid 3 > x > -8\}$$

Review Exercise 7

Q3. Answer the following short questions.

i) Define a linear inequality in one variable.

Ans. Linear Inequality in one variable

Let a, b be real numbers, then a is greater than b if the difference $a - b$ is positive and we denote this order relation by the inequality $a > b$. An equivalent statement is that b is less than a , symbolized by $b < a$. Similarly, if $a - b$ is negative, then a is less than b and expressed in symbols as $a < b$.

ii) State the trichotomy and transitive properties of inequality.

Ans. Trichotomy Property of inequality

For any $a, b \in \mathbb{R}$, one and only one of the following statements is true.

$$a < b \text{ or } a = b, \text{ or } a > b$$

Transitive Property of inequality

Let $a, b, c \in \mathbb{R}$

i) If $a > b$ and $b > c$, then $a > c$

ii) If $a > b$ and $b < c$, then $a < c$

- iii) The formula relating degrees Fahrenheit to degrees Celcius is $F = \frac{9}{5}C + 32$. For what value of C is $F < 0$.

Ans. According to formula "F" will be zero, if $\frac{9}{5}C + 32 = 0$

$$\frac{9}{5}C = -32$$

$$C = -\frac{32}{9} \times 5$$

$$C = -\frac{160}{9}$$

To get $F < 0$ i.e. negative $C < -\frac{160}{9}$

- iv) Seven times the sum of an integer and 12 is at least 50 and at most 60. Write and solve the inequality that expresses this relationship.

Ans. Let the required integer be x then

$$50 \leq x + 12 \leq 60$$

$$50 \leq x + 12 \text{ and } x + 12 \leq 60$$

$$50 - 12 \leq x \text{ and } x \leq 60 - 12$$

$$38 \leq x \text{ and } x \leq 48$$

$$38 \leq x \leq 48$$

- v) Solve each of the following and check for extraneous solution, if any.

$$\sqrt{2t+4} = \sqrt{t-1}$$

Squaring both sides

$$(\sqrt{2t+4})^2 = (\sqrt{t-1})^2$$

$$2t + 4 = t - 1$$

$$2t - t = -1 - 4$$

$$t = -5$$

ck:

$$\sqrt{2t+4} = \sqrt{t-1}$$

$$\sqrt{2(-5)+4} = \sqrt{-5-1}$$

$$\sqrt{-10+4} = \sqrt{-6}$$

$\sqrt{-6} = \sqrt{-6}$ Which is true, so
solution Set = $\{-5\}$

ii) $\sqrt{3x-1} - 2\sqrt{8-2x} = 0$

$$\sqrt{3x-1} = 2\sqrt{8-2x}$$

Squaring both sides

$$(\sqrt{3x-1})^2 = (2\sqrt{8-2x})^2$$

$$3x - 1 = 4(8 - 2x)$$

$$3x - 1 = 32 - 8x$$

$$3x + 8x = 32 + 1$$

$$11x = 33$$

$$x = \frac{33}{11}$$

$$x = 3$$

Check:

$$\sqrt{3x-1} - 2\sqrt{8-2x} = 0$$

$$\sqrt{3(3)-1} - 2\sqrt{8-2(3)} = 0$$

$$\sqrt{9-1} - 2\sqrt{8-6} = 0$$

$$\sqrt{8} - 2\sqrt{2} = 0$$

$$2\sqrt{2} - 2\sqrt{2} = 0$$

$$0 = 0 \text{ Which is true, so}$$

solution set = $\{3\}$

Q5. Solve for x

i) $|3x+14| - 2 = 5x$

$$|3x+14| = 5x + 2$$

$$\pm(3x+14) = 5x + 2$$

$$3x + 14 = \pm(5x + 2)$$

$$3x + 14 = 5x + 2 \text{ or } 3x + 14 = -5x - 2$$

$$3x - 5x = 2 - 14 \text{ or } 3x + 5x = -2 - 14$$

$$-2x = -12 \text{ or } 8x = -16$$

$$x = \frac{12}{2} \text{ or } x = -\frac{16}{8}$$

$$x = 6 \text{ or } x = -2$$

Check:

Put $x = 6$ in

$$|3x+14|-2=5x$$

$$|3(6)+14|-2=5(6)$$

$$|18+14|-2=30$$

$$|32|-2=30$$

$$30-2=30$$

$$30=30, \text{ which is true}$$

Now put $x = -2$

$$|3(-2)+14|-2 \neq 5(-2)$$

$$|-6+14|-2 \neq -10$$

$$|8|-2 \neq -10$$

$$8-2 \neq -10$$

$$6 \neq -10 \text{ which is not true}$$

So, Solution Set = $\{6\}$

$$\text{ii) } \frac{1}{3}|x-3| = \frac{1}{2}|x+2|$$

$$\frac{|x-3|}{|x+2|} = \frac{3}{2}$$

$$\frac{|x-3|}{|x+2|} = \frac{3}{2}$$

$$\pm \left(\frac{x-3}{x+2} \right) = \frac{3}{2}$$

$$\frac{1}{3}|-3| = \frac{1}{2}|2|$$

$$\frac{3}{3} = \frac{2}{2}$$

$$1=1, \text{ which is true}$$

So, Solution Set = $\{-12, 0\}$

Q6. Solve the following inequality.

$$\text{i) } -\frac{1}{3}x + 5 \leq 1$$

$$-\frac{1}{3}x \leq 1-5$$

$$-\frac{1}{3}x \leq -4$$

Multiplying both sides by -3

$$x \geq 12$$

Solution Set = $\{x / x \geq 12\}$

$$\text{or } \frac{x-3}{x+2} = \pm \frac{3}{2}$$

$$\frac{x-3}{x+2} = \frac{3}{2} \quad \text{or} \quad \frac{x-3}{x+2} = -\frac{3}{2}$$

$$2(x-3) = 3(x+2) \quad \text{or} \quad 2(x-3) = -3(x+2)$$

$$2x-6 = 3x+6 \quad \text{or} \quad 2x+3x = 6-6$$

$$-x = 12 \quad \text{or} \quad 5x = 0$$

$$x = -12 \quad \text{or} \quad x = 0$$

Check:

Put $x = -12$

$$\frac{1}{3}|x-3| = \frac{1}{2}|x+2|$$

$$\frac{1}{3}|-12-3| = \frac{1}{2}|-12+2|$$

$$\frac{1}{3}|-15| = \frac{1}{2}|-10|$$

$$\frac{15}{3} = \frac{10}{2}$$

$$5 = 5, \text{ which is true}$$

Now put $x = 0$

$$\frac{1}{3}|0-3| = \frac{1}{2}|0+2|$$

$$\text{ii) } -3 < \frac{1-2x}{5} < 1$$

$$-3 < \frac{1-2x}{5} \quad \text{and} \quad \frac{1-2x}{5} < 1$$

Multiplying both sides by 5

$$-15 < 1-2x \quad \text{and} \quad 1-2x < 5$$

$$-15-1 < -2x \quad \text{and} \quad -2x < 5-1$$

$$-16 < -2x \quad \text{and} \quad -2x < 4$$

Multiplying both sides by -1

$$16 > 2x \quad \text{and} \quad 2x > -4$$

$$\frac{16}{2} > x \quad \text{and} \quad x > \frac{-4}{2}$$

$$8 > x \quad \text{and} \quad x > -2$$

$$8 > x > -2$$

Solution Set = $\{x / 8 > x > -2\}$

Objective

1. Which of the following is the solution of the inequality

$$3 - 4x \leq 11?$$

(a) $x \geq -8$

(b) $x \geq -2$

(c) $x \geq \frac{-14}{4}$

(d) None of these

2. A statement involving any of the symbols $<$, $>$ or $\leq$ or $\geq$ is called:

(a) Equation (b) Identity

(c) Inequality (d) Linear equation

3. $x = \underline{\hspace{2cm}}$ is a solution of the inequality $-2 < x < \frac{3}{2}$

(a) -5 (b) 3

(c) 0 (d) $\frac{5}{2}$

4. If x is not larger than 10, then

(a) $x \geq 8$ (b) $x \leq 10$

(c) $x < 10$ (d) $x > 10$

5. If the capacity c of an elevator is at most 1600 pounds, then

(a) $c < 1600$ (b) $c \geq 1600$

(c) $c \leq 1600$ (d) $c > 1600$

6. $x = 0$ is a solution of the inequality

(a) $x > 0$ (b) $3x + 5 < 0$

(c) $x + 2 < 0$ (d) $x - 2 < 0$

7. The linear equation in one variable x is:

(a) $ax + b = 0$

(b) $ax^2 + bx + c = 0$

(c) $ax + by + c = 0$

(d) $ax^2 + by^2 + c = 0$

8. An inconsistent equation is that whose solution set is:

(a) Empty (b) Not empty

(c) Zero (d) None of these

9. Absolute value of a real number a

(a) $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$

(b) $|a| = \begin{cases} a & \text{if } a \leq 0 \\ -a & \text{if } a > 0 \end{cases}$

(c) $|a| = \begin{cases} a & \text{if } a > 0 \\ -a & \text{if } a < 0 \end{cases}$

(d) None of these

10. $|x| = a$ is equivalent to:

(a) $x = a$ or $x = -a$

(b) $x = \frac{1}{a}$ or $x = \frac{-1}{a}$

(c) $x = a$ or $x = \frac{-1}{a}$

(d) None of these

11. A linear inequality in one variable x is:

(a) $ax + b > 0, a \neq 0$

(b) $ax^2 + bx + c < 0, a \neq 0$

(c) $ax + by + c > 0, a \neq 0$

(d) $ax^2 + by^2 + c < 0, a \neq 0$

12. Law of Trichotomy is ...

($a, b \in \mathbb{R}$)

(a) $a < b$ or $a = b$ or $a > b$

(b) $a < b$ or $a = b$

(c) $a < b$ or $a > b$

(d) None of these

13. Transitive law is ____
- (a) $a < b$ and $b < c$, then $a < c$
 (b) $a > b$ and $b < c$, then $a > c$
 (c) $a > b$ and $b < c$, then $a > c$
 (d) None of these
14. If $a > b$, $c > 0$ then:
- (a) $ac < bc$ (b) $ac > bc$
 (c) $ac = bc$ (d) None
15. If $a > b$, $c > 0$ then:
- (a) $\frac{a}{c} > \frac{b}{c}$ (b) $\frac{a}{c} < \frac{b}{c}$
 (c) $\frac{a}{c} = \frac{b}{c}$ (d) $\frac{b}{c} \neq \frac{b}{c}$
16. If $a > b$, $c < 0$, then:
- (a) $\frac{a}{c} < \frac{b}{c}$ (b) $\frac{a}{c} > \frac{b}{c}$
 (c) $\frac{a}{c} = \frac{b}{c}$ (d) $\frac{a}{c} \leq \frac{b}{c}$
17. If $a, b \in \mathbb{R}$ then:
- (a) $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$ (b) $|ab| = \frac{|a|}{|b|}$
 (c) $\left| \frac{b}{a} \right| = \frac{|b|}{|a|}$ (d) None of these
18. When the variable in an equation occurs under a radical, the equation is called a ____ equation.
- (a) Radical (b) Absolute value
 (c) Linear (d) None of these
19. $|x|=0$ has only ____ solution.
- (a) one (b) two
 (c) three (d) none of these
20. The equation $|x|=2$ is equivalent to
- (a) $x=2$ or $x=-2$
 (b) $x=-2$ or $x=-2$

- (c) $x=2$ or $x=\frac{1}{2}$
 (d) $x=2$ or $x=\frac{-1}{2}$
21. An ____ is equation that is satisfied by every number for which both sides are defined:
- (a) Identity (b) Conditional
 (c) Inconsistent (d) None
22. An ____ equation is an equation whose solution set is the empty set:
- (a) Identity (b) Conditional
 (c) Inconsistent (d) None
23. A ____ equation is an equation that is satisfied by atleast one number but is not an identity:
- (a) Identity (b) Conditional
 (c) Inconsistent (d) None
24. $x+4=4+x$ is ____ equation:
- (a) Identity (b) Conditional
 (c) Inconsistent (d) None
25. $2x+1=9$ is ____ equation:
- (a) Identity (b) Conditional
 (c) Inconsistent (d) None
26. $x=x+5$ is ____ equation:
- (a) Identity (b) Conditional
 (c) Inconsistent (d) None
27. Equations having exactly the same solution are called ____ equations.
- (a) equivalent (b) Linear
 (c) Inconsistent (d) None
28. A solution that does not satisfy the original equation is called ____ solution:
- (a) Extraneous (b) Root
 (c) General (d) None

ANSWER KEY

1.	b	2.	c	3.	c	4.	b	5.	c
6.	d	7.	a	8.	a	9.	a	10.	a
11.	a	12.	a	13.	a	14.	b	15.	a
16.	a	17.	a	18.	a	19.	a	20.	a
21.	a	22.	c	23.	b	24.	a	25.	b
26.	c	27.	a	28.	a				