

LINEAR GRAPHS AND THEIR APPLICATION

An Ordered Pair of Real Numbers

An ordered pair of real numbers x and y is pair (x, y) in which elements are written in specific order. i.e.,

(i) (x, y) is an ordered pair in which first element is x and second is y such that $(x, y) \neq (y, x)$ for example:

$(2, 3)$ and $(3, 2)$ are two different ordered pairs.

(ii) $(x, y) = (m, n)$ if and only if $x = m$ and $y = n$.

Cartesian Plane

The Cartesian plane establishes one-to-one correspondence between the set of ordered pairs $R \times R = \{(x, y) \mid x, y \in R\}$ and the points of the Cartesian plane.

In plane two mutually perpendicular straight lines are drawn. The lines are called the coordinate axes. The point O , where the two lines meet is called origin. This plane is called the coordinate plane or the Cartesian plane.

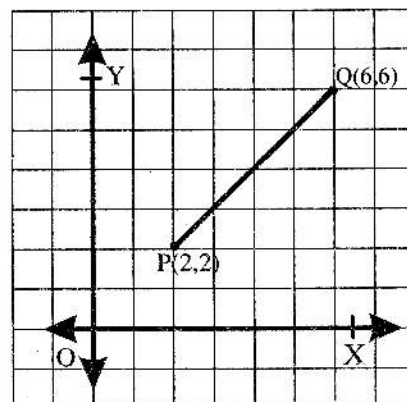
Drawing different geometrical Shapes in Cartesian Plane

(a) Line-Segment

Example:

Let $P(2, 2)$ and $Q(6, 6)$ be two points.

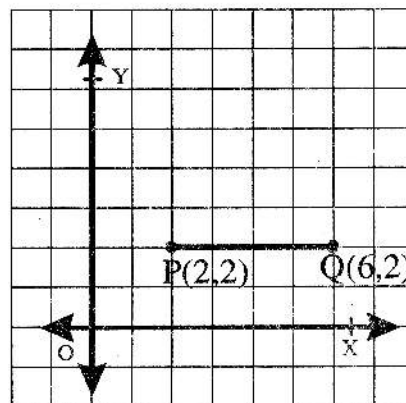
1. Plot points P and Q .
2. Join the points P and Q , we get the line segment PQ . It is represented by $\overline{PQ}$.



Example:

Plot points $P(2, 2)$ and $Q(6, 2)$. By joining them, we get a line segment PQ parallel to x -axis,

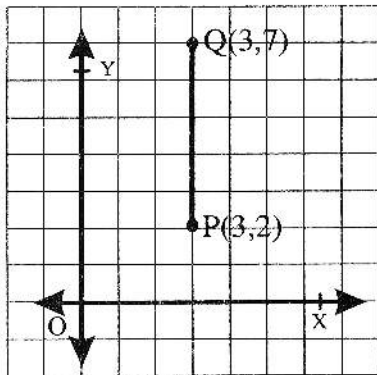
Where ordinate of both points is equal.



Example:

Plot points $P(3, 2)$ and $Q(3, 7)$. By joining them, we get a line segment PQ parallel to y -axis.

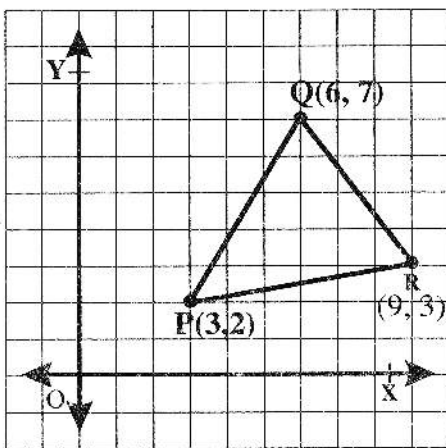
In this graph abscissas of both points are equal.



(b) Triangle

Example:

Plot the points P(3, 2), Q(6, 7) and R(9, 3). By joining them, we get a triangle PQR.



Example:

For points O(0, 0), P(3, 0) and R(3, 3), the triangle OPR is constructed.

Construction of a Table for Pairs of Values Satisfying a Linear Equation in Two Variables.

Let $2x + y = 1$ (i)

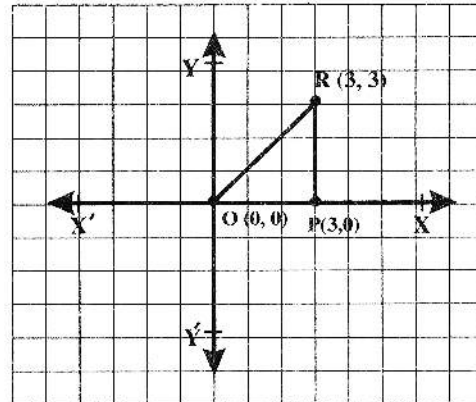
Be a linear equation in two variables x and y.

The ordered pair (x, y) satisfies the equation and by varying x, corresponding y is obtained.

We express (i) in the form

$y = 1 - 2x$ (ii)

The pairs (x, y) which satisfy (ii) are tabulated below.



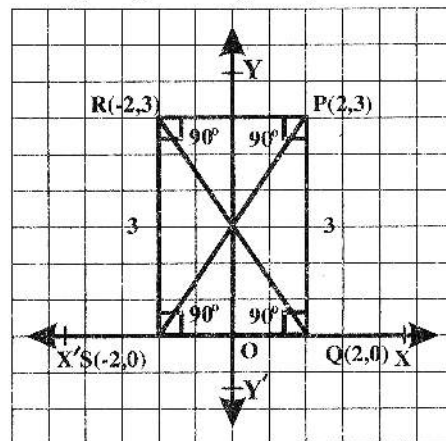
(c) Rectangle

Example

Plot the points P(2, 3), Q(2, 0), S(-2, 0) and R(-2, 3). Joining the points P, Q, S and R, we get a rectangle PQSR.

Along y-axis,

$2 \text{ (Length of square)} = 1 \text{ unit}$



x	y	(x, y)
-1	3	(-1, 3)
0	1	(0, 1)
1	-1	(1, -1)
3	-5	(3, -5)

$$\text{at } x = -1, y = (-2)(-1) + 1 = 2 + 1 = 3$$

$$\text{at } x = 0, y = (-2)(0) + 1 = 0 + 1 = 1$$

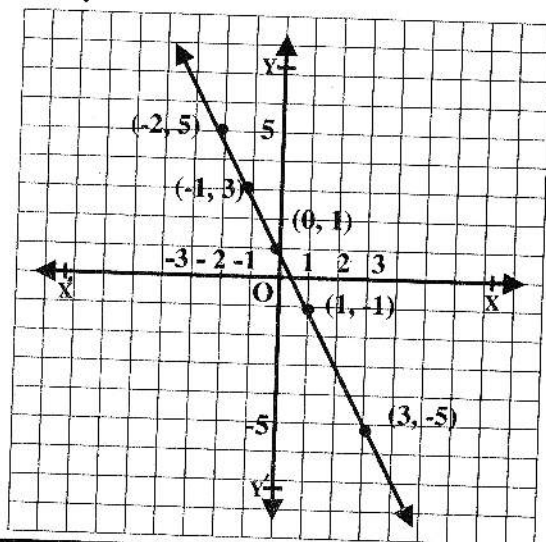
$$\text{at } x = 1, y = (-2)(1) + 1 = -2 + 1 = -1$$

$$\text{at } x = 3, y = -2(3) + 1 = -6 + 1 = -5$$

Similarly all the points can be computed, the ordered pairs of which do satisfy the equation (i)

Plotting the points to get the graph

Now we plot the points obtained in the table. Joining these points we get the graph of the equation. The graph of $2x + y = 1$



Example:

Equation $y = x + 16$ shows the relationship between the age of father and

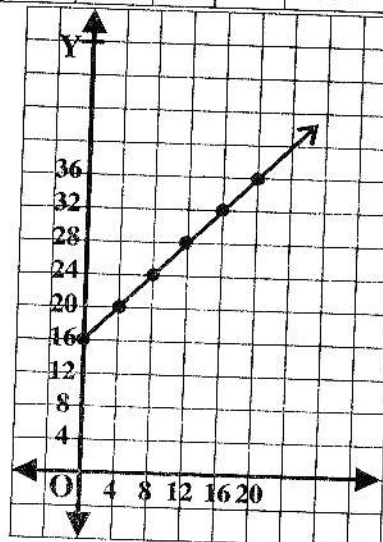
son i.e., if the age of son is x , then the father's age is y . Draw the graph.

Solution:

We know that $y = x + 16$

Table of points for equation is given as:

x	0	4	8	12	16	20
y	16	20	24	28	32	36



Exercise 8.1

1. Determine the quadrant of the coordinate plane in which the following points lie.

Ans. (i) P (-4, 3) II quadrant

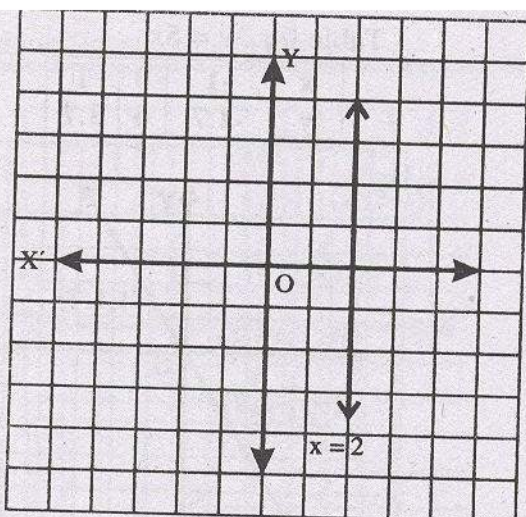
(ii) Q (-5, -2) III quadrant

(iii) P (2, 2) I quadrant

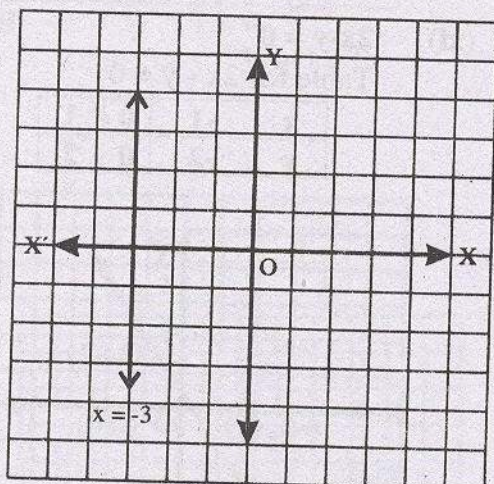
(iv) S(2, -6) IV quadrant

2. Draw the graph of each of the following.

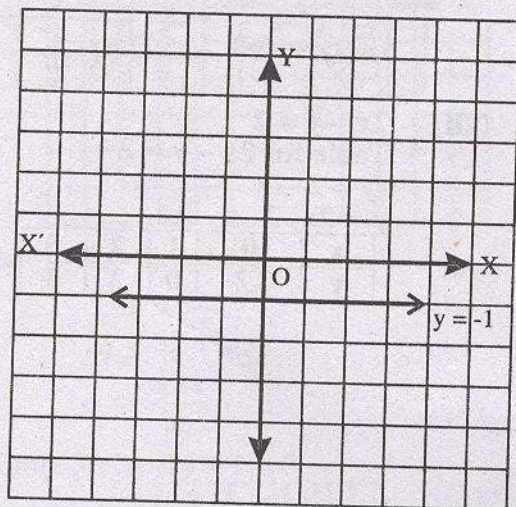
(i) $x = 2$



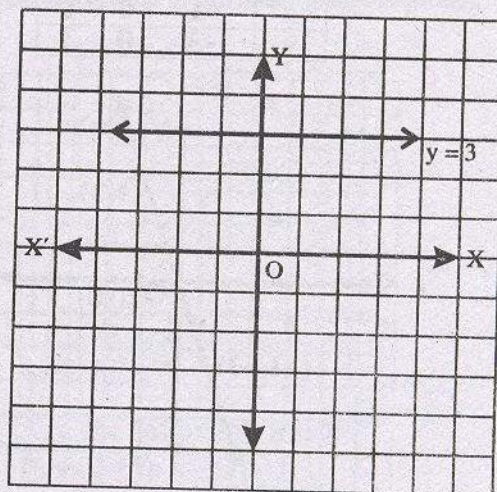
(ii) $x = -3$



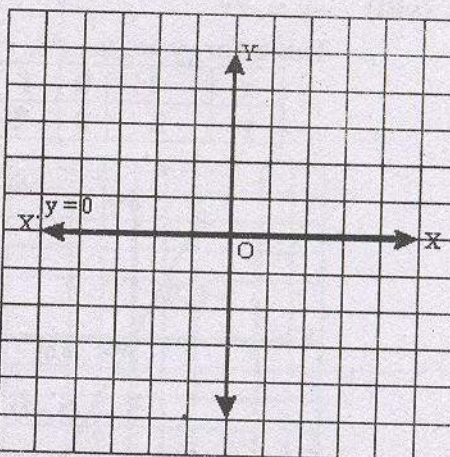
(iii) $y = -1$



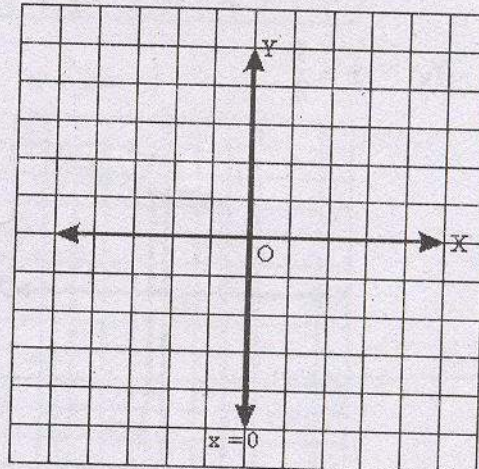
(iv) $y = 3$



(v) $y = 0$



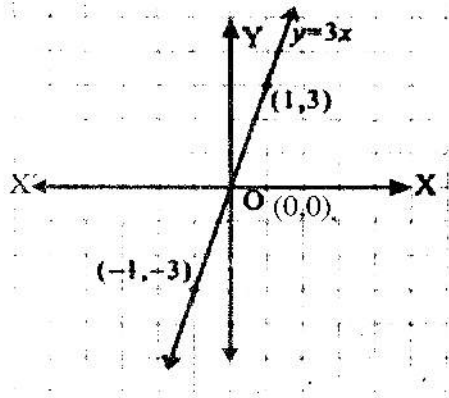
(vi) $x = 0$



(vii) $y = 3x$

Table for $y = 3x$

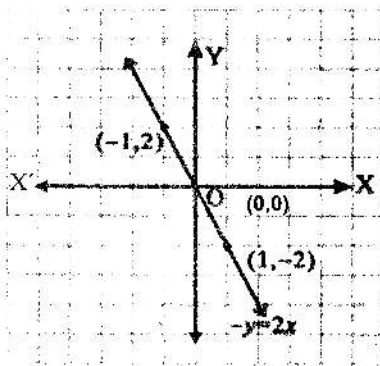
x	-1	0	1
y	-3	0	3



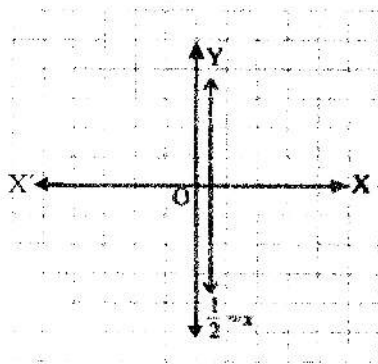
(viii) $-y = 2x$

Table for $-y = 2x$

x	-1	0	1
y	2	0	-2



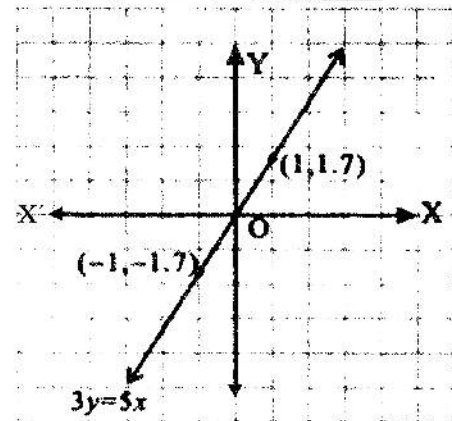
(ix) $x = \frac{1}{2}$



(x) $3y = 5x$

Table for $3y = 5x$

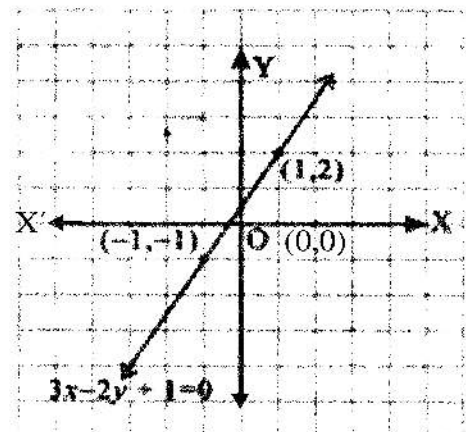
x	-1	0	1
y	-1.7	0	1.7



(xi) $2x - y = 0$

Table for $2x - y = 0$

x	-1	0	1
y	-2	0	2



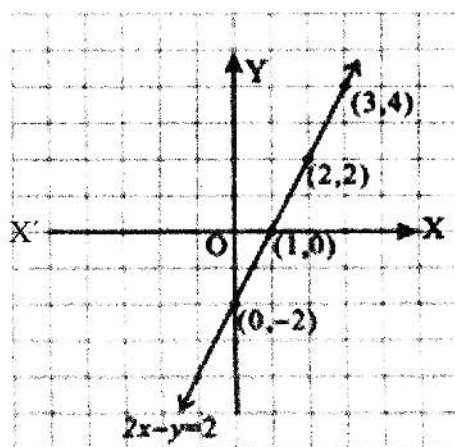
(xii) $2x - y = 2$

Table for $2x - y = 2$

$-y = 2 - 2x$

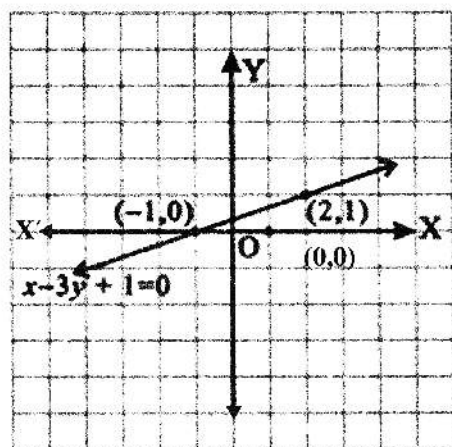
$y = 2x - 2$

x	0	1	2	3
y	-2	0	2	4



- (xiii) $x - 3y + 1 = 0$
 Table for $x - 3y + 1 = 0$
 $-3y = -x - 1$
 $3y = x + 1$
 $y = \frac{x+1}{3}$

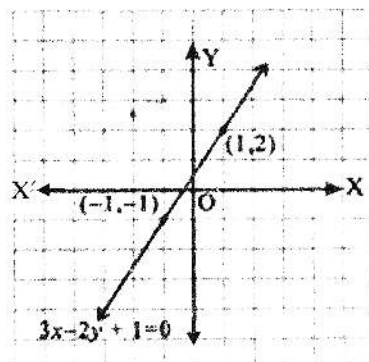
x	-1	2
y	0	1



- (xiv) $3x - 2y + 1 = 0$
 $-2y = -3x - 1$
 $2y = 3x + 1$
 $y = \frac{3x+1}{2}$

Table for $3x - 2y + 1 = 0$

x	-1	1
y	-1	2



Q.3 Are the following lines:

(i) **Parallel to x-axis**

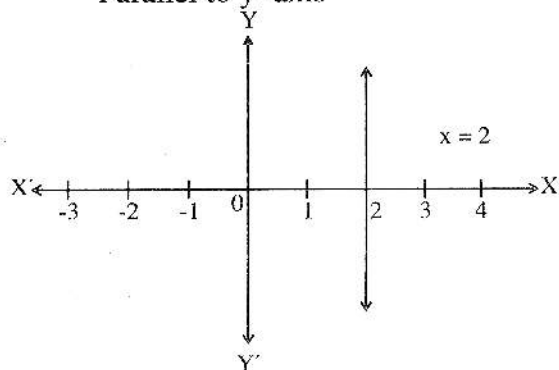
(ii) **Parallel to y-axis**

(i) $2x - 1 = 3$

$$2x = 3 + 1$$

$$x = \frac{4}{2} = 2$$

Parallel to y-axis

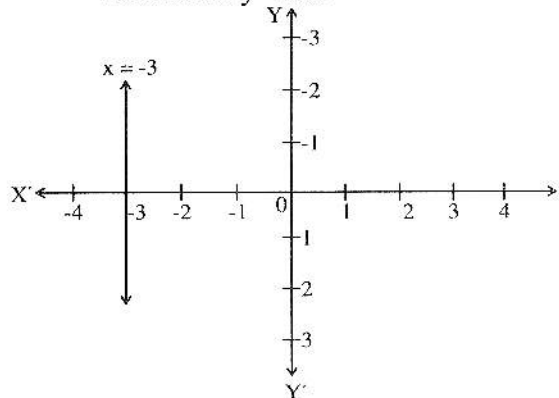


(ii) $x + 2 = -1$

$$\Rightarrow x = -1 - 2$$

$$x = -3$$

Parallel to y-axis

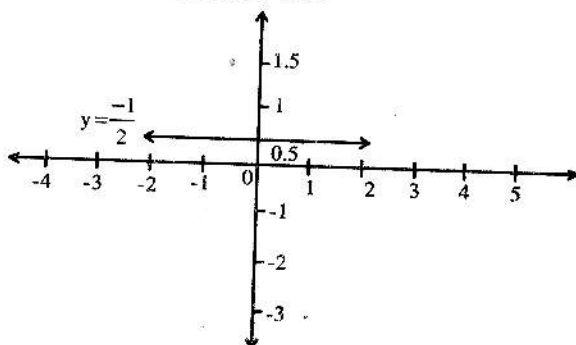


(iii) $2y + 3 = 2$

$\Rightarrow 2y = 2 - 3$

$y = -\frac{1}{2}$

Parallel to x-axis



(iv) $x + y = 0$

$\Rightarrow x = -y$

neither

(v) $2x - 2y = 0$

$2x = 2y$

$x = y$

neither

Q.4 Find the value of m and c of the following lines by expressing them in the form $y = mx + c$

(a) $x - 2y = -2$

$-2y = -2 - x$

$2y = 2 + x$

$y = \frac{2+x}{2}$

$y = 1 + \frac{1}{2}x$

$y = \frac{1}{2}x + 1 \dots\dots(1)$

$y = mx + c \dots\dots(2)$

comparing (1) and (2) we get

$m = \frac{1}{2}$ and $c = 1$

(b) $2x + 3y - 1 = 0$

$3y = -2x + 1$

$y = \frac{-2x+1}{3}$

$y = \frac{-2}{3}x + \frac{1}{3} \dots\dots(1)$

$y = mx + c \dots\dots(2)$

comparing (1) and (2) we get

$m = \frac{-2}{3}$ and $c = \frac{1}{3}$

(c) $3x + y - 1 = 0$

$y = -3x + 1 \dots\dots(1)$

Also $y = mx + c \dots\dots(2)$

Comparing (1) and (2)

$m = -3$ and $c = 1$

(d) $2x - y = 7$

$-y = 7 - 2x$

$y = -7 + 2x$

$y = 2x - 7 \dots\dots(1)$

also $y = mx + c \dots\dots(2)$

comparing (1) and (2)

$m = 2$ and $c = -7$

(e) $3 - 2x + y = 0$

$y = -3 + 2x$

$y = 2x - 3 \dots\dots(1)$

Also $y = mx + c \dots\dots(2)$

Comparing (1) and (2) we get

$m = 2$ and $c = -3$

(f) $2x = y + 3$

$y = 2x - 3 \dots\dots(1)$

Also $y = mx + c \dots\dots(2)$

Comparing (1) and (2) we get

$m = 2$ and $c = -3$

Q.5 Verify whether the following points lies on the line $2x - y + 1 = 0$ or not.

Ans. $2x - y + 1 = 0$

(i) $(2, 3) \Rightarrow x = 2, y = 3$

$2x - y + 1 = 0$

$\Rightarrow 2(2) - 3 + 1 = 0$

$4 - 3 + 1 \neq 0$

$2 \neq 0$ Point $(2, 3)$ does not lie on the line

(ii) $(0, 0) \Rightarrow x = 0, y = 0$

$2x - y + 1 = 0$

$\Rightarrow 2(0) - 0 + 1 = 0$

$1 \neq 0$

Point $(0, 0)$ does not lie on the line

(iii) $(-1, 1) \Rightarrow x = -1, y = 1$

$2x - y + 1 = 0$

$\Rightarrow 2(-1) - (1) + 1 - 0 = 0$

$-2 - 1 + 1 = 0$

$-2 \neq 0$

Point $(-1, 1)$ does not lie on the line

(iv) $(2, 5) \Rightarrow x = 2, y = 5$

$2x - y + 1 = 0$

$\Rightarrow 2(2) - 5 + 1 = 0$

$4 - 5 + 1 = 0$

$-1 + 1 = 0$

$0 = 0$

Yes the Point $(2, 5)$ lies on the line

(v) $(5, 3) \Rightarrow x = 5, y = 3$

$2x - y + 1 = 0$

$\Rightarrow 2(5) - 3 + 1 = 0$

$10 - 2 = 0$

$8 \neq 0$

The point $(5, 3)$ does not lie on the line

(a) **Example: (Kilometre (Km) and Mile (M) Graphs)**

To draw the graph between kilometre (Km) and Miles (M), we use the following relation:

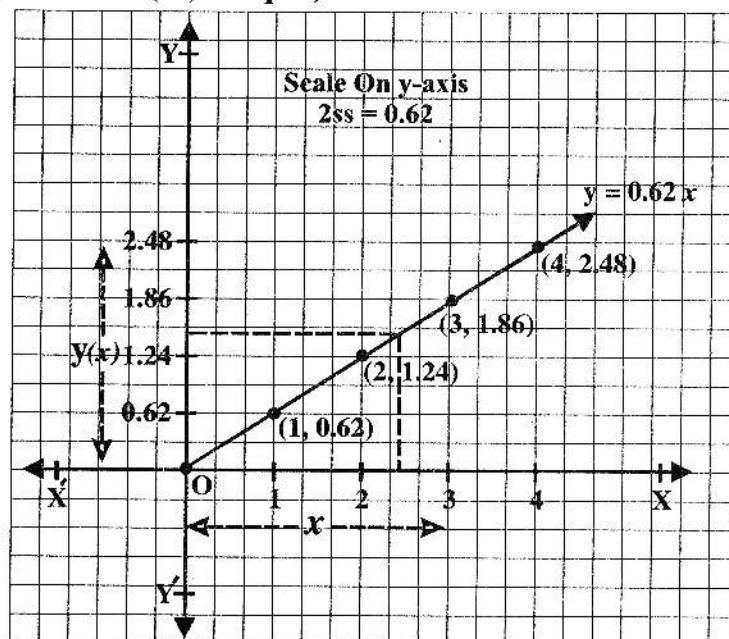
One kilometre = 0.62 miles,
(approximately)

And One mile = 1.6 km
(approximately)

(i) The relation of mile against kilometre is given by the linear equation,

$y = 0.62x,$

If y is a mile and x is a kilometre, then we tabulate the ordered pairs (x, y) as below;



x	0	1	2	3	4
y	0	0.62	1.24	1.86	2.48

The ordered pairs (x, y) corresponding to $y = 0.62x$ are represented in the Cartesian plane. By joining them we get the desired graph of miles against kilometers.

For each quantity of kilometre x along x -axis their corresponding mile along y -axis.

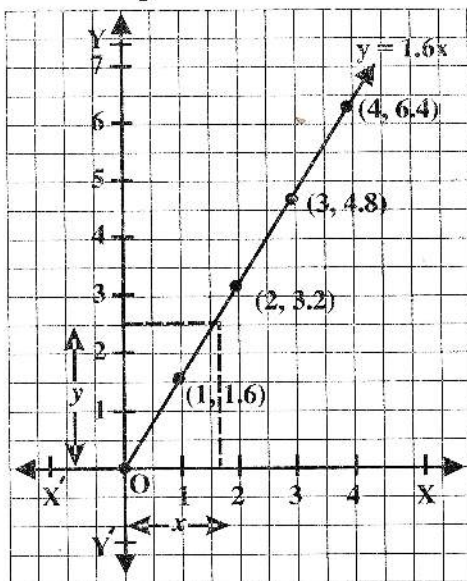
(ii) The conversion graph of kilometer against mile is given by

$$y = 1.6x \quad (\text{approximately})$$

If y represents kilometers and x a mile, then the values x and y are tabulated as:

x	0	1	2	3	4
y	0	1.6	3.2	4.8	6.4 ...

We plot the points in the xy -Plane corresponding to the ordered pairs. $(0,0)$, $(1, 1.6)$, $(2, 3.2)$ $(3, 4.8)$ and $(4, 6.4)$ as shown in figure.



By joining the points we actually find the conversion graph of kilometers against miles.

(b) Conversion Graph of Hectares and Acres

(i) The relation between Hectare and Acre is defined as:

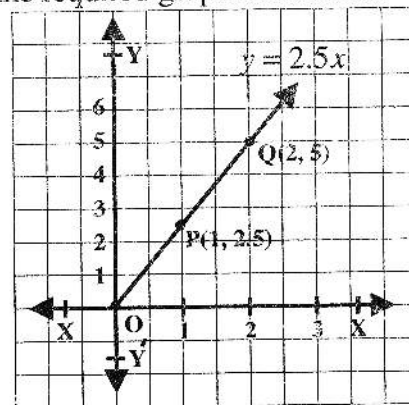
$$\begin{aligned} \text{Hectare} &= \frac{640}{259} \text{ Acres} \\ &= 2.5 \text{ Acres (approximately)} \end{aligned}$$

In case when hectare = x and acre = y , then relation between them is given by the equation, $y = 2.5x$

If x is represented as hectare along the horizontal axis and y as Acre along y -axis, the values are tabulated below:

x	0	1	2	3	4
y	0	2.5	5.0	7.5	10

The ordered pairs $(0, 0)$, $(1, 2.5)$, $(2,5)$ etc., are plotted as points in the xy -plane as below and by joining the points the required graph is obtained:



b – (i)

(ii) Now the conversion graph

$$\text{Acre} = \frac{1}{2.5} \text{ Hectare is simplified as,}$$

$$\text{Acre} = \frac{10}{25} \text{ Hectare}$$

$$= 0.4 \text{ Hectare (approximately)}$$

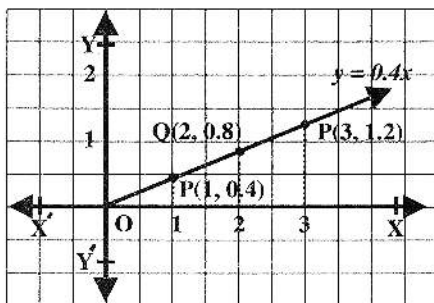
If Acre is measured along x -axis and hectare along y -axis then

$$y = 0.4x$$

The ordered pairs are tabulated in the following table:

x	0	1	2	3 ...
y	0	0.4	0.8	1.2 ...

The corresponding ordered pairs $(0, 0)$, $(1, 0.4)$, $(2, 0.8)$ etc., are plotted in the xy -plane, join of which will form the graph of (b)-ii as a conversion graph of (b)-i:



b – (ii)

(c) **Conversion Graph of Degrees Celsius and Degrees Fahrenheit**

(i) The relation between Celsius (C) and degree Fahrenheit (F) is given by

$$F = \frac{9}{5}C + 32$$

The value of F at C = 0 is obtained as

$$F = \frac{9}{5} \times 0 + 32 = 0 + 32 = 32$$

Similarly,

$$F = \frac{9}{5} \times 10 + 32 = 18 + 32 = 50,$$

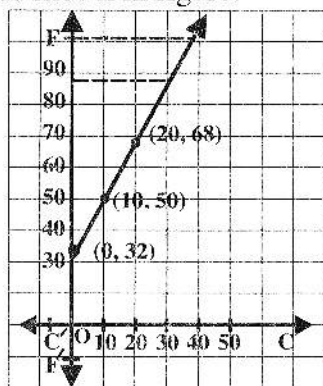
$$F = \frac{9}{5} \times 20 + 32 = 36 + 32 = 68,$$

$$F = \frac{9}{5} \times 100 + 32 = 180 + 32 = 212$$

We tabulate the values of C and F.

C	0°	10°	20°	50°	100°...
F	32°	50°	68°	122°	212°...

The conversion graph of F with respect to C is shown in figure.



10° = length of square

(d) **Conversion graph of US\$ and Pakistani Currency**

The daily News, on a particular day informed the conversion rate of Pakistani currency to the US\$ currency as.

$$1 \text{ US\$} = 66.46 \text{ Rupees}$$

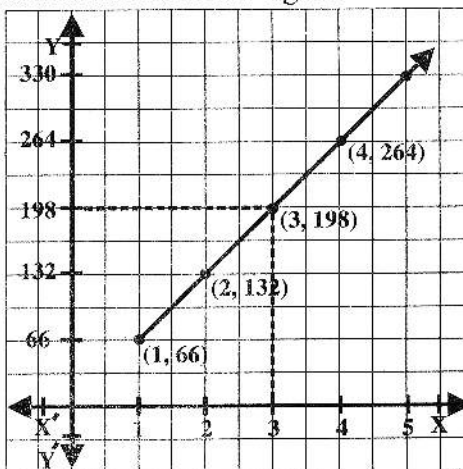
If the Pakistani currency y is an expression of US\$ x, expressed under the rule

$$y = 66.46x \approx 66x \text{ (approximately)}$$

Then draw the conversion graph.

x	1	2	3	4 ...
y	66	132	198	264 ...

Plotting the points corresponding to the ordered pairs (x, y) from the above table and joining them provides the currency linear graph of rupees against dollars as shown in the figure.



Conversion graph $x = \frac{1}{66}y$ of $y = 66x$ can

be shown by interchanging x-axis to y-axis and vice versa.

